Lecture 8
WDM Network Design

Homework #3:
8.2 Ramaswami
8.3 Ramaswami
8.4 Ramaswami
8.7 Ramaswami
LTD and RWA Problems

- Look at a design problem where
  - The underlying fiber topology does not pose any constraint
  - All lightpaths are bi-directional
  - Each node has an IP router with at most $\Delta$ ports connecting to other IP routers
  - Number of lightpaths in the network is therefore constrained on $\Delta$
LTD and RWA Problems

⇒ Assume a statistical model for IP traffic
  ⇒ Arrival rate for packets for source-destination (s-d) pair \((s,d)\) is \(\lambda_{sd}\).
  ⇒ \(s,d = 1, \ldots, m\)
  ⇒ \(b_{ij}, i,j = 1, \ldots, n, i \neq j\) are binary variables for each possible lightpath \((i,j)\).
  ⇒ Solution to the lightpath design problem is to specify the set \(\{b_{ij}\}\)
  ⇒ \(a_{ij}^{sd}\) is the fraction of traffic between pair \((s,d)\) routed over link \((i,j)\)
  ⇒ \(\lambda_{ij}^{sd} = a_{ij}^{sd} \lambda_{sd}\) is the traffic in packets/sec between pair \((s,d)\) routed over link \((i,j)\)
  ⇒ Total traffic over all pairs routed over link \((i,j)\) is

\[
\lambda = \sum_{sd} \lambda_{ij}^{sd}
\]
Congestion Parameter and Queuing Delay

Define congestion parameter \( \lambda_{\text{max}} = \max_{ij}\{\lambda_{ij}\} \)

We must determine variables \( \lambda_{ij}^{sd}, \lambda_{\text{max}} \) and \( \lambda_{ij} \)

The process of determining \( \lambda_{ij}^{sd}, \lambda_{\text{max}} \) and \( \lambda_{ij} \) is equivalent to the problem of finding a routing algorithm

Example:

Packet transmission times are exponentially distributed (Poisson Process) with mean time \( 1/\mu \) seconds.

Model each link as an M/M/1 queue, where it is assumed that traffic offered to a link in the network is independent of the traffic offered to other links.

The average queuing delay under these assumptions is

\[
d_{ij} = \frac{1}{\mu - \lambda_{ij}}
\]
Throughput

⇒ *Throughput* is defined as minimum value of the offered load for which any link experiences infinite delay, which happens when

\[ \lambda_{\text{max}} = \max_{ij} \lambda_{ij} = \mu \]

⇒ Performance objective

\[
\text{Minimize } \left( \lambda_{\text{max}} \right)
\]

⇒ Subject to the following flow conservation at each node

\[
\sum_j \lambda_{ij}^{sd} - \sum_j \lambda_{ji}^{sd} = \begin{cases} 
\lambda^{sd} & \text{if } s = i, \\
-\lambda^{sd} & \text{if } d = i, \\
0 & \text{otherwise},
\end{cases} \quad \text{for all } s, d, i
\]

⇒ Subject to the total flow on a logical link

\[
\lambda_{ij} = \sum_{s,d} \lambda_{ij}^{sd}, \quad \text{for all } i, j,
\]

\[
\lambda_{ij} \leq \lambda_{\text{max}}, \quad \text{for all } i, j,
\]

\[
\lambda_{ij}^{sd} \leq b_{ij} \lambda^{sd}, \quad \text{for all } i, j,
\]
Throughput

\( \Rightarrow \) Subject to degree constraints

\[ \sum_i b_{ij} \leq \Delta, \quad \text{for all } j, \]
\[ \sum_j b_{ij} \leq \Delta, \quad \text{for all } i. \]

\( \Rightarrow \) Subject to bidirectional lightpath constraint

\[ b_{ij} = b_{ji}, \quad \text{for all } i, j. \]

\( \Rightarrow \) Subject to Nonnegativity and integer constraints

\[ \lambda_{ij}^{sd}, \lambda_{ij}, \lambda_{\text{max}} \geq 0, \quad \text{for all } i, j, s, d \]
\[ b_{ij} \in \{0, 1\}, \quad \text{for all } i, j. \]
Throughput

We associate packets routed between pair (s,d) as the flow of a commodities.

The flow conservation constraint at node i yields the net flow out of node i for one commodity (s,d).

The net flow is the difference between the outgoing and ingoing flows.

The flow conservation is 0 if a node is neither a source or destination of that particular commodity.

If node i is the source of the flow (i=s) the net flow = \( \lambda_{^s}^d \)

If node i is the destination of the flow (i=d) the net flow = -\( \lambda_{^s}^d \)