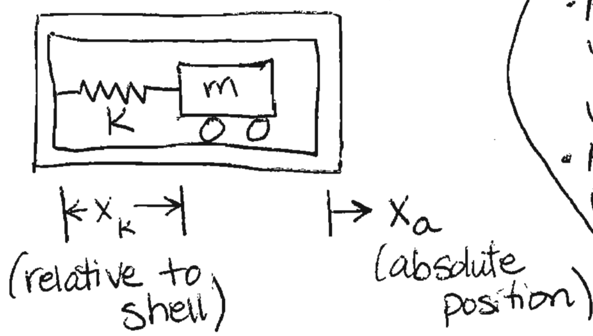
Sensors – Important to consider both freq. response & noise characteristics.

Accelerometer (e.g. MEMS)



- Assume x_a is the "system input" we wish to measure.
- Assume sensor output is proportional to a gap (measured capacitively).

Force balance (Newton's 2nd law...)

$$\sum F = ma$$

$$\boxed{\underbrace{-kx_K}_{\text{relative position}} = m\underbrace{(\ddot{x}_a + \ddot{x}_K)}_{\text{total acceleration (absolute + relative)}}}$$

$$-m\ddot{x}_K - kx_K = m\ddot{x}_a$$

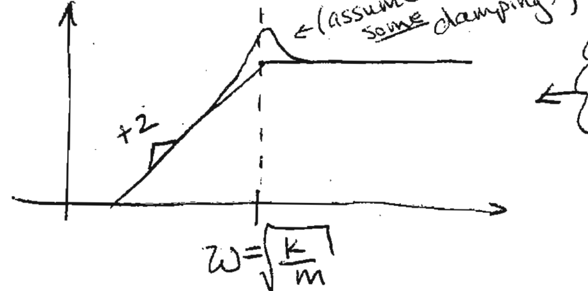
$$-(ms^2 + k)\cdot X_K(s) = (ms^2)\cdot X_a(s)$$

$$\frac{X_K(s)}{X_a(s)} = \frac{-(ms^2)}{(ms^2 + k)}$$

$$\boxed{V_{sense} = A_{sense} \cdot X_K}$$ ← Sensor output voltage is proportional to relative displacement, X_K

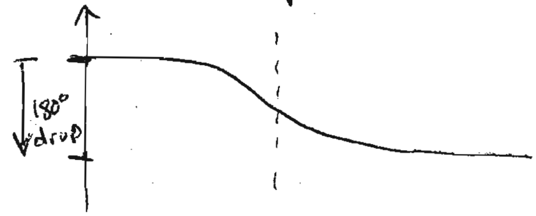
So: $$\frac{V_{sense}}{X_a} = A_{sense} \cdot \frac{X_K}{X_a} = -A \frac{ms^2}{(ms^2 + K)}$$

$$\left|\frac{V_{sense}(s)}{X_a(s)}\right|$$



$$-A\,\frac{ms^2}{(ms^2+K)}$$

$$\measuredangle\left(\frac{V_s}{X_a}\right)$$

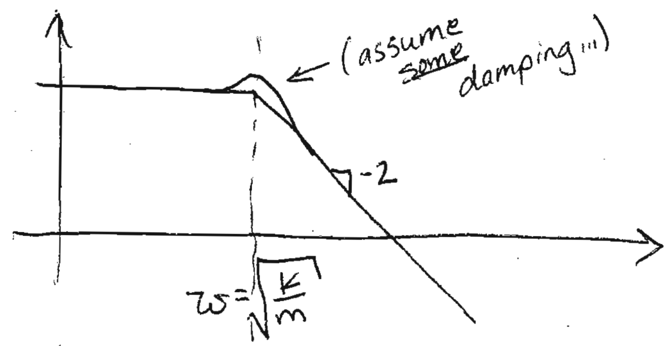


★ So, at "high frequencies", we have a nice, flat measurement of POSITION of the outer shell of the accelerometer...

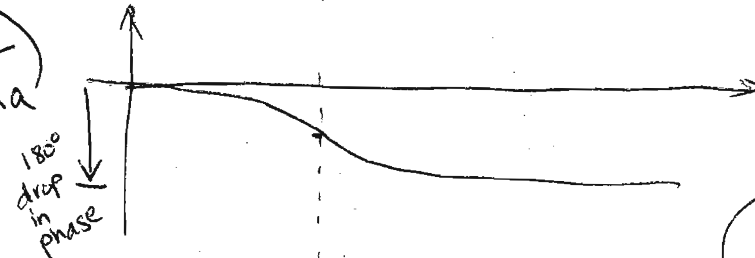
But this is called an "accelerometer", right?
What is the relationship between V_{sense} & $\ddot{X}_a$??

$$\frac{V_s(s)}{\ddot{X}_a(s)} = \frac{V_s(s)}{s^2 X_a(s)} = -A\,\frac{ms^2}{s^2(ms^2+K)} = -A\,\frac{m}{ms^2+K}$$

$$\left|\frac{V_s}{s^2 X_a}\right|$$



$$\measuredangle\left(\frac{V_s}{s^2 X_a}\right)$$

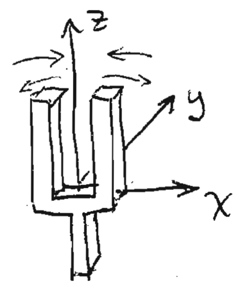


So, at low frequencies, we have a good measurement of ACCELERATION.

Note - I am intentionally being "sloppy" in labeling "phase", b/c sense constant "A" can be > 0 or < 0...

<u>Gyro</u> (e.g., MEMS) – A gyroscope gives an output proportional to <u>angular velocity</u>. Inertial measurement units (IMU's) typically do something akin to integration to turn this into an estimate of <u>angular position</u>.

(Coriolis effect... $a_c = 2(\Omega \times v)$)

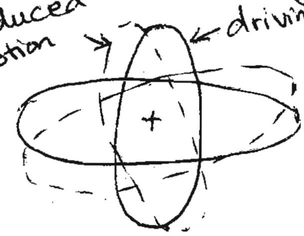


$F_c = m \cdot a_c$

← Drive ± x (local coordinate). Rotation about $\vec{z}$ is sensed, b/c out-of-plane motion (in $\vec{y}$) is induced.

"tuning fork" version

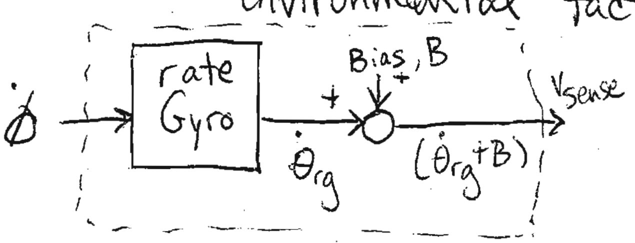
<u>Alternate form:</u> Wineglass mode.

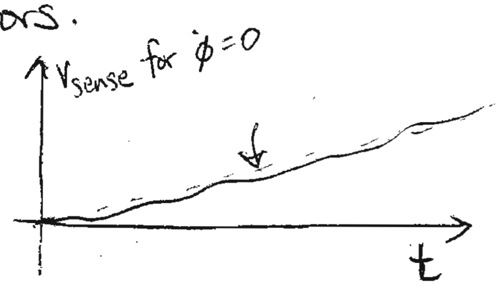


mode that you hear when you rub the rim of a wineglasss.

$\boxed{\text{Drift}}$ →

- Gyro sensor will have noise at all frequencies, but the most problematic noise is <u>low frequency</u>, i.e., $\omega = 0$. (Steady-state offset.)
- Unfortunately, this offset is (generally) sensitive to <u>temperature</u> and other environmental factors.





Sensor Fusion: Complementary Filters

Problem – One sensor may work well over a particular range of frequencies, while another works well over a different range. Can we blend these?

Example – Gyro has drift, so it performs poorly at low frequency estimates of θ (position, via integration).

– Accelerometer can work as a "tilt sensor", but it is only effective for low frequency estimates.

$$V_{acc} = C_a \cdot \sin\theta + N_{acc}$$
$$\approx C_a \cdot \theta + N_{acc} \leftarrow \text{accelerometer noise}$$

$$V_{gyro} = C_g \cdot \dot{\theta} + N_{gyro}$$
$\uparrow$ includes steady-state bias!

- So, let's use a low-pass filter on V_{acc}:

$$\theta \longrightarrow \boxed{\text{Accel'r}} \xrightarrow{V_{acc}} \boxed{\frac{1}{\tau s+1}} \longrightarrow \left(\frac{1}{\tau s+1}\right)\cdot C_a\theta + \left(\frac{1}{\tau s+1}\right)\cdot N_{acc}(s)$$

- What is a good choice of filter for V_{gyro}?

(For now, assume $C_a = 1$, $C_g = 1$ ...)

$$\underbrace{\left(\frac{1}{\tau s+1}\right)}_{\text{Accelerometer filter}} \cdot \theta_{(a)} + \underbrace{\left(\; ? \; \right)}_{\text{Gyro filter}} \cdot \dot{\theta}_{(g)} = \theta$$

$$\left(\frac{1}{\tau s+1}\right) \cdot \theta + \left(\; ? \; \right) \cdot s \cdot \theta = \theta \cdot \overbrace{\left(\frac{\tau s+1}{\tau s+1}\right)}^{\text{just "1"...}}$$

Answer:

$$\left(\; ? \; \right) \cdot s \cdot \theta = \left[\frac{\tau s+1}{\tau s+1} - \frac{1}{\tau s+1}\right] \cdot \theta$$

$$(?) \cdot s = \frac{\tau s}{\tau s+1}$$

$$\boxed{? = \frac{\tau}{\tau s+1}}$$

Higher-order filters work, too:

$$\left(\frac{3s^2+3s+1}{s^3+3s^2+3s+1}\right) \cdot \theta_a + \left(\frac{s^2}{s^3+3s^2+3s+1}\right) \cdot s \cdot \theta_g = \theta \cdot 1$$

- Set filter to attentuate noise of each sensor.

→ Next class: Observers

Kalman Filters

Extended Kalman Filters (EKF)