The Pendubot: A Mechatronic System for Control Research and Education

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Abstract[1]

In this paper we describe the *Pendubot*, a mechatronic device for use in control engineering education and for research in nonlinear control and robotics. This device is a two–link planar robot with an actuator at the shoulder but no actuator at the elbow. With this system, a number of fundamental concepts in nonlinear dynamics and control theory may be illustrated. The pendubot complements previous mechatronic systems, such as the Acrobot [3] and the inverted pendulum of Furuta [4].

1 Introduction

In this paper we discuss the design, and control of the *Pendubot*, a two–link, underactuated robotic mechanism that we are using for research in nonlinear control and to educate students in various concepts in nonlinear dynamics, robotics, and control system design. The novelty of our system lies in the ease with which we are able to demonstrate advanced concepts such as partial feedback linearization and zero dynamics, both as a vehicle for research, and as an instructional device.

2 Description of the Hardware

Figure 1 shows a drawing of the Pendubot.

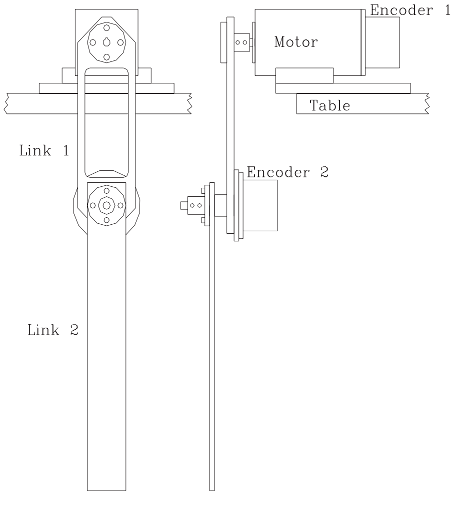

Figure 1: a. Front and Side Perspective Drawings of The Pendubot

The Pendubot consists of two rigid aluminum links of lengths 14in and 8in, respectively. Link 1 is directly coupled to the shaft of a 90V permanent magnet DC motor mounted to the end of a table. The motor mount and bearings are then the support for the entire system. Link 1 also includes the bearing housing for joint two. Needle roller bearings riding on a ground shaft were used to construct the revolute joint for link 2. The shaft extends out both directions of the housing allowing coupling to the second link and to an optical encoder mounted on link one. The design gives both links full 360^o of rotational motion. Link 2 is constructed of a $\frac{1}{4}$–inch (0.635 cm) thick length of aluminum with a coupling that attaches to the shaft of joint two.

All of our control computations are performed on a Dell 486DX2/50 PC workstation with a D/A card and an encoder interface card. Using the standard software library routines supplied with these interface cards we are able to program control algorithms directly in C.

3 Dynamics

Since our device is a two link robot (with only one actuator) its dynamic equations can be found in numerous robotics textbooks as

$$\begin{aligned} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + h_1 + \phi_1 &= \tau &\quad (1)\\ d_{21}\ddot{q}_1 + d_{22}\ddot{q}_2 + h_2 + \phi_2 &= 0 &\quad (2) \end{aligned}$$

where q_1, q_2 are the joint angles and τ is the input torque. The important distinction then between the system (1)–(2) and a standard two–link robot is, of course, the absence of a control input torque to the second equation (2). Underactuated mechanical systems generally have equilibria which depend on both their kinematic and dynamic parameters. If the Pendubot is mounted so that the joint axes are perpendicular to gravity, then there will be a continuum of equilibrium configurations, each corresponding to a constant value, $\bar{\tau}$, of the input torque τ. These equilibria are characterized by the second link vertical for any position of the first link.

4 Identification

The dynamic parameters of the Pendubot were identified two ways: using the AutoCAD drawings, and on–line using the Hamiltonian based approach of Gautier and Khalil [2]. Because of the low friction in our device

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the Hamilton based approach to identification works remarkably well.

4 The Pendubot as an Inverted Pendulum

In this section we discuss the application of the Pendubot as an inverted pendulum in which the motion of the actuated first link is used to balance the second link. This system is distinct from the more classical cart–pole system in which the linear motion of the cart is used to balance a pendulum. Our system may also be contrasted with the recent and very elegant variation due to Furuta [4] which mounts the pendulum at the end of a horizontally rotating first link. One may think of Furuta's device as a two link underactuated robot arm with the two joint axes orthogonal, while our Pendubot has the joint axes parallel.

4.1 Controllability and Balancing

The balancing problem for the Pendubot may be solved by linearizing the equations of motion about an operating point and designing a linear state feedback controller, very similar to the classical cart–pole problem, so we will only give a few remarks here. One very interesting distinction of the Pendubot over both the classical cart–pole system and Furuta's system is the continuum of balancing positions. This feature of the Pendubot is pedagogically useful in several ways, to show students how the Taylor series linearization is operating point dependent and for teaching controller switching and gain scheduling. Students can also easily understand physically how the linearized system becomes uncontrollable at $q_1 = 0, \pm\pi$. As the Pendubot approaches this uncontrollable configuration, the controllability matrix becomes increasingly ill–conditioned.

4.2 Swing Up Control

The problem of swinging the Pendubot up from the downward configuration to the inverted configuration is an interesting and challenging nonlinear control problem. With this problem one may illustrate the nonlinear control ideas of nonlinear relative degree, partial feedback linearization and zero dynamics.

We have used the method of partial feedback linearization to swing up the Pendubot, i.e., to move from the stable equilibrium $q_1 = -\pi/2$, $q_2 = 0$, to the inverted position $q_1 = \pi/2$, $q_2 = 0$. The same approach can easily be modified to swing the system to any point on its equilibrium manifold. The control is switched to a second control to balance the Pendubot about the equilibrium whenever the swing up controller moves it into the basin of attraction of the balancing controller. The actual control design and analysis is similar to our previous work with the Acrobot [3].

The experimental results have been very good. As a balancing controller we have used linear quadratic methods. Figure 2 shows the Pendubot swinging and balancing at its top position using a partial feedback linearization swingup strategy.

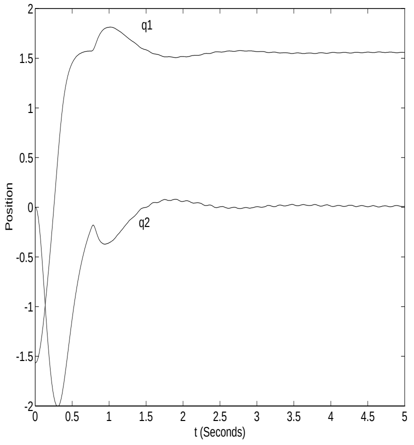

Figure 2: Swing-Up and Balance Control at the Top Position

5 Conclusions

This paper presents our concept of a two link underactuated planar revolute robot, named the Pendubot. This system is useful both in research and for instruction in controls. Students at all levels may benefit from this system. Various aspects of control theory from local linear state feedback to balance the Pendubot in one position to more complex global nonlinear controllers to swing up the Pendubot are easily illustrated. The reader is referred to [1] for additional details. In addition the full version of this paper is available upon request.

References

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