ECE 235: Problem Set 2

Assigned: Monday October 5
Due: Monday October 12
Reading: Hajek, Chapter 1, Sections 3.1-3.4
Topics: Probability and random variables; jointly Gaussian random variables
Reminder: Probability quiz in class on Tuesday, October 13
Agenda for this week: We will consolidate our probability review on Tuesday and then start on Chapter 3. We will start on Chapter 2 after covering part of Chapter 3.

Practice problems (not to be turned in): The even numbered problems in Chapter 1, up to Problem 1.30. Compare your solutions with the solutions provided to make sure you understand the concepts.

Problem 1: Consider the random variable X with CDF F_X as in Problem 1.11.
(a) Given a random variable U which is uniform over $[0,1]$, specify in detail the transformation $g(U) = F_X^{-1}(U)$ that maps it to a random variable with CDF F_X.
(b) Use simulations (e.g., using matlab) to estimate $E[X]$. Compare the result with the exact answer.
(c) Once you are happy that your program works, use them to estimate $E[\cos(X^3)]$, a quantity that is difficult to compute analytically.

Problem 2: Two random variables X and Y have joint density

$$p_{X,Y}(x,y) = \begin{cases} Ke^{-\frac{2x^2+y^2}{2}} & xy \geq 0 \\ 0 & xy < 0 \end{cases}$$

(a) Find K.
(b) Show that X and Y are each Gaussian random variables.
(c) Express the probability $P[X^2 + X > 2]$ in terms of the Q function.
(d) Are X and Y jointly Gaussian?
(e) Are X and Y independent?
(f) Are X and Y uncorrelated?
(g) Find the conditional density $p_{X|Y}(x|y)$. Is it Gaussian?

Problem 3: The random vector $\mathbf{X} = (X_1 X_2)^T$ is Gaussian with mean vector $\mathbf{m} = (-2,1)^T$ and covariance matrix $\mathbf{K}$ given by

$$\mathbf{K} = \begin{pmatrix} 9 & -2 \\ -2 & 4 \end{pmatrix}$$

(a) Let $Y_1 = X_1 + 2X_2$, $Y_2 = -X_1 + X_2$. Find $cov(Y_1, Y_2)$.
(b) Write down the joint density of Y_1 and Y_2.
(c) Express the probability $P[Y_1 > 2Y_2 + 1]$ in terms of the Q function.

Problems 4-6: Problems 3.1, 3.3, 3.5