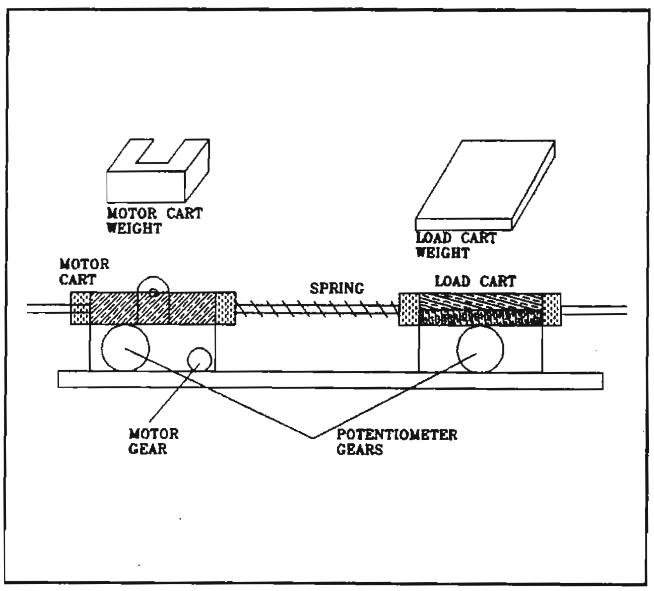
3.2.1 DESCRIPTION

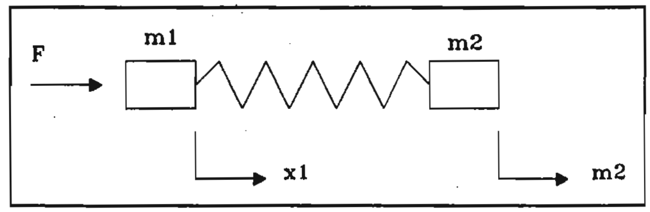
The system consists of two carts as shown in Figure LFJ1. Both carts slide on a ground stainless steel shaft and are coupled via a spring. The cart on the left is equipped with a motor and a potentiometer and is the same cart that is used in the servo experiment. The other cart only has a potentioemeter. The shafts of these elements are coupled to a rack and pinion mechanism in order to input the driving force to the system and to measure the two cart positions. The motor shaft is connected to a 0.5" diameter gear while the potentiometer shafts are attached to 1.166" diameter gears. When the motor turns, the torque created at the output shaft it translated to a linear force which results in the cart's motion. When the cart moves, the potentiometer shaft turns and the voltage measured from the potentiometer can be calibrated to obtain the track position. Masses are available for attachment to each cart. The mass for the motor cart is used in order to reduce slippage. The masses for the load cart are for assessing the robustness of the controller and the effects of variations in parameters.



LFJ 1 Experimental setup for linear flexible joint

3.2.2 MATHEMATICAL MODEL

The simplified model of the system is shown in Figure LFJ2.



LFJ 2 Simplified model

where:

F = input force to the cart (N)
m1 = mass of motor cart (Kg)
m2 = mass of load cart (Kg)
K = spring stiffness (N/m)

The states of the system are:

x_1 = position of motor cart (m)
$\dot{x}_1$ = velocity of motor cart(m/sec)
x_2 = position of load cart (m)
$\dot{x}_2$ = velocity of load cart(m/sec)

Note the directions defined for these variables. Also note that two coordinate systems are defined, one for the motor cart and one for the load cart. Using this approach the length of the spring is not required for linear modelling.

Using the value given, the state description with Force as input is given as:

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k}{m_1} & 0 & \frac{k}{m_1} & 0 \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & 0 & -\frac{k}{m_2} & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix} F$$

which with the given values results in (rounded values)

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -45 & 0 & 45 & 0 \\ 0 & 0 & 0 & 1 \\ 76 & 0 & -76 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1.02 \\ 0 \\ 0 \end{bmatrix} F$$

To convert to voltage input we had obtained:

$$F = \frac{K_m K_g}{Rr} V - \frac{K_m^2 K_g^2}{Rr^2} \dot{x}$$

substituting this into the matrix equation we have

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dot{x}_2 \\ \ddot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -45 & -7.4 & 45 & 0 \\ 0 & 0 & 0 & 1 \\ 76 & 0 & -76 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1.73 \\ 0 \\ 0 \end{bmatrix} V$$

Which is the desired representation.

3.2.3 CONTROL SYSTEM DESIGN

Using MATLAB LQR DESIGN and the performance index:

$$J = \int (x'Qx + rV^2)\, dt$$

with

$$Q = \begin{bmatrix} 1000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

and r = 0.02

results in:

K = [277 14 -53 13] Volts/m units

or

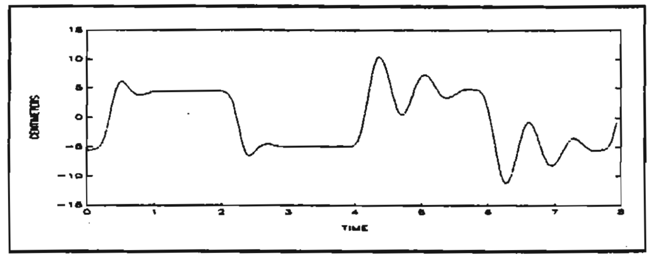
K = [2.72 .13 -.48 .134] Volts/cm units

The closed loop eigenvalues are at
13.2 +/- j 14.3.
-2.8 +/- j 8.3

3.2.4 RESULTS

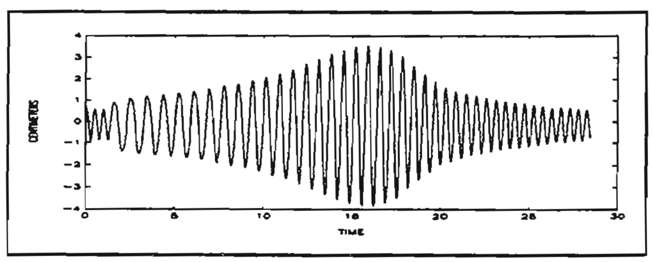
The system should perform quite well with the gains given above. Further tuning could change the response to suit various behaviour specifications. We have chosen to reduce k_3 to -0.2 instead of -0.53 to reduce steady state error that results from stiction and deadband.

Figure LFJ 3 shows the step response at the output cart using two types of controllers. The first portion of the trace shows the response using the full state feedback designed above while while the second part of the trace shows the response when only motor cart feedback is used (K2=K4=0). Clearly full state feedback performs better.

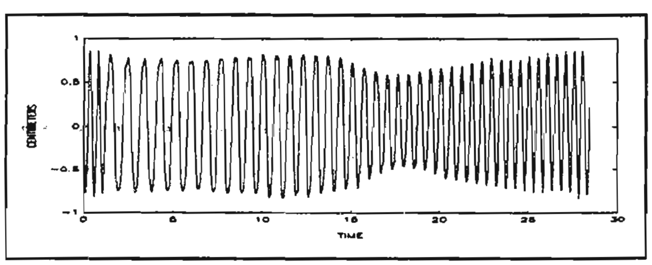


LFJ 3 Step response at output cart with full state feddback in the first two steps compared to motor cart feedback only (k2=k4 =0) in the following two steps.

Also, it is interesting to observe the frequency responses of the system. Figures LFJ4 and LFJ5 show the frequency responses at the output and motor cart when K2=K4=0 and are obtained by sweeping the command from 1 Hz. to 2 Hz. in steps of 0.025 Hz. This means that the slowest cycle is at 1 Hz, the second at 1.025 Hz and so on. The amplitude of the command is 1 cm. in both cases. Note the resonant peak in the output cart at around 1.5 Hz and the anti-resonance in the motor cart.



LFJ 4 Sine sweep response at output cart with K2=K4=0

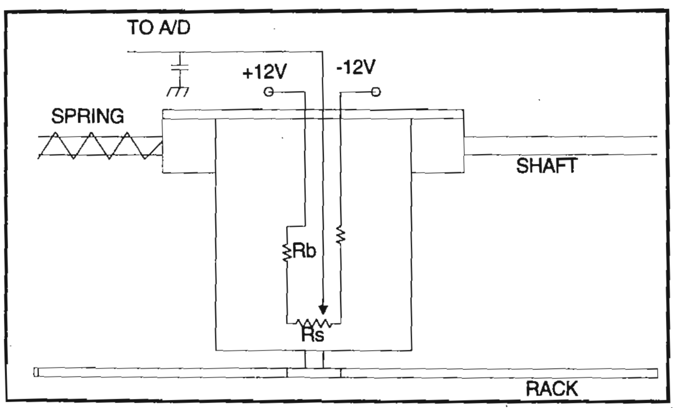


LFJ 5 Sine sweep response at input cart with K2=K4=0

5.2.2 LINEAR FLEXIBLE JOINT / LINFLEX

PARAMETER	SYMBOL	VALUE	UNITS	
MASS FOR MOTOR CART		0.455	Kg	TOTAL MASS m1 = 0.96 Kg
EXTRA MASS FOR MOTOR CART		0.380	Kg	
SPRING FIXTURE MASS (1 FOR EACH CART)		0.125	Kg	
SECOND CART MASS		0.24	Kg	TOTAL MASS m2 = 0.615 Kg
EXTRA MASSES FOR LOAD CART (2 AVAILABLE)		0.13 (each)	Kg	
SPRING FIXTURE MASS (1 FOR EACH CART)		0.125	Kg	
POTENTIOMETER	SAME AS MOTOR CART			
SPRING STIFFNESS	Ks	38.25	N/m	

The equivalent circuit that is achieved using the Quick Connect Module (QCM) is shown below in figure WI 8.



WI- 8 Wiring the load cart for the Linear Flexible Joint