Lab 2b : Frequency-Domain Control Design

Due: Monday, April 19 (with Lab2a from 4/7/2010)

Topics:
- Position control of motor-driven cart
- Loop shaping design of a compensator (lead-lag)
- Integrator wind-up
- Relation between transfer function (bandwidth; integral action) and time response

Procedure:
Design a lead-lag compensator, $C(s) = \left[ \frac{(s + K_i)}{s} \right] \cdot \left[ K_{lead} \frac{(s + z_{lead})}{(s + p_{lead})} \right]$, for the motor-cart system you analyzed last week in lab. You may use your own model for the plant, or you may simply assume:

$$P(s) = \frac{1.24}{s(s + 10)}$$

Your design specifications are to achieve a forward path, $G(s) = C(s) \cdot P(s)$, where:

$$\omega_c = 50 \text{ (rad/s) [crossover frequency]}$$

and

$$\phi_m = 45^\circ \text{ [phase margin]}.$$

Deliverables:
Questions 1-6 can be done in groups. 7 must be completed individually.

1. Solve for $K_i$, $K_{lead}$, $z_{lead}$, and $p_{lead}$.

2. On a single MATLAB plot, show both the negative loop transmission,

$$G(s) = C(s) \cdot P(s)$$

and the closed-loop system, $G_{closed} = \frac{G}{G + 1}$.

3. Convert your continuous-time controller into a discrete-time implementation.

Keep the lag and lead part SEPARATE, as shown in the example model in Figure 1. What are the discrete-time transfer functions for both the lag and lead parts?

4. Is a lag compensator really required for your modeled plant? What about for the true (nonlinear) plant? What if the true plant has a second pole at $s = -0.2$ (replacing the pole at $s = 0$), for instance due to the effects of the cables?
5. Command a constant reference of $x_r = 0$ and push on the cart for varying lengths of time. You should notice the effects of “integrator windup”. Find appropriate “saturation limits” for the integrator. What values did you use?

6. Provide a few (2-4) time responses of the cart tracking a small amplitude sine wave (e.g., .05 meters or less) at each of several frequencies (within the controlled bandwidth), to demonstrate performance. Also provide two step response plots: one with the integrator (with or without your saturation limits) and one with the integrator removed. Allow a long time (e.g., 20-30 seconds) to observe if the controller is really driving steady state error to zero or not. **WARNING: Use a small value for the step!** 1cm (.01 meters) should be about right.

7. (To be completed individually!) Write a MATLAB m-file that will automatically solve for a lead-lag compensator for user-defined values of $\omega_c$ (assume a range of possible values from 10 to 50 rad/s) and $\phi_m$ (assume a range of possible values from 45 to 75 degrees). Your code should output both the continuous time transfer functions for both the lag and lead compensators as well as the z-domain coefficients required to implement both compensators as discrete-time transfer functions. (Assume a sampling rate of $T=.001$ seconds.)

**Submission:**
- Assignment 2 (“Lab2a: System Identification” plus “Lab2b: Frequency-Domain Control Design”) is due: **Monday, April 19 at midnight.**

- **You must submit part 7 electronically**, using the ECE 238 website drop box, located here: [http://www.ece.ucsb.edu/courses/ECE238/238_S10Byl/homework/](http://www.ece.ucsb.edu/courses/ECE238/238_S10Byl/homework/)
  (Check your class email or contact me to get the username and password.)

- Other parts may be submitted via either
  i. Electronic dropbox website, at link above
  ii. Physical dropbox, outside 3120A HFH.

- Although I do not demand any specific formatting instructions for the assignments, clarity and brevity will certainly earn you “style” points. (A well-written report will also put your instructor in a much better mood during grading, which will undoubtedly work in your favor, as well!)
Figure 1. Use a separate “sub-system” block (here, labeled “leadlag”) to contain your controller. Also, keep the lag and lead separate, as illustrated, and implement your lag (“PI” control, since the pole is just at zero) with a “Discrete-Time Integrator”. This way, you can experiment and set integrator saturations limits easily (or even “turn it off”).