UCSB Fall 2009

ECE 235: Problem Set 3

Assigned: Friday October 16
Due: Monday October 26
Reading: Hajek, Chapter 3
Topics: MMSE Estimation; Joint Gaussianity

Practice problems (not to be turned in): The even numbered problems in Chapter 3, up to Problem 3.30. Compare your solutions with the solutions provided to make sure you understand the concepts.

Problems 1-5: Problems 3.3, 3.5, 3.7, 3.9, 3.19

Problem 6: Consider a scalar Kalman filitering problem:
State equation: $x_{k+1} = \frac{2}{3}x_k + w_k$, where $w_k \sim N(0,1)$ are i.i.d., and $x_0 \sim N(0,1)$.
Measurement equation: $y_k = x_k + v_k$, where $v_k \sim N(0,\sigma^2)$ are i.i.d.
Assume that x_0, $\{w_k\}$ and $\{v_k\}$ are independent.

(a) Write down the Kalman filtering updates for $\hat{x}_{k|k}$ in terms of $\hat{x}_{k-1|k-1}$ and y_k, expressing the gains involved in the updates in terms of the error variance sequence $\{\Sigma_{i|i}\}$.
(b) Write down the recursion for $\Sigma_{k|k}$ in terms of $\Sigma_{k-1|k-1}$.
(c) For $\sigma^2 = 16$, find the limiting value of the error variance. How does it compare with the error variance of the instantaneous estimate $\hat{x}_k = y_k$?
(d) Find the limiting value of the Kalman gains involved in (a) for $\sigma^2 = 16$.
(e) Run a matlab simulation and plot the state trajectory $\{x_k\}$ and the estimated trajectory $\{\hat{x}_{k|k}\}$ for $k = 0, 1, ..., 1000$. Also plot the instantaneous estimate $\hat{x}_k = y_k$. Attach plots for 2-3 simulation runs.
Remark: You should precompute the error variances and Kalman gains (since they do not depend on the realization).
(f) For the simulated trajectory in (c), find an empirical estimate for the error variance. How does it compare with the limiting value in (c).

Problem 7 (optional, for extra credit): Consider the scalar tracking problem mentioned in class, where p_k, v_k denote the position and velocity of the target at time k. The evolution of the system is as follows:

$$p_{k+1} = p_k + v_k$$
$$v_{k+1} = v_k + a_k$$

where the acceleration sequence is modeled as i.i.d. $N(0,1)$. Let us set our initial condition as $p_0 \sim N(0,100)$ and $v_0 \sim N(0,25)$, which means that we know very little about where the target is or how fast it is moving before we make our first measurement.
The tracking system makes noisy measurements of the target position:

$$y_k = p_k + v_k$$

where $v_k \sim N(0,\sigma^2)$.
As usual, assume that $\{v_k\}$, $\{a_k\}$, p_0 and v_0 are independent.
(a) Set up a Kalman filtering framework for estimating $\hat{p}_{k|k} = E[p_k|y_0,...,y_k]$ and $\hat{v}_{k|k} = E[v_k|y_0,...,y_k]$, specifying all the relevant recursions.
(b) For $\sigma^2 = 16$, do offline computations of the error covariances and gains.
(c) Simulate the position and velocity trajectories, and plot them along with their estimates from the Kalman filter. Comment on how the Kalman filter performs relative to naive estimates (e.g., $\hat{p}_k = y_k$ and $\hat{v}_k = y_k - y_{k-1}$).