



# Lecture 8

# WDM Network Design



Homework #3:

8.2 Ramaswami

8.3 Ramaswami

8.4 Ramaswami

8.7 Ramaswami

# LTD and RWA Problems



- ⇒ Look at a design problem where
  - ⇒ The underlying fiber topology does not pose any constraint
  - ⇒ All lightpaths are bi-directional
  - ⇒ Each node has an IP router with at most  $\Delta$  ports connecting to other IP routers
  - ⇒ Number of lightpaths in the network is therefore constrained on  $\Delta$

# LTD and RWA Problems

- ⇒ Assume a statistical model for IP traffic
  - ⇒ Arrival rate for packets for source-destination (s-d) pair  $(s,d)$  is  $\lambda^{sd}$ ,  $s,d = 1, \dots, m$
  - ⇒  $b_{ij}$ ,  $i,j = 1, \dots, n$ ,  $i \neq j$  are binary variables for each possible lightpath  $(i,j)$ .
  - ⇒ Solution to the lightpath design problem is to specify the set  $\{b_{ij}\}$
  - ⇒  $a_{ij}^{sd}$  is the fraction of traffic between pair  $(s,d)$  routed over link  $(i,j)$
  - ⇒  $\lambda_{ij}^{sd} = a_{ij}^{sd} \lambda^{sd}$  is the traffic in packets/sec between pair  $(s,d)$  routed over link  $(i,j)$
  - ⇒ Total traffic over all pairs routed over link  $(i,j)$  is

$$\lambda = \sum_{sd} \lambda_{ij}^{sd}$$

# Congestion Parameter and Queuing Delay

- ⇒ Define **congestion** parameter  $\lambda_{\max} = \max_{ij}\{\lambda_{ij}\}$
- ⇒ We must determine variables  $\lambda_{ij}^{sd}$ ,  $\lambda_{\max}$  and  $\lambda_{ij}$
- ⇒ The process of determining  $\lambda_{ij}^{sd}$ ,  $\lambda_{\max}$  and  $\lambda_{ij}$  is equivalent to the problem of finding a routing algorithm

⇒ Example:

- ⇒ Packet transmission times are exponentially distributed (Poisson Process) with mean time  $1/\mu$  seconds.
- ⇒ Model each link as an M/M/1 queue, where it is assumed that traffic offered to a link in the network is independent of the traffic offered to other links.
- ⇒ The average queuing delay under these assumptions is

$$d_{ij} = \frac{1}{\mu - \lambda_{ij}}$$

# Throughput

⇒ **Throughput** is defined as minimum value of the offered load for which any link experiences infinite delay, which happens when

$$\lambda_{\max} = \max_{ij} \lambda_{ij} = \mu$$

⇒ Performance objective

$$\text{Minimize } (\lambda_{\max})$$

⇒ Subject to the following flow conservation at each node

$$\sum_j \lambda_{ij}^{sd} - \sum_j \lambda_{ji}^{sd} = \begin{cases} \lambda^{sd} & \text{if } s = i, \\ -\lambda^{sd} & \text{if } d = i, \\ 0 & \text{otherwise,} \end{cases} \quad \text{for all } s, d, i$$

⇒ Subject to the total flow on a logical link

$$\lambda_{ij} = \sum_{s,d} \lambda_{ij}^{sd}, \quad \text{for all } i, j,$$

$$\lambda_{ij} \leq \lambda_{\max}, \quad \text{for all } i, j,$$

$$\lambda_{ij}^{sd} \leq b_{ij} \lambda^{sd}, \quad \text{for all } i, j,$$

# Throughput

⇒ Subject to degree constraints

$$\sum_i b_{ij} \leq \Delta, \quad \text{for all } j,$$

$$\sum_j b_{ij} \leq \Delta, \quad \text{for all } i.$$

⇒ Subject to bidirectional lightpath constraint

$$b_{ij} = b_{ji}, \quad \text{for all } i, j.$$

⇒ Subject to Nonnegativity and integer constraints

$$\lambda_{ij}^{sd}, \lambda_{ij}, \lambda_{\max} \geq 0, \quad \text{for all } i, j, s, d$$

$$b_{ij} \in \{0, 1\}, \quad \text{for all } i, j.$$

# Throughput



- ⇒ We associate packets routed between pair  $(s,d)$  as the *flow* of a commodities.
- ⇒ The flow conservation constraint at node  $i$  yields the net flow out of node  $i$  for one commodity  $(s,d)$ .
- ⇒ The net flow is the difference between the outgoing and ingoing flows.
- ⇒ The flow conservation is 0 if a node is neither a source or destination of that particular commodity.
- ⇒ If node  $i$  is the source of the flow ( $i=s$ ) the net flow =  $\lambda^{sd}$
- ⇒ If node  $i$  is the destination of the flow ( $i=d$ ) the net flow =  $-\lambda^{sd}$