Magnetic Bearing Lab # 4:
Lead Controller Design for a Magnetic Bearing System*

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In the lab that follows, we will consider the design of a lead controller to stabilize one loop of the MBC 500 Magnetic Bearing system. We will first develop a general method for designing lead controllers for unstable systems. Then we will show how such a controller can be implemented using operational amplifiers, resistors and capacitors. After designing and building our lead controller, we will verify and fine tune its transfer function using the dynamic signal analyzer. Finally, we will connect the controller C with the previously designed notch filter N to the bearing system G as shown in Figure 1 and tune it to achieve stability and performance.
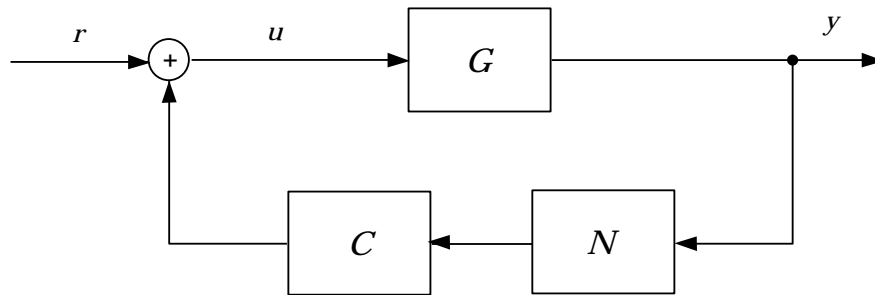

Figure 1: Magnetic Bearing Closed-loop Configuration

4.1   Lead Controller Design

In *Magnetic Bearing Lab # 2* entitled *Magnetic Bearing System Identification*, in Exercises 1-8, we collected data describing the frequency response of the magnetic bearing system from output OUT_{11} to output OUT_{21} as shown in Figure 2. A vector **g** of this complex frequency
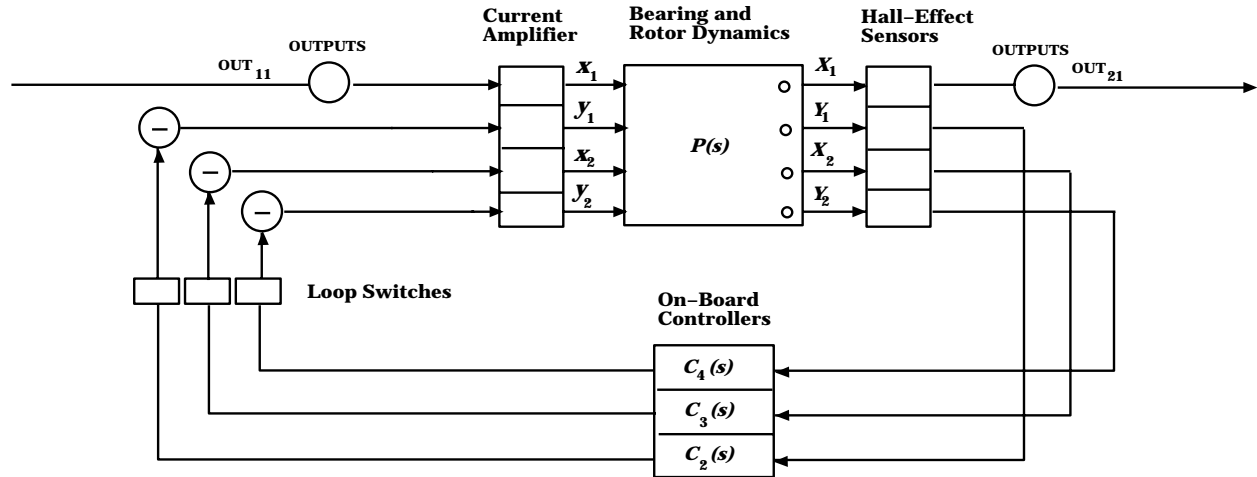

Figure 2: Magnetic Bearing Block Diagram

response and a vector `rps` of frequency points in rad/sec was obtained in MATLAB and will be used again in this lab. If this system identification lab was not previously performed, it will need to be performed before continuing with the current lab. In Exercise 9 of the system identification lab, a transfer function model fit to this data was obtained. Notice that this transfer function has one unstable pole. In *Magnetic Bearing Lab # 3* entitled *Notch Filtering of Resonant Modes*, we saw that for stability of such systems, the Nyquist Criterion requires that the Nyquist plot of the loop transfer function for the system encircle the point $(-1, 0j)$ once in the counter-clockwise direction. Given the system configuration shown in Figure 1, the loop transfer function is $G(s)N(s)C(s)$. Thus, we will need to design our controller $C(s)$ to produce one counter-clockwise encirclement of the point $(-1, 0j)$ by the product $G(j\omega)N(j\omega)C(j\omega)$ as ω varies from $-\infty$ to ∞.

Exercise 1: Use the system frequency response vector **g** created in Exercise 9 of *Magnetic Bearing Lab # 2* and the MATLAB routine below to plot the Nyquist plot of the bearing system response.

```
plot(-1,0,'go')

hold

plot(real(g),imag(g),'y')

plot(real(g),-imag(g),'m')

for i=1:100:800

plot(real(g(i)),imag(g(i)),'rx')

end

hold
```

Notice that this routine plots an "x" on the Nyquist plot for every one hundred frequency points. The x's allow us to track the path of the Nyquist plot as a function of frequency. The resulting plot should look something the plot shown in Figure 3.

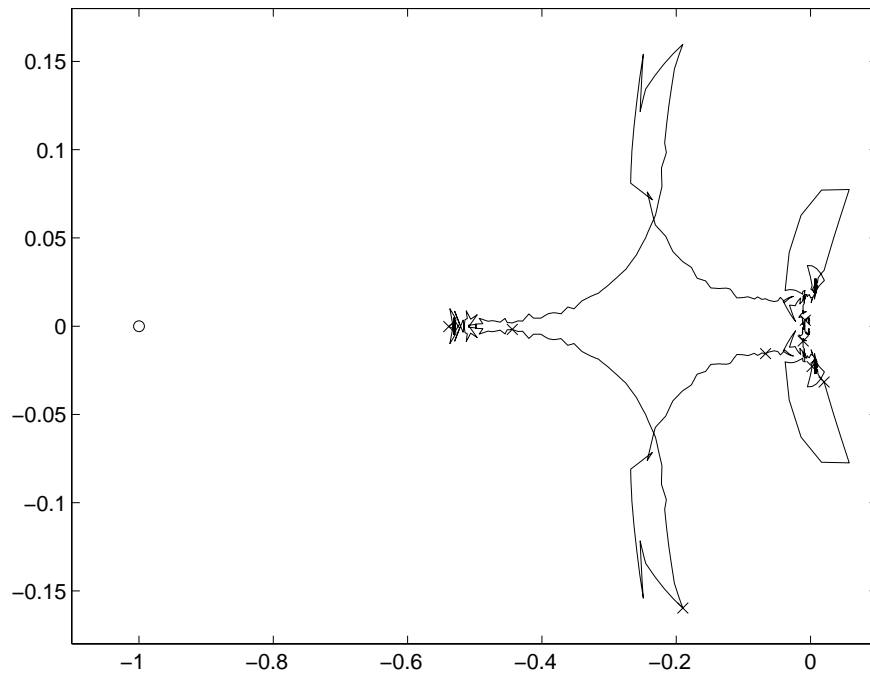Figure 3: Nyquist Plot of Bearing System

Our design objective is to choose $C(s)$ so that the Nyquist plot of the product $G(s)N(s)C(s)$ encircles the point $(-1, 0j)$ once in the counter-clockwise direction with a margin for error. Looking at the Nyquist plot of the system, we can determine the characteristics that a controller C(s) must have in order to satisfy this objective and hence stabilize the system. The

controller must have fairly high gain at DC in order to start the Nyquist plot outside of the point $(-1, 0j)$. Then, as the magnitude of the product $G(s)N(s)C(s)$ decreases, the phase must increase so that the Nyquist plot will circle widely around the $(-1, 0j)$ point. The wider the encirclement; the higher the gain and phase margins are for our system. For this reason, we consider a lead type of controller. The Bode plot for a typical lead controller is shown in Figure 4. Notice that the lead controller adds positive phase to the system in a given frequency range.
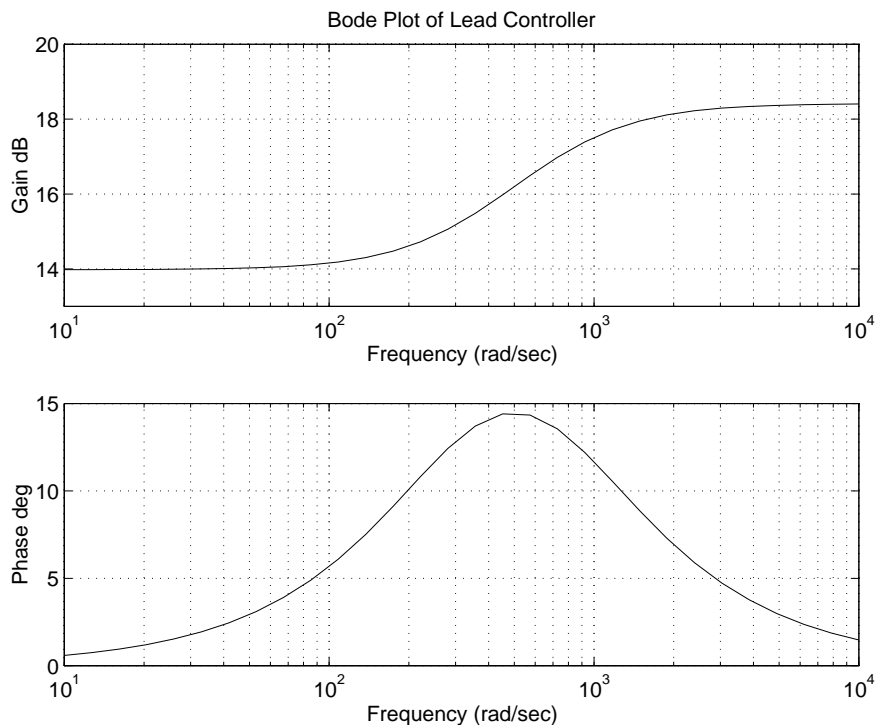
Figure 4: Transfer Function of a Lead Controller

We now consider the design of a simple lead controller. The general transfer function form of a lead controller is as follows:

$$C_{lead}(s) = k \frac{Ts + 1}{\alpha Ts + 1}$$

where $\alpha < 1$. The DC gain of this controller is the value k. Since the phase of $G(s)$ at $s = 0$ is -180°, the phase of $N(s)$ at $s = 0$ is 0°, and the phase of $C_{lead}(s)$ at $s = 0$ is 0°, the plot of $G(s)N(s)C_{lead}(s)$ will intersect the negative real axis at $s = 0$. Thus, we will first need to choose k to guarantee that our Nyquist plot begins comfortably outside the point $(-1, 0j)$.

Next we will plot the Nyquist plot for the system $G(s)N(s)k$ where the controller is equal to the constant gain k. From this Nyquist plot we will determine the new gain and phase margins for the system. Comparing the available phase margin to the desired phase margin, we can determine the additional phase which the controller must supply the system. If we call this additional phase ϕ, the following relationship holds:

$$\phi = desired\ phase\ margin - available\ phase\ margin.$$

Define ϕ_m to be the maximum phase over all frequencies which the lead controller can produce. ϕ_m satisfies the following relationship:

$$\sin\phi_m = \frac{1-\alpha}{1+\alpha}.$$

We then choose $\phi_m = \phi$ and use the above relationship to solve for the controller parameter α as follows:

$$\alpha = \frac{1-\sin\phi}{1+\sin\phi}.$$

We have yet to specify the frequency at which this maximum phase will occur for our controller. We will use our last undefined controller parameter T to specify this. We will denote the frequency at which the controller phase is maximum as ω_m. The relationship between ω_m and T is the following:

$$\omega_m = \frac{1}{\sqrt{\alpha}\,T}.$$

At the frequency ω_m, the controller supplies a gain of $\frac{1}{\sqrt{\alpha}}$. Thus, in order to identify the new gain cross-over frequency for our system, we must identify ω such that the magnitude

$$|G(j\omega)N(j\omega)k| = \sqrt{\alpha}.$$

If we choose ω_m to be this frequency, we will apply maximum controller phase to the new gain crossover point for our system. We can now solve for T from the relationship:

$$T = \frac{1}{\sqrt{\alpha}\,\omega_m}.$$

We summarize the steps for designing a lead controller below.

- Choose k so that $|G(s)N(s)k|$ at $s = 0$ is comfortably greater than one. For example, a DC gain of 2 is reasonable.

- Plot the Nyquist plot corresponding to $G(s)N(s)k$ and note the new gain and phase margins.

- Decide on a desired phase margin and compute

$$\phi = desired\ phase\ margin - available\ phase\ margin.$$

- Solve for $\alpha = \frac{1 - \sin \phi}{1 + \sin \phi}$.

- Find ω_0 such that $|G(j\omega_0)N(j\omega_0)k| = \sqrt{\alpha}$ and let $\omega_m = \omega_0$.

- Solve for $T = \frac{1}{\sqrt{\alpha}\omega_m}$.

Notice from the Bode plot of a typical lead controller given in Figure 4 that the magnitude of a lead controller never decays. Thus, high frequency noise is magnified by such a controller and will negatively effect the performance of the system. Thus, we would like to add an additional high frequency pole to our lead controller design to reduce the gain at high frequencies i.e. to filter out high frequencies. We will refer to this type of two-pole, one zero controller as a lead/filter controller and label our controller $C_{l/f}$. The lead/filter controller has a transfer function of the following form:

$$C_{l/f}(s) = k\frac{Ts + 1}{(\alpha Ts + 1)(T_0 s + 1)}$$

where typically T_0 is a decade or so greater than the lead controller pole.

Exercise 2: Using the procedure outlined above, design a lead/filter controller for the bearing system. For the bearing system frequency response $G(s)$, use the frequency response vector **g** and the frequency vector **rps**. For the purposes of this design, we will assume that the effect of the notch is insignificant at low frequencies which are of primary interest for stabilization. Therefore, for this exercise we assume $N(j\omega) = 1$ for all ω. Use MATLAB to plot the Nyquist plot and to check the gain and phase margins of the completed design. Iterate on the design process until the desired gain and phase

margins have been achieved. Now plot and print out the Bode plot of the resulting controller and note the frequency (in Hz), the magnitude and the phase of the controller at certain key frequency points. (For example, the following frequencies could be chosen: 10Hz, 100Hz, 300Hz, 700Hz, 1kHz, and 10kHz.) This information will be used later to tune your controller circuit.

4.2   Building the Lead/Filter Controller

One of the fundamental building blocks of analog controller circuits is the op amp in an inverting configuration as shown in Figure 5. Hence, we begin with a review of the transfer function corresponding to such circuits. In deriving the voltage properties of the op amp circuit of Figure 5, we assume that the voltage at the negative op amp input is virtually the same as the voltage at its positive input and yet no current passes into the op amp at either terminal. Therefore, the voltage at the negative input to the op amp is virtually zero volts and $i_2 = i_1$. Hence, we can write

$$i_2 = \frac{V_o - 0}{Z_2} = \frac{0 - V_i}{Z_1} = i_1.$$

Taking the Laplace transform of this relationship gives us the transfer function for this circuit. The Laplace transform of the impedances Z_1 and Z_2 are $Z_1(s)$ and $Z_2(s)$, respectively. Similarly, the Laplace transform of the voltages V_o and V_i are $V_o(s)$ and $V_i(s)$, respectively. The transfer function for Figure 5 is

$$T(s) = V_o(s)/V_i(s) = -\frac{Z_2(s)}{Z_1(s)}.$$

Next, since our lead/filter controller design makes use of capacitances in the inverting op amp configuration shown above, we will need to determine the Laplace transform representation of the impedance of a capacitor. Recall that the relationship between voltage, current, and capacitance is the following:

$$i = C\frac{dv}{dt}.$$

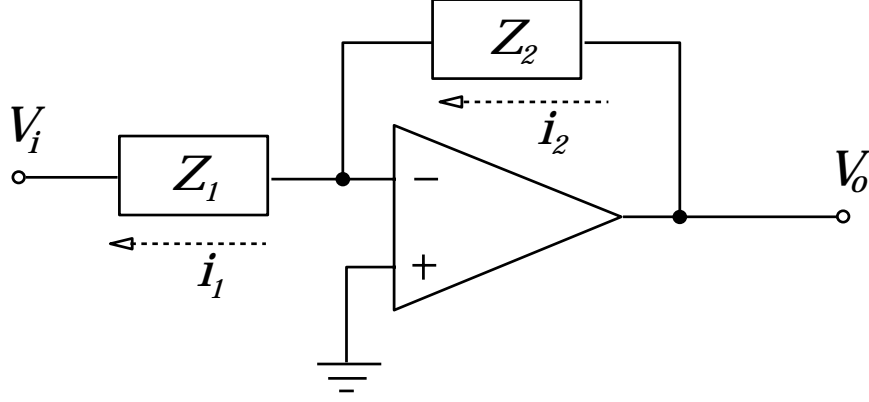Therefore, if we find the Laplace transform of this relationship, we have

$$I(s) = CsV(s)$$

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Figure 5: Inverting Op Amp Configuration

where $I(s)$ and $V(s)$ are the Laplace transforms of i and v, respectively. Thus, the impedance of a capacitor C expressed as a Laplace transform is

$$Z(s) = V(s)/I(s) = \frac{1}{sC}.$$

The circuit we have chosen for implementing our lead/roll-off controller is shown in the schematic of Figure 6. This same transfer function can be implemented in many different
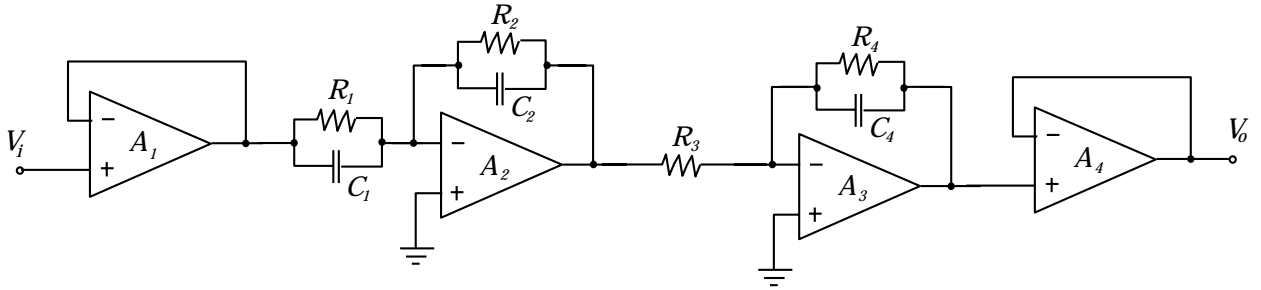

Figure 6: Lead/Roll-off Controller Schematic

op amp configurations; however, the configuration in Figure 6 has the advantage of being simple to tune. In order to derive the transfer function of this circuit, we will first convert the circuit to its Laplace domain representation. The two op amps A_1 and A_4 of Figure 6 function as unity gain buffer/followers which prevent the bearing system and notch filter from loading our lead/filter controller circuit. Thus, op amps A_1 and A_4 can be neglected in our transfer function derivation. The Laplace domain representation of our controller circuit is given in Figure 7 where Z_1, Z_2, and Z_4 are parallel combinations of a resistor and a capacitor. From our derivation above, these impedances are equal to
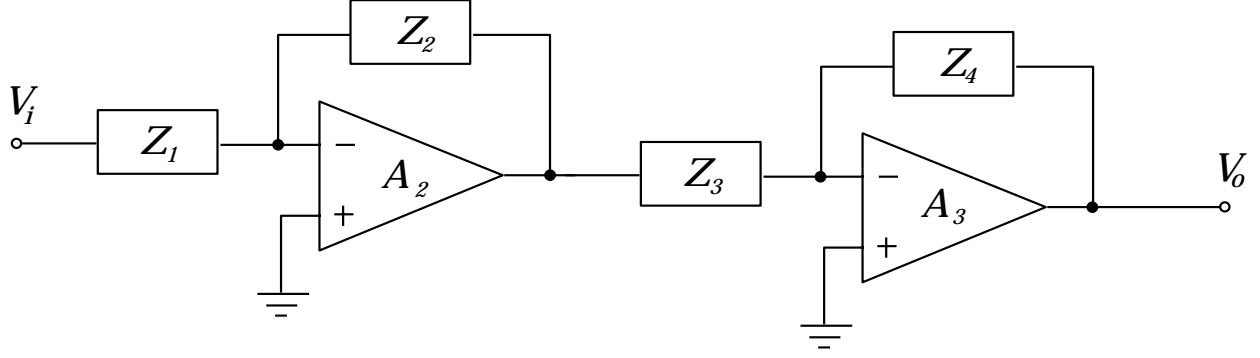
Figure 7: Laplace Domain Representation of Lead/Roll-off Controller

$$Z_k = \frac{1}{\frac{1}{(\frac{1}{sC_k})} + \frac{1}{R_k}} = \frac{R_k}{R_k C_k s + 1}$$

for $k = 1$, 2, and 4. This circuit configuration is simply a series connection of two inverting op-amps of the type shown in Figure 5. Thus, its transfer function is the product of two transfer functions of the form given for Figure 5. Thus, we have

$$C_{l/f}(s) = \frac{Z_2}{Z_1}\frac{Z_4}{Z_3} = \frac{R_2 R_4}{R_1 R_3}\frac{R_1 C_1 s + 1}{(R_2 C_2 s + 1)(R_4 C_4 s + 1)}.$$

Comparing this expression to our previous $C_{l/f}$ transfer function

$$C_{l/f}(s) = k\frac{Ts + 1}{(\alpha Ts + 1)(T_0 s + 1)}$$

we can see that the following mapping holds:

$$
\begin{aligned}
T &= R_1 C_1 \\
\alpha T &= R_2 C_2 \\
T_0 &= R_4 C_4 \\
k &= \frac{R_2 R_4}{R_1 R_3}.
\end{aligned}
$$

We have the following procedure for selecting our circuit components.

- Select C_1 such that R_1 is in the 1k to 10k range. Then select $R_1 = T/C_1$.

- Choose C_2 such that R_2 is in the 1k to 10k range. Then select $R_2 = \alpha T/C_2$.

- Now choose C_4 such that R_4 is in the 10k to 100k range. Note that C_4 may be very small. Typically it will be in the nanoFarad or picoFarad range. Then choose $R_4 = T_0/C_4$.

- Finally, select $R_3 = \frac{R_2 R_4}{R_1 k}$.

Each of the resistances should be implemented using potentiometers so that each parameter in the controller circuit can be easily adjusted during tuning. Note that resistor R_1 can be used to tune T, resistor R_2 can be used to tune αT, resistor R_4 allows us to adjust T_0, and resistor R_3 allows us to set the gain k.

Exercise 3: Build your controller using the schematic shown in Figure 6 choosing circuit components according to the method given above. Be sure to label each potentiometer with its circuit name R_1, R_2, R_3, or R_4.

Exercise 4: Now to verify the controller transfer function, we will plot its Bode plot using the dynamic signal analyzer. Connect the controller input to the signal analyzer "Source" signal. Also, connect the signal analyzer "Channel 1" input to the "Source" signal. Finally, connect the controller output to "Channel 2" of the signal analyzer. Using the procedure outlined in *Magnetic Bearing Lab # 2*, perform a swept sine analysis of your controller. Use the marker function of the signal analyzer to check the values of controller magnitude and phase at the frequencies you noted in Exercise 2. Make adjustments to the resistances R_1, R_2, R_3, and R_4 as necessary and repeat the swept sine analysis until the desired transfer function has been achieved.

4.3   Closing the Loop

When the controller has been designed and its transfer function has been verified using the dynamic signal analyzer, the real test of the controller can begin. It is time to put the controller in feedback with the bearing system and observe its performance.

Exercise 5: Connect the controller output to IN_1 of the bearing system and the output of the notch filter to the input of the controller. In addition, the bearing output OUT_{21}

should be connected to the input of the notch filter. Make sure that the Loop Switch for the outermost loop (x_1 to X_1) is in its open position. Now turn on the power to your controller and then turn on power to the bearing system. Observe the performance of the system.

At this point, if the rotor is levitated and spins freely when touched, you have achieved at least some form of stability. If the rotor is firmly fixed to one side of the bearings or another, then you have not yet achieved stability. The most likely cause of this type of instability is a gain that is either too high or two low. The gain can be adjusted by altering the R_3 potentiometer. Another type of instability that can occur involves unstable oscillations which cause the rotor to contact the touchdown bushings. The cause of this type of instability is likely to be higher frequency dynamics in the bearing/controller system. An adjustment to the controller transfer function will have to be made.

Exercise 6: If the bearing/controller system is not even stable briefly upon turning on the system, a controller redesign is necessary. A good first adjustment to make to the controller is to change the gain of the controller. Adjust the gain to the full extent of its range and observe the controller performance. Choose the gain which seems "best". If nominal stability is still not achieved for any gain setting, the other controller parameters will need to be adjusted as well. For the adjusting of the parameters, it is best to alter only one at a time. This provides some kind of organization to the tuning and allows the designer to make orderly comparisons of system performance for various parameter values. Continue tuning until nominal stability has been achieved.

Finally, even if nominal stability is achieved, the stability may not be robust. In other words, if disturbed slightly from the nominal operating conditions, the system may become unstable. One disturbance which commonly causes instability is the excitation of one of the flexible modes of the rotor. These modes can become excited during normal system operation or by an external disturbance to the rotor.

Exercise 7: Now we will test the robustness of our controller design and tune the design to improve the robustness if possible. First, turn on the bearing/controller system and

allow it to run for an extended period of time. If no instability occurs, gently turn the rotor and allow it to continue to run. If instability occurs because of extended operation, this can be caused by the bearing dynamics changing due to system components heating up. The dynamics will change only slightly, but if the bearing/controller stability is not robust, the change can be enough to cause instability. The controller may be tuned again at this point to improve the robustness of the system.

Exercise 8: A more stringent test of the robustness of the system can be performed by tapping the rotor (e.g. with a screwdriver handle) to apply an impulse disturbance. By tapping the rotor in its center, the first flexible mode of the rotor (and higher modes to a lesser extent) will be excited. Secondly, tapping the rotor at one fourth of its length from the end will excite primarily the second flexible mode. Perform each of these tests on your bearing/controller system. Vary the strength of your tap and note what happens to the system. Now run the bearing system with the on-board controller running. (In other words, close the first Loop Switch and disconnect your controller.) Perform the same tests and compare the response of the on-board controller to the response of your controller design. Continue to tune your controller until an acceptable level of robustness has been achieved. Now measure and record the final resistance values and note the capacitance values used for this final implementation. Calculate the resulting controller transfer function by determining the values of the parameters k, α, T and T_0.