Magnetic Bearing Lab # 3: Notch Filtering of Resonant Modes*

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In the following lab we will begin to investigate the design of a controller to stabilize the magnetic bearing system. We will review the Nyquist Criterion and its application to design and analysis of unstable systems such as the magnetic bearing. Using this criterion, we will see how resonant modes can threaten the stability of the closed loop system. Consequently, we will design and build a notch filter to cancel the effect of the first resonant flexible rotor mode. This notch filtering is the first phase of our controller design.

3.1 The Nyquist Criterion

The magnetic bearing system can be described by the block diagram shown in Figure 1 where $G(s)$ is the magnetic bearing system shown in Figure 2 and $C(s)$ is the controller we will design to stabilize it. For such a system configuration, the closed-loop transfer function can be written

$$T(s) = \frac{y}{r} = \frac{G(s)}{1 + G(s)C(s)}.$$

If we define polynomials n_G, d_G, n_C and d_C as follows: $G(s) \triangleq n_G/d_G$ and $C(s) \triangleq n_C/d_C$, then we can write

$$T(s) = \frac{\frac{n_G}{d_G}}{1 + \frac{n_G n_C}{d_G d_C}} = \frac{\frac{n_G}{d_G}}{\frac{d_G d_C + n_G n_C}{d_G d_C}} = \frac{n_G d_C}{d_G d_C + n_G n_C}.$$

*The equipment needed to perform this lab includes an MBC 500 Magnetic Bearing, Hewlett Packard 3562A Dynamic Signal Analyzer, a Burr-Brown UAF42 14-pin DIP chip, a ±12V or ±15V power supply and basic circuit building equipment including a breadboard, assorted potentiometers and fixed value resistors, circuit wire, various coaxial cables and adaptors.

Thus, the zeros of $1+G(s)C(s) = \frac{d_G d_C + n_G n_C}{d_G d_C}$ are equal to the poles of the closed-loop transfer function $T(s)$.

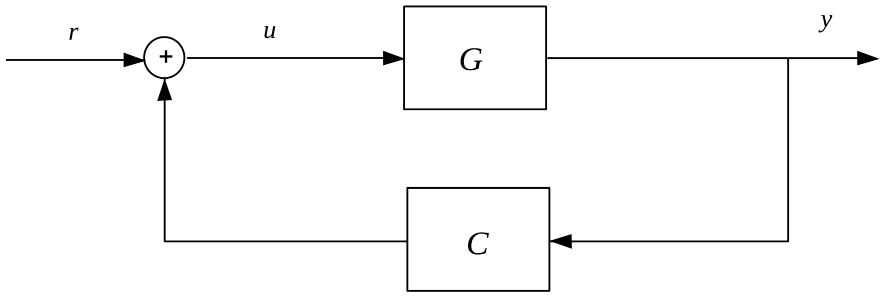

Figure 1: Magnetic Bearing Closed-loop Configuration

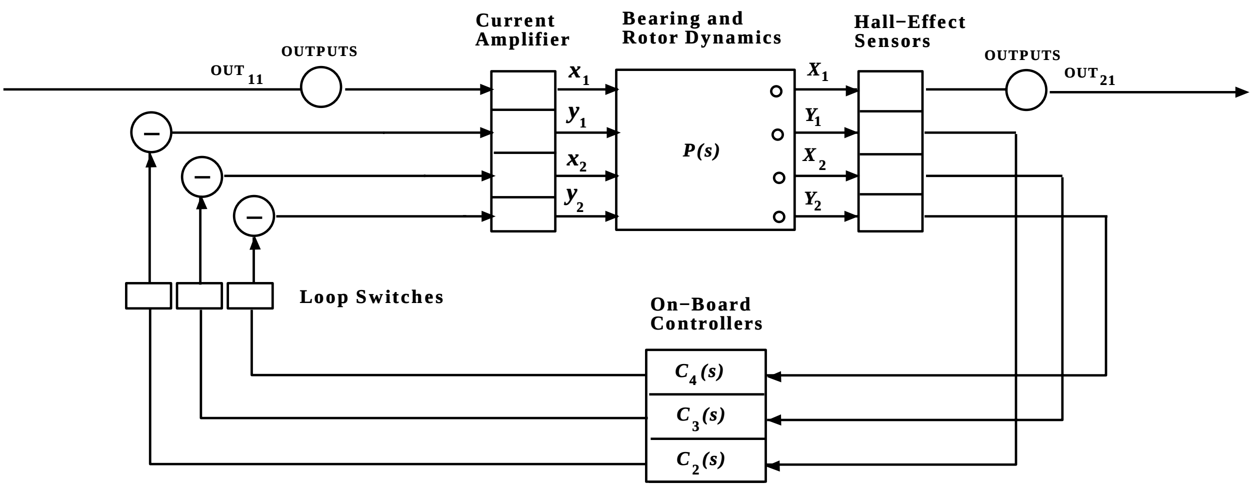

Figure 2: Magnetic Bearing System

The Nyquist Criterion provides a check on the location of the zeros of $1 + G(s)C(s)$, or equivalently the closed-loop system poles, and can therefore be used to determine stability of the closed-loop system. In performing this check, the open-loop transfer function, $G(s)C(s)$, is plotted in the complex plane as s takes values along the imaginary axis, i.e. $s = j\omega$ where ω varies from $-\infty$ to $+\infty$. Such a plot is called a Nyquist plot of $G(s)C(s)$. The Nyquist Criterion can be stated as follows:

The Nyquist Criterion *For the closed-loop system of Figure 1 to be stable, the Nyquist plot of $G(s)C(s)$ must encircle the point $(-1, 0j)$ the same number of times as the number of right half plane poles of $G(s)C(s)$, and the encirclements, if any, must be in the counterclockwise direction.* [1] [2]

These results may be summarized as follows:

$N =$ *the net number of clockwise encirclements of the point* $(-1, 0j)$ *by* $G(s)C(s)$ *with* $s = j\omega$ *as* ω *varies from* $-\infty$ *to* $+\infty$*. Counterclockwise encirclements are counted as negative encirclements.*

$P =$ *the number of poles of* $G(s)C(s)$ *that are in the right half of the complex plane.*

With these definitions, the Nyquist Criterion can be stated: *The closed-loop system is stable if and only if*

$$N + P = 0.$$

We know from previous identification experiments that our system $G(s)$ has one unstable pole; therefore, for any stable $C(s)$ which stabilizes our system, the Nyquist plot of $G(s)C(s)$ must encircle the point $(-1, 0j)$ one time in the counterclockwise direction. This fact gives us insight into how to choose the magnitude and phase characteristics of $C(s)$ to achieve stability. Now let's consider the effect of the rotor resonant modes on the stability of the closed-loop system. A Bode plot of the bearing frequency characteristic will show a large increase in magnitude and wide fluctuation in phase around the resonances. This constrains our choice of $C(s)$ because the controller must maintain one encirclement of the point $(-1, 0j)$ in the Nyquist plot of $G(s)C(s)$ despite these wide fluctuations. Thus, we would have more flexibility in the design of $C(s)$ if the resonances were not in the system. One approach to controller design of systems with resonances is to "notch out" or approximately cancel the effect of the resonances by use of a notch filter near the resonant frequencies. In the controller design to follow, we will attempt to notch the first and largest rotor resonance, the one near 800 Hz.

3.2 Notch Filter Design

Exercise 1: Repeat Exercises 1-4 of *Magnetic Bearing Lab # 2* to obtain a Bode plot on the dynamic signal analyzer of the bearing system $G(s)$ except this time plot the response magnitude in MAG (LOG) instead of MAG (dBm). The sequence of steps required is included in Tables 1 and 2 below for your convenience. After setting up the signal analyzer, start the sine sweep by pressing the START key in the CONTROL section

hard key	*soft key*	*value*	*soft key*
MEAS MODE	SWEPT SINE		
SELECT MEAS	FREQ RESP		
SOURCE	SOURCE LEVEL	500	mV
FREQ	START FREQ	10	Hz
	STOP FREQ	10	kHz

Table 1: Key Sequence for Initiating Swept Sine Mode

hard key	*soft key*
A	
MEAS DISP	FREQ RESP
COORD	MAG (LOG)
B	
MEAS DISP	FREQ RESP
COORD	PHASE
A&B	

Table 2: Key Sequence for Bode Plot Display

of the front panel remembering to adjust the "Source" signal amplitude as needed to prevent the rotor from hitting its physical limits. With the swept sine data displayed, we will now use the marker capability of the dynamic signal analyzer to determine more accurately the frequency of the first rotor flexible mode. In the MARKERS section of the signal analyzer front panel, select the key labeled X. This positions a round dot on the Bode plot at the first frequency point and indicates to the top left of each plot the frequency, magnitude and phase at the marked location on the plot. The round knob in the MARKERS section may now be used to shift the marker to a different frequency point on the plot. In particular, move the marker to the magnitude peak corresponding to the first rotor flexible mode. This should be near 800 Hz. Note the frequency of the peak from the marker information at the top left of the screen.

Now that we have identified the frequency corresponding to the first resonant mode, we would like to use the signal analyzer to identify this mode more exactly.

Exercise 2: After Exercise 1, the signal analyzer should already be set up for swept sine identification of the bearing and display of the Bode plot of the system frequency response. If it is not, follow the procedure of Exercise 1 above to set up the signal analyzer. We will now narrow the range of our frequency sweep to focus on the first resonant mode. We will want to span a frequency range of 100 to 150 Hz centered at the resonant frequency determined in Exercise 1. We can make this adjustment by pressing the FREQ key in the MEASUREMENT portion of the front panel. Following the procedure outlined in Exercise 1, we can select a new start frequency and stop frequency to achieve the desired frequency range. This new frequency sweep is initiated by pressing the START key in the CONTROL section of the front panel. When the measurement has been completed, use the X marker to note the frequency at which the response peaks. Note that this frequency may be slightly different from the one determined in Exercise 1 because the trace we have just performed has higher resolution. This frequency in Hz we will denote as f_n in the following. Also, note the magnitude at the peak of the resonance response curve. This value we will denote as A_{peak}. Now select two points f_1 and f_2 on either side of the peak and on the portion of the plot unaffected by the resonance (See Figure 3.) such that

$$\frac{f_1}{f_n} = \frac{f_n}{f_2}$$

or equivalently

$$f_2 = \frac{f_n^2}{f_1}.$$

This places f_1 and f_2 equidistant from f_n when measured on a log scale. Note the magnitudes at these two points on the plot and call them A_1 and A_2. Now form an average of these two magnitudes to calculate what the magnitude of the response at f_n would be if the resonance were not in the system. Call this average magnitude A_{base}. In other words,

$$A_{base} = \frac{A_1 + A_2}{2}.$$

With the information obtained in Exercise 2, we will attempt to estimate the damping of the first resonant mode. The transfer function for a general resonance pole-pair, assuming

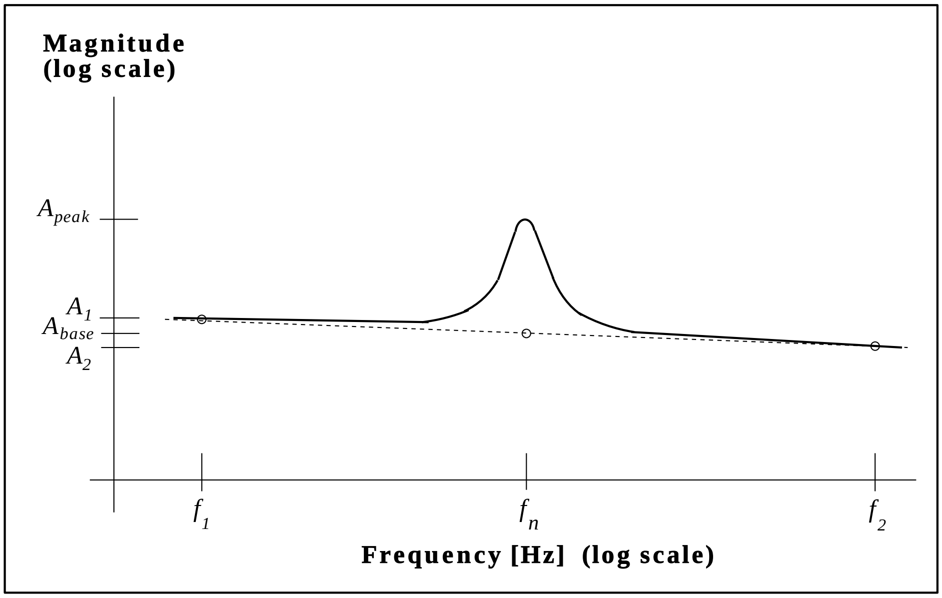

Figure 3: Approximating Resonance Q from Frequency Data

unity gain at DC, can be written in the form:

$$G_{RES}(s) = \frac{\omega_n^2}{s^2 + s\frac{\omega_n}{Q} + \omega_n^2}$$

where ω_n is the resonant frequency in radians per second. If we evaluate this resonance transfer function at the resonant frequency, ω_n, we have:

$$G_{RES}(j\omega_n) = \frac{\omega_n^2}{j\frac{\omega_n^2}{Q}}$$

and

$$|G_{RES}(j\omega_n)| = Q.$$

If we express the total bearing system transfer function as a product of the resonance transfer function and the rest of the system, we find that:

$$|G(j\omega_n)| = |G_{REST}(j\omega_n)G_{RES}(j\omega_n)| = |G_{REST}(j\omega_n)|Q.$$

Exercise 2 gives us ω_n as follows: $\omega_n = 2\pi f_n$. Also, from Exercise 2 we know that

$$|G(j\omega_n)| = A_{peak}$$

and

$$|G_{REST}(j\omega_n)| \cong A_{base}.$$

Therefore, we may approximate Q as follows:

$$Q \cong A_{peak}/A_{base}.$$

Exercise 3: Using the information obtained from Exercise 2 and the equations above, determine ω_n and an approximate Q for the first resonant mode.

Now that we have specified ω_n and Q for the resonance, we may design a notch filter with the same natural frequency and damping to cancel the effect of the resonance. Therefore, the same ω_n and Q will describe the notch filter as well as the resonance.

3.3 Building the Notch Filter

For the implementation of the notch filter, we will use a Burr-Brown Universal Active Filter UAF42 chip (14 pin DIP version) [1]. A schematic of the notch filter we will build is shown in Figure 4. This circuit design, using the Burr-Brown UAF42 chip, was chosen to reduce the sensitivity of the filter response to component variations. When connected in the configuration shown, the UAF42 chip has three filter outputs. Pin 1 corresponds to a low-pass output, pin 7 is a bandpass output, and pin 13 is a high-pass output. The transfer functions corresponding to these three outputs are listed in Table 3. The notch filter transfer function is

$$N(s) = \frac{A_N(s^2 + \omega_n^2)}{s^2 + s\frac{\omega_n}{Q} + \omega_n^2}$$

and is created by summing the low-pass and high-pass filter outputs. This summing is done using the auxiliary op amp located on the UAF42 chip in pins 4-6 connected as a unity gain non-inverting summer. In the UAF42 filter configuration shown in Figure 1, we have $A_{LP} = A_{HP}$; therefore, we also have

$$A_N = A_{LP} = A_{HP}.$$

At this point in the design process, we will not attempt to design the gain of the notch filter. The filter will be incorporated into the overall controller and the gain will be fixed at that

[1] To order a Burr-Brown Integrated Circuits Data Book which describes in more detail the use and specifications of the UAF42 chip, you may write to: Burr-Brown Corporation, PO Box 11400, Tucson, AZ 85734 or call 1-800-548-6132.

Pin Number	*Filter Type*	*Transfer Function*
1	low-pass	$\frac{A_{LP}\omega_n^2}{s^2+s\frac{\omega_n}{Q}+\omega_n^2}$
7	bandpass	$\frac{A_{BP}(\omega_n/Q)s}{s^2+s\frac{\omega_n}{Q}+\omega_n^2}$
13	high-pass	$\frac{A_{HP}s^2}{s^2+s\frac{\omega_n}{Q}+\omega_n^2}$

Table 3: UAF42 Filter Options

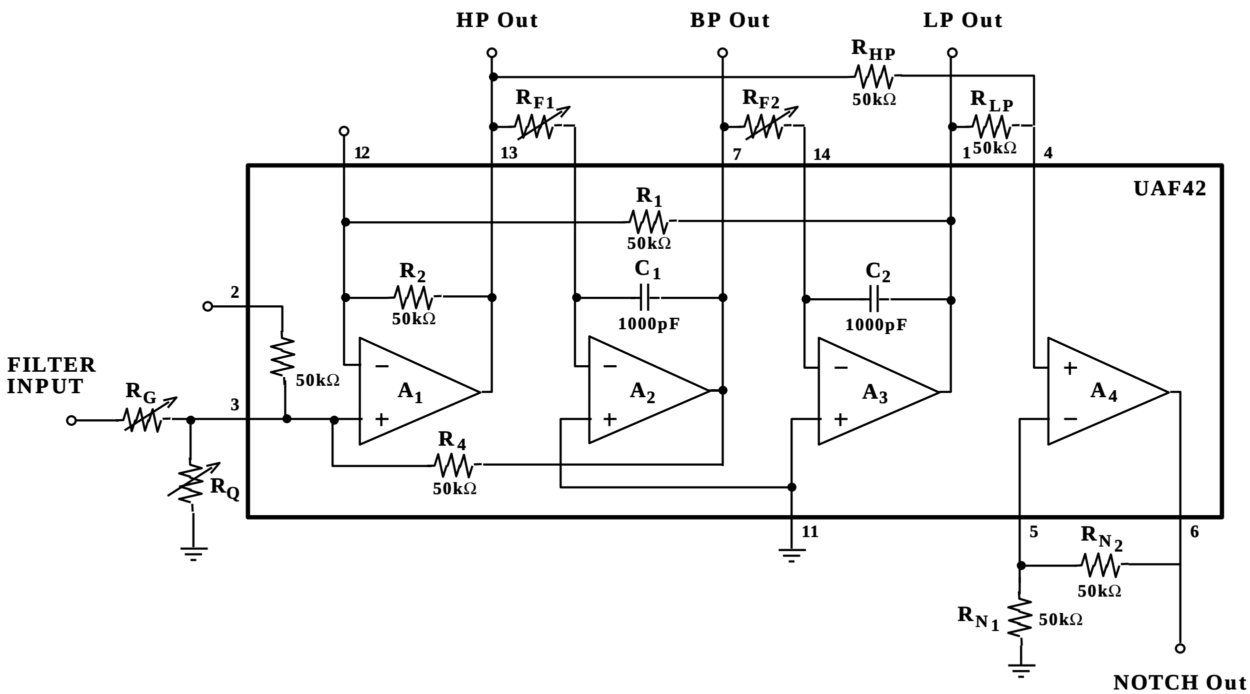

Figure 4: Notch Filter Schematic Diagram

stage. Because of the way in which the UAF42 chips implement the notch, for most values of Q it will have a DC magnitude near one. The ω_n and Q for all of the filters are determined by choice of the resistances R_{F1}, R_{F2}, R_G and R_Q. This relationship is described by the following equations:

$$
\begin{aligned}
\omega_n^2 &= \frac{10^{18}}{R_{F1}R_{F2}} \\
Q &= \frac{1}{2} + (2.5 \times 10^4)\frac{R_G + R_Q}{R_G R_Q}\left(\frac{R_{F1}}{R_{F2}}\right)^{1/2}.
\end{aligned}
$$

From these equations, we see that we have four parameters to tune and two equations to satisfy. Thus, we have extra flexibility in choosing these parameters. We will use this flexibility to make our resistances fall within the range 1k ohm to 1M ohm [2]. The first equation above shows us that ω_n is determined by the values of R_{F1} and R_{F2}. However, from the second equation we see that the value of Q is also affected by choice of R_{F1} and R_{F2}. We will select the values of R_{F1} and R_{F2} in two stages. First, we will use the second equation to select the ratio $\frac{R_{F1}}{R_{F2}}$ so that the values of R_G and R_Q may fall within the desired range of 1k ohm to 1M ohm. Then we will fix the values of R_{F1} and R_{F2} to give the correct ω_n based on the first equation. Once R_{F1} and R_{F2} are fixed, we can use the second equation to choose R_G and R_Q.

Exercise 4: Assume $R_G = R_Q$ and set them equal to some nominal resistance value (e.g. $R_G = R_Q = 50$k ohm) in the Q equation above. Given the value of Q determined in Exercise 3, solve for the ratio $\eta \triangleq \frac{R_{F1}}{R_{F2}}$.

Exercise 5: Now substitute the relationship $R_{F1} = \eta R_{F2}$ into the equation for ω_n^2 given above. Using this new equation and the value for ω_n from Exercise 3, solve for R_{F2}. Solve $R_{F1} = \eta R_{F2}$ to find R_{F1}. Check to see that both values fall within the range 1k ohm to 1M ohm . If they do not, then repeat Exercises 4 and 5 with the value of $R_G = R_Q$ adjusted appropriately.

Exercise 6: Build the notch circuit shown in the schematic of Figure 4 using the resistor values calculated in Exercises 4 and 5. Use potentiometers for R_{F1}, R_{F2}, R_G, and R_Q. Be sure to physically label each of the potentiometers so that they are easily distinguishable for tuning purposes. Also, it is important to keep your wiring neat and simple. This may seem to take more time in the circuit building phase, but it will save even more time in the debugging phase.

Let us now consider the effect of ω_n on two of the filter responses available from the UAF42 chip. In particular, let us focus on the bandpass filter and notch filter responses. The transfer function of a bandpass filter is given as

$$T_{BP}(s) = \frac{A_{BP}(\omega_n/Q)s}{s^2 + s\frac{\omega_n}{Q} + \omega_n^2}.$$

[2]We choose this range because resistances below 1k ohm will cause excessive output leveling on the op amps; resistances above 1M ohm, when multiplied by offset currents of the op amp give large offset voltages.

For now we will assume that $A_{BP} = 1$. This gives the magnitude of the bandpass response at frequency ω as follows:

$$|T_{BP}(j\omega)| = \frac{\frac{\omega_n \omega}{Q}}{\sqrt{(\omega_n^2 - \omega^2)^2 + \frac{\omega_n^2 \omega^2}{Q^2}}}.$$

Notice that at $\omega = \omega_n$, $|T_{BP}(j\omega)| = 1$. Notice also that the maximum value of $|T_{BP}(j\omega)|$ over all frequencies is 1. Thus, the magnitude of the bandpass response achieves its peak value at $\omega = \omega_n$. Now let us consider the notch filter response. The notch transfer function is as follows:

$$T_N(s) = \frac{A_N(s^2 + \omega_n^2)}{s^2 + s\frac{\omega_n}{Q} + \omega_n^2}.$$

Again, let us consider the case $A_N = 1$. The magnitude of the notch filter response can be written

$$|T_N(j\omega)| = \frac{|\omega_n^2 - \omega^2|}{\sqrt{(\omega_n^2 - \omega^2)^2 + \frac{\omega_n^2 \omega^2}{Q^2}}}.$$

Thus, at $\omega = \omega_n$ we have $|T_N(j\omega)| = 0$. In other words, the notch filter completely rejects signals of frequency ω_n. Figure 5 shows Bode diagrams of band-pass and notch filter responses for the case of $\omega_n = 1$ and $Q = 1$.

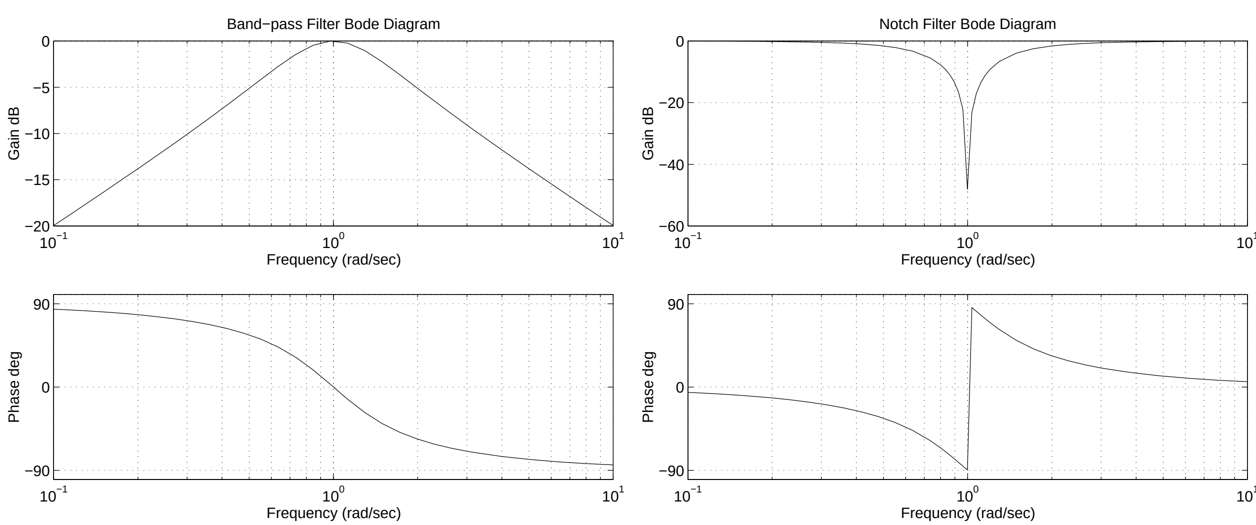

Figure 5: Band-Pass and Notch Filter Bode Diagrams

Exercise 7: We will now use the signal analyzer to tune ω_n by identifying the UAF42 band-pass filter response. As explained earlier, the resistances R_{F1} and R_{F2} determine the value of ω_n for all of the filter responses of the UAF42 chip. Therefore, the ω_n for the bandpass filter is the same as the ω_n for the notch filter. Connect the signal analyzer SOURCE signal to the FILTER INPUT as well as to CHANNEL 1 of the signal analyzer. The UAF42 bandpass filter output, pin 7 of the UAF42, should be connected to CHANNEL 2 of the signal analyzer. Follow the procedure outlined in Exercise 1 for setting up the signal analyzer for swept sine analysis if it has not already been set up. Use the same frequency range that you used in Exercise 2 to focus on the peak of the bandpass filter response. Perform the swept sine identification. In the MARKERS section of the signal analyzer front panel press the SPCL MARKER key followed by the MRKR $\rightarrow$ PEAK soft key to place the marker at the peak of the bandpass filter response. The equation for the frequency in Hz at this point is $f_n = \omega_n/(2\pi)$. Using the ω_n^2 equation given above, we can now adjust R_{F1} and R_{F2} appropriately to achieve our desired ω_n. Repeat the swept sine identification and the adjusting of R_{F1} and R_{F2} until the UAF42 filter ω_n is very close to the desired value.

Exercise 8: Use the new R_{F1} and R_{F2} values determined in Exercise 7 and the Q equation above to determine a new value for $R_G = R_Q$. Adjust the R_G and R_Q potentiometers in the notch filter circuit to make them equal to this new value.

In considering the role of the parameter Q on the various filter responses, let us look at the transfer function denominator common to all of the filters,

$$D(s) = s^2 + s\frac{\omega_n}{Q} + \omega_n^2.$$

The roots of this polynomial are

$$s = \frac{-\frac{\omega_n}{Q} \pm \sqrt{\frac{\omega_n^2}{Q^2} - 4\omega_n^2}}{2} = \frac{-\omega_n}{2Q} \pm \omega_n\sqrt{\frac{1}{4Q^2} - 1}.$$

Let us call these two roots p_1 and p_2. Notice that if $Q \leq \frac{1}{2}$, the roots are real; however, if $Q > \frac{1}{2}$, the roots are complex conjugate with real parts equal to $-\omega_n/(2Q)$. Thus, as Q becomes large, the real part of the complex conjugate roots becomes very small. The denominator can be rewritten as $D(s) = (s - p_1)(s - p_2)$ and the magnitude of the denominator as

$|D(s)| = |s - p_1||s - p_2|$. Now consider the magnitude of the vector $j\omega - p_1$ as ω varies from 0 to ∞. As shown in Figure 6, the magnitude of this vector becomes very small as ω approaches ω_n, and the smaller the real part of the root p_1, the faster the magnitude changes as the frequency varies near ω_n. In particular, since $D(s)$ is the denominator for both the bandpass and notch filters, for very large Q values, the bandpass filter has a sharp peak at ω_n and the notch filter has a steep notch at ω_n in their magnitude Bode plots. Thus, the size of Q determines the steepness of the response at ω_n of the notch filter we are designing.

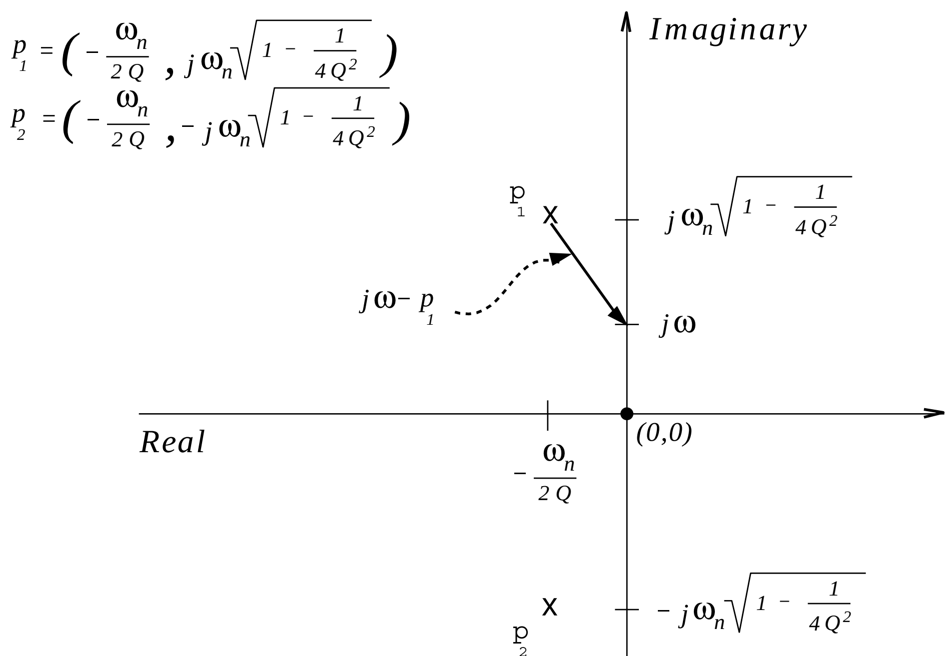

Figure 6: The Relationship Between Magnitude and Q Value

Similar to ω_n, the resistances R_G and R_Q as well as R_{F1} and R_{F2} determine a Q value that holds for all of the UAF42 filter responses. Since R_{F1} and R_{F2} were fixed in the determination of ω_n, we will adjust the resistances R_G and R_Q to tune the Q value. From our swept sine analysis, we were able to identify the ω_n of the first resonant mode fairly accurately; however, unlike the ω_n, our estimate of the resonance Q value was fairly inaccurate. Thus, in our tuning of Q we will not attempt to match our Q design value, but rather tune Q to the actual system to best notch the resonant mode.

Exercise 9: Repeat the signal analyzer sine sweep performed in Exercise 7 with the updated values of R_G and R_Q obtained in Exercise 8. Choose ω_3 to be some frequency (in rad/sec) in the passband of the bandpass filter other than the resonance frequency ω_n. See Figure 7. Now, from the bandpass filter swept sine response, use the signal analyzer markers to evaluate the magnitude of the bandpass filter response at ω_3. Now define M to be the bandpass filter magnitude at ω_3. In other words,

$$M \triangleq \left| \frac{A_{BP}(\omega_n/Q)s}{s^2 + s\frac{\omega_n}{Q} + \omega_n^2} \right|_{s=j\omega_3} = \frac{A_{BP}\frac{\omega_n\omega_3}{Q}}{\sqrt{(\omega_n^2 - \omega_3^2)^2 + \frac{\omega_n^2\omega_3^2}{Q^2}}}.$$

Simple algebraic manipulation yields the following relationship:

$$Q = \pm\frac{\sqrt{A_{BP}^2 - M^2}}{M}\frac{\omega_n\omega_3}{\omega_n^2 - \omega_3^2}.$$

The UAF42 Data Book[1] gives the equation $A_{BP} = 50k\Omega/R_G$. Use the above equations to solve for Q from the signal analyzer bandpass filter data. Check to see that this Q value is roughly the designed value chosen in Exercise 3. If the filter Q is significantly different from the designed value, then check the calculations performed in Exercises 4 and 5. Adjust R_G and R_Q as needed to make the filter Q approximately equal to the designed value of Q. For simplicity, you may want to keep $R_G = R_Q$.

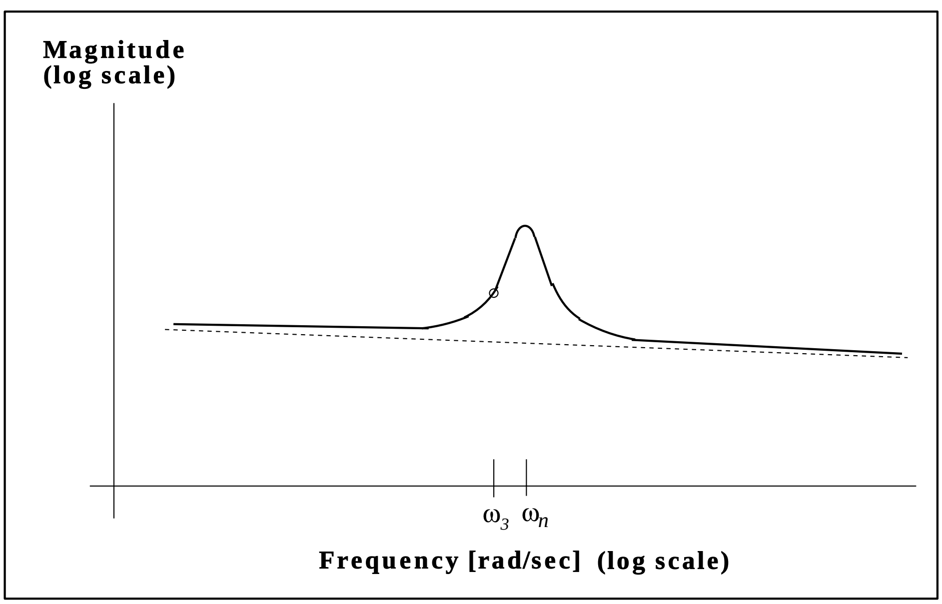

Figure 7: Calculation of Filter Q from Frequency Data

In tuning Q for best notching of the first resonant mode, we will connect the notch filter so that it is in series with the bearing system as shown in Figure 8. In other words, we will connect the bearing output OUT_{21} to the notch circuit FILTER INPUT. We will then apply our signal analyzer SOURCE signal to input IN_1; however, we will consider output OUT_{11} to be the input to the series connection for determining its transfer function. Hence, we will connect OUT_{11} to the signal analyzer Channel 1 input. Finally, we will connect the notch filter output, NOTCH Out, to the signal analyzer Channel 2 output. As we discussed in *Magnetic Bearing Lab # 2*, this type of identification scheme can be biased by system noise. However, for interactive tuning of the parameter Q, a one step identification scheme is fastest and accurate enough to provide satisfactory tuning.

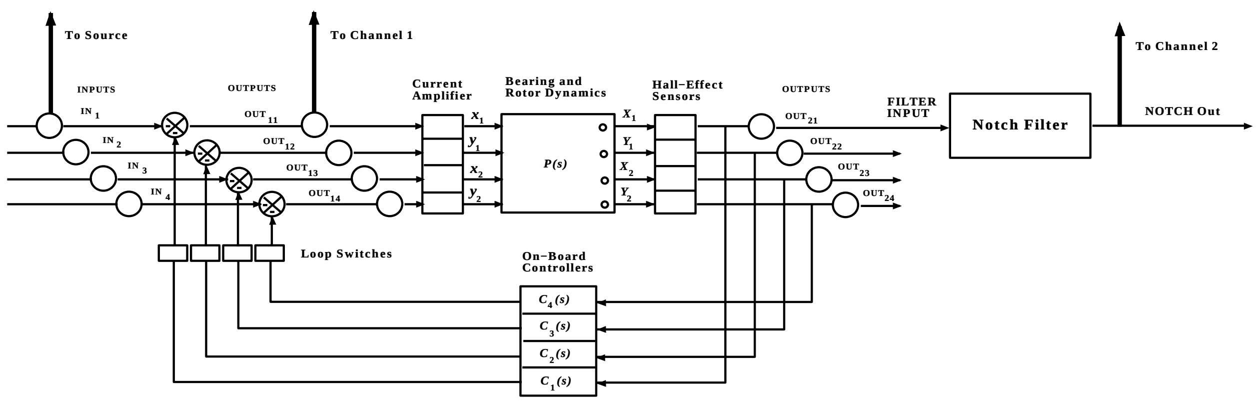

Figure 8: Series Connection of Bearing and Notch Filter

Exercise 10: Connect the notch filter in series with the bearing system as described above and as shown in Figure 7. Perform a swept sine identification of the bearing/notch combination using the frequency range chosen in Exercise 2 which focuses on the first resonant mode. Adjust $R_G = R_Q$ about their initial value performing new sine sweeps of the notched resonance after each adjustment and noting the maximum value of the resonant peak with each value. Choose the value for $R_G = R_Q$ which gives the best notching of the resonance. You should expect to achieve roughly an order of magnitude decrease in the peak magnitude from the unnotched to the notched resonance. Typical data obtained from the swept sine analysis of the first resonant mode of both the notched and the unnotched systems are shown in Figure 9.

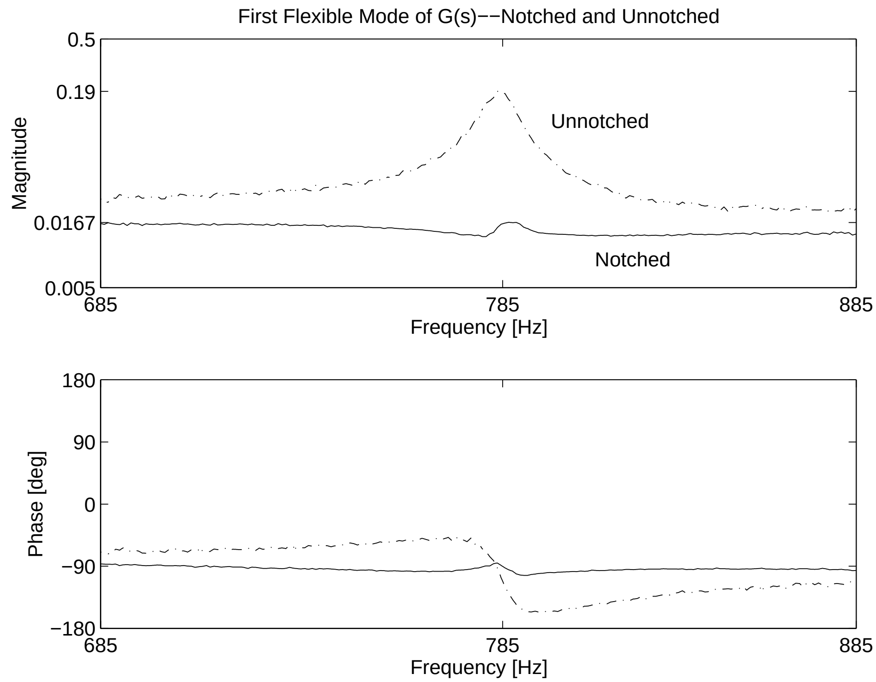

Figure 9: First Resonant Mode of Notched and Unnotched Systems

For comparison purposes, we also include the following plot of swept sine data obtained from the notch filter used to obtain the notched system response shown in Figure 9. This data is shown in Figure 10.

References

[1] Kuo, Benjamin C., **Automatic Control Systems**, 5th ed., Prentice-Hall, Inc., 1987, p. 568.

[2] Franklin, Gene F., J. David Powell and Abbas Emami-Naeini, **Feedback Control of Dynamic Systems**, 2nd ed., Addison-Wesley Publishing Co., 1991, pp. 265-270.

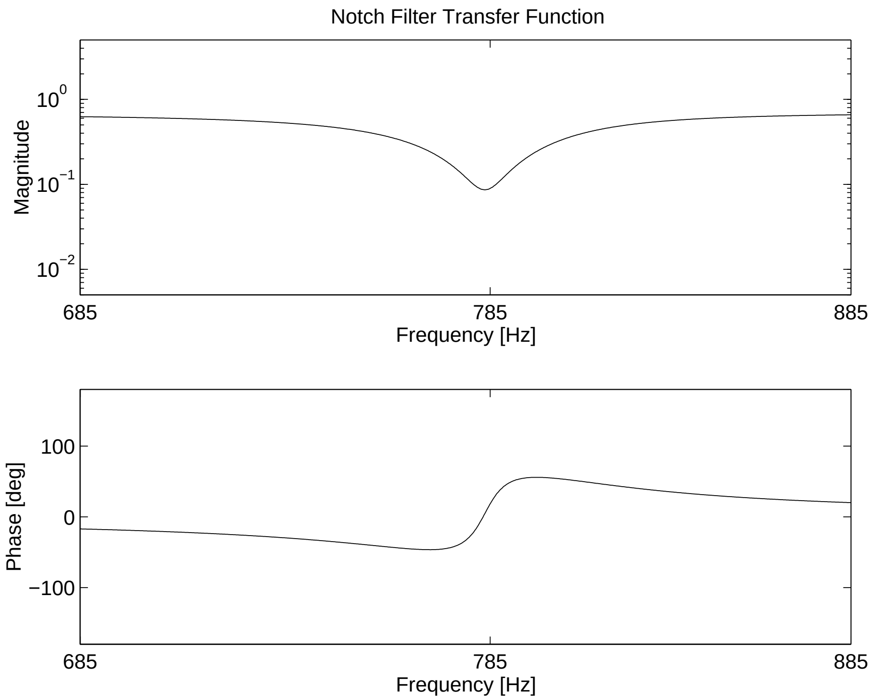

Figure 10: Notch Filter Transfer Function

Version	*Date*	*Modifications*
4	4/24/96	Definition for N modified on p. 2
		Statement of Nyquist Criterion changed on p. 3
		'clockwise' changed to 'counterclockwise' on p. 3
		Figure 4 modified to change inverting summer to a non-inverting summer on p. 8
		Op amp summer description similarly modified on p. 7
		Figure 6 modified for clarity
		Plots of typical data added in Figures 9 and 10
		Minor text modifications made