Lecture 9

Homework

Homework #3:

8.2 Ramaswami

8.3 Ramaswami

8.4 Ramaswami

8.7 Ramaswami

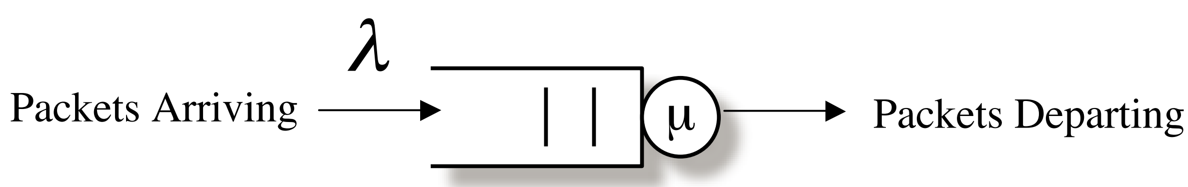
Intro to Queueing Theory

⇨ A simple queue model is shown below. It handles data packets that arrive at an average rate of λ packets/time.

⇨The packets queue up for service in the buffer and are served (according to some service discipline) to the output at an average rate of μ packets/time.

⇨ Define link ***utilization*** or ***traffic intensity (ρ)***as

$$\rho \equiv \frac{\lambda}{\mu}$$

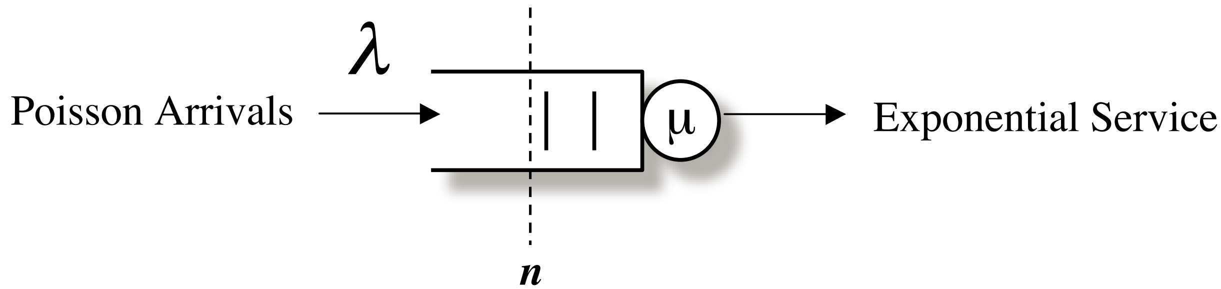


Performance

- ⇨ We are interested the dependence on μ and the buffer size on the time delay, blocking performance and packet throughput.
- ⇨ These performance parameters depend on the ***probability of state*** of the queue.
- ⇨ In order to determine the probability of state of the queue we need to know
  - ⇨ The ***packet arrival process*** (or statistics)
  - ⇨ The ***packet length distribution***
  - ⇨ The queue ***service discipline*** (for example, FCFS, FIFO, LCFS)

M/M/1 Queue

- ⇨ An M/M/1 queue is a single server queue, that assumes Poisson packet arrival, exponential service-time statistics and FIFO service
- ⇨ A/B/C denotes A = Arrival distribution, B = Service distribution and C = Number of servers used.
- ⇨ M stands for Markov process which translates to Poisson arrival statistics and exponential inter-packet arrival (or transmission).
- ⇨ Probabilities of state are denoted p_n, which is the probability that there are n customers (packets) at the queue. This number includes that packet that is currently being served.
- ⇨ We assume in steady state that the p_n do not change with time.

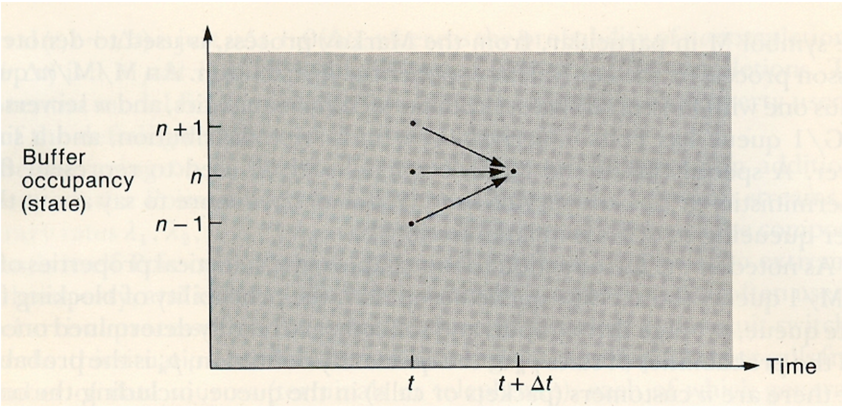


Buffer Occupancy State

⇨ The probability $p_n(t+\Delta t)$ that there are ***n*** customers in the queue at time t+ Δt, derives from the probability $p_n(t)$.

⇨ If queue was in state ***n*** at time ***t+Δt***, it only could have been in states ***n-1, n or n+1*** at time ***t***.

⇨ The probability of being in state ***n*** at time ***t+Δt*** is given by the sum of the mutually exclusive probabilities that the queue was in state ***n-1, n or n+1*** at time ***t*** each weighted by the independent probability of arriving at state ***n*** in ***Δ t*** units of time.



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M/M/1 Queue Occupancy State

$$p_n(t+\Delta t) = p_n(t)\left[(1-\lambda\Delta t)(1-\mu\Delta t) + \mu\Delta t \cdot \lambda\Delta t + O(\Delta t)\right]$$

$$+p_{n-1}(t)\left[\lambda\Delta t\right](1-\mu\Delta t) + O(\Delta t)\Big]$$

$$+p_{n+1}(t)\left[\mu\Delta t\right](1-\lambda\Delta t) + O(\Delta t)\Big]$$

Dropping terms $O(\Delta t)$

$$p_n(t+\Delta t) = \left[1-(\lambda+\mu)\Delta t\right]p_n(t) + \lambda\Delta t p_{n-1}(t) + \mu\Delta t p_{n+1}(t)$$

Expanding $p_n(t+\Delta t)$ in a Taylor Series, we can write as a difference function

$$p_n(t+\Delta t) = p_n(t) + \frac{dp_n(t)}{dt}\Delta t$$

$$\frac{dp_n(t)}{dt} = -(\lambda+\mu)p_n(t) + \lambda p_{n-1}(t) + \mu p_{n+1}(t)$$

For steady state

$$\frac{dp_n(t)}{dt} = 0$$

$$(\lambda+\mu)p_n = \lambda p_{n-1} + \mu p_{n+1}, n \geq 1$$

Steady State

⇨ The solution to the steady state equation yields a Geometric Distribution for the steady M/M/1 infinite queue state probability distribution.

$$\lambda p_n = \mu p_{n+1}$$

or for n processes

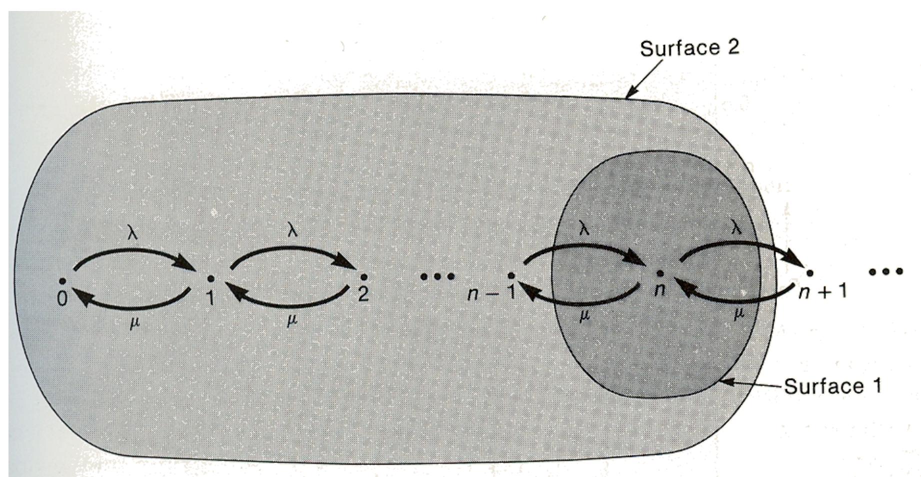
$$p_n = \rho^n p_0$$

$$\rho \equiv \frac{\lambda}{\mu}$$

or for an infinite queue

$$p_0 = (1 - \rho)$$

$$p_n = (1 - \rho)\rho^n \text{ for } \rho \equiv \frac{\lambda}{\mu} < 1$$



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