Problem 1: Consider the random variable $X$ with CDF $F_X$ as in Problem 1.11.
(a) Given a random variable $U$ which is uniform over $[0, 1]$, specify in detail the transformation $g(U) = F_X^{-1}(U)$ that maps it to a random variable with CDF $F_X$.
(b) Use simulations (e.g., using matlab) to estimate $E[X]$. Compare the result with the exact answer.
(c) Once you are happy that your program works, use them to estimate $E[\cos(X^3)]$, a quantity that is difficult to compute analytically.

Problem 2: Two random variables $X$ and $Y$ have joint density

$$p_{X,Y}(x, y) = \begin{cases} Ke^{-\frac{x^2+y^2}{2}} & xy \geq 0 \\ 0 & xy < 0 \end{cases}$$

(a) Find $K$.
(b) Show that $X$ and $Y$ are each Gaussian random variables.
(c) Express the probability $P[X^2 + X > 2]$ in terms of the $Q$ function.
(d) Are $X$ and $Y$ jointly Gaussian?
(e) Are $X$ and $Y$ independent?
(f) Are $X$ and $Y$ uncorrelated?
(g) Find the conditional density $p_{X|Y}(x|y)$. Is it Gaussian?

Problem 3: The random vector $X = (X_1X_2)^T$ is Gaussian with mean vector $m = (-2, 1)^T$ and covariance matrix $K$ given by

$$K = \begin{pmatrix} 9 & -2 \\ -2 & 4 \end{pmatrix}$$

(a) Let $Y_1 = X_1 + 2X_2$, $Y_2 = -X_1 + X_2$. Find $cov(Y_1, Y_2)$.
(b) Write down the joint density of $Y_1$ and $Y_2$.
(c) Express the probability $P[Y_1 > 2Y_2 + 1]$ in terms of the $Q$ function.

Problems 4-6: Problems 3.1, 3.3, 3.5