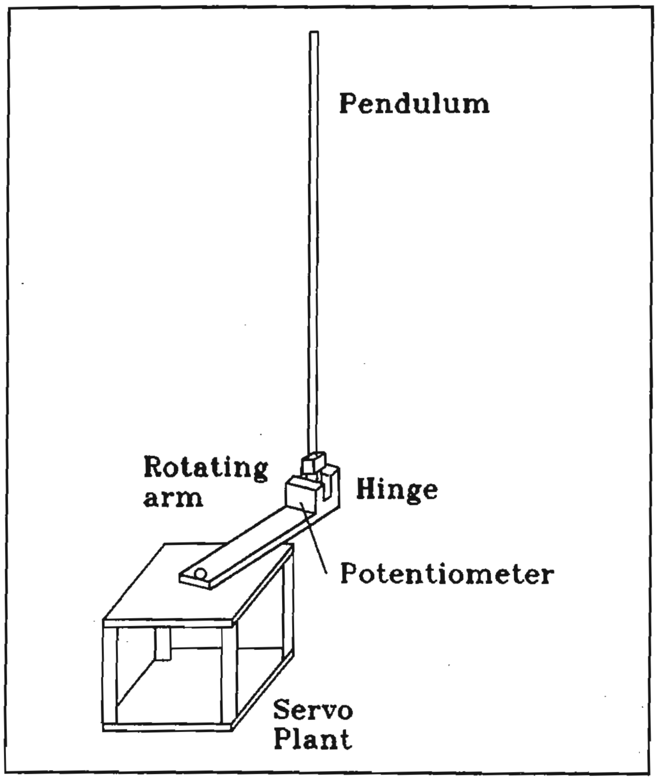
2.3.1 DESCRIPTION

The Rotary Inverted Pendulum module shown in Figure RP1 consists of a flat arm at the end of which is a hinged potentiometer. The inverted pendulum is mounted to the hinge. Measurement of the pendulum angle is obtained using a potentiometer. The objective of the experiment is to design a feedback control system that positions the arm as well as maintains the inverted pendulum vertical. This problem is similar to the classical inverted pendulum except that the trajectory is circular.



RP 1 Schematic of Rotary Inverted Pendulum attached to Servo plant SRV-02

2.3.2 MATHEMATICAL MODEL

Consider the simplified diagram in Figure RP2. The kinetic and potential energies in the system are given by:

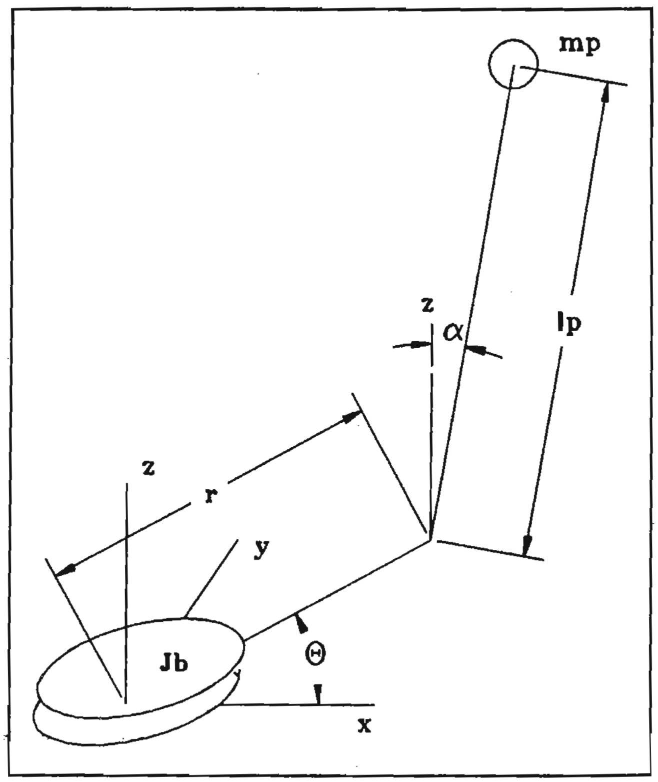
$$PE_{pen} = m_p \, g \, l_p \cos(\alpha)$$
$$KE_{pen} = 0.5 \, m_p \, [(\dot{\theta} r + \dot{\alpha} \, l_p \cos(\alpha))^2 + (\dot{\alpha} \, lp \sin(\alpha))^2]$$
$$KE_{base} = 0.5 \, J_b \, \dot{\theta}^2$$

Using the above and the Lagrangian formulation (SEE ROTPEN.MAP" we obtain the differential equations of the system:

$$(m_p r^2 + J_b)\,\ddot{\theta} + m_p r \ddot{\alpha} \, l_p \cos(\alpha) - m_p r \dot{\alpha}^2 l_p \sin(\alpha) = T$$
$$m_p l_p \cos(\alpha)\,\ddot{\theta}\, r - m_p l_p \sin(\alpha)\,\dot{\alpha}\,\dot{\theta}\, r + m_p \ddot{\alpha}\, l_p^2 - m_p g \, l_p \sin(\alpha) = 0$$

as well as the linear model given below:

$$\begin{bmatrix}\dot{\theta}\\ \dot{\alpha}\\ \ddot{\theta}\\ \ddot{\alpha}\end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -\dfrac{m_p r g}{J_b} & 0 & 0\\ 0 & g\dfrac{J_b + m_p r^2}{l_p J_b} & 0 & 0 \end{bmatrix} \begin{bmatrix}\theta\\ \alpha\\ \dot{\theta}\\ \dot{\alpha}\end{bmatrix} + \begin{bmatrix} 0\\ 0\\ \dfrac{1}{J_b}\\ -\dfrac{r}{l_p J_b}\end{bmatrix} T$$



RP 2 Simplified model for rotary inverted pendulum

Using the constants given for the system we obtain the state description with a Torque as input is given as:

$$\begin{bmatrix}\dot\theta\\ \dot\alpha\\ \ddot\theta\\ \ddot\alpha\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -67 & 0 & 0\\ 0 & 64 & 0 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\\ \dot\theta\\ \dot\alpha\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 258\\ -62\end{bmatrix}\tau$$

The torque is generated by a DC motor with the following equations:

$$V = I R_m + K_m \omega_m = I R + K_m K_g \omega \text{ ; since } \omega_m = K_g \omega$$

$$\text{Thus} \quad I = \frac{V}{R} - \frac{K_m K_g}{R}\omega$$

$$\text{But} \quad \tau = K_g \tau_m = K_g K_m I$$

$$\text{Then} \quad \tau = \frac{K_m K_g}{R} V - \frac{K_m^2 K_g^2}{R}\omega$$

substituting this and the system parameters into the matrix equation we have:

$$\begin{bmatrix}\dot\theta\\ \dot\alpha\\ \ddot\theta\\ \ddot\alpha\end{bmatrix} = \begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ 0 & -67 & -25 & 0\\ 0 & 64 & 12 & 0\end{bmatrix}\begin{bmatrix}\theta\\ \alpha\\ \dot\theta\\ \dot\alpha\end{bmatrix} + \begin{bmatrix}0\\ 0\\ 47\\ -22\end{bmatrix} V$$

which is the desired representation.

2.3.4 CONTROL SYSTEM DESIGN

Using MATLAB (lqr design) we obtain the optimal feedback gain K for the feedback law:

$$V_{in} = -(k_1\,\theta + k_2\,\alpha + k_3\,\theta' + k_4\,\alpha')$$

such that the closed loop system:

$$A_c = A\text{-}bK$$

minimizes the quadratic performance index:

$$J = \int (x'Qx + rV^2)\,dt$$

with

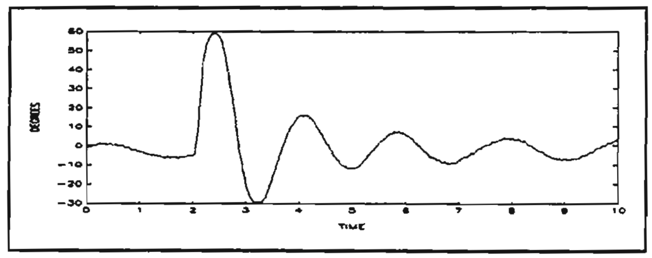
$$Q = \begin{bmatrix}0.25 & 0 & 0 & 0\\ 0 & 4 & 0 & 0\\ 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0\end{bmatrix} \quad \text{and} \quad r = 0.05$$

The optimal gain is given by:

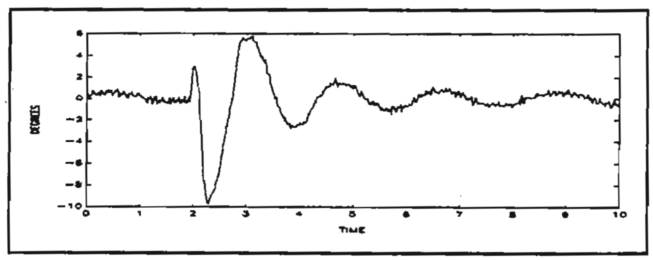
K = [-2.23 -31.24 -2.01 -5.0] V/rad.
K = [-.0391 -.54 -.035 -.087] V/deg.

2.3.5 RESULTS

The disturbance response of the system is shown in Figures 3 and 4. The system is responding to a tap to the pendulum. Figure RP3 is the response of the arm while Figure RP4 is the pendulum angle. Note the non-minimum phase response in the pendulum angle. The pendulum is tapped such that it falls around 2.5 degrees which causes the arm to move towards the falling direction. This results in the pendulum swinging to about 10 degrees in the opposite direction. The system recovers in about 5 seconds.



RP 3 Disturbance response of Rotary Inverted Pendulum arm to a tap on the pendulum.



RP 4 Disturbance response of pendulum to a tap. Note the initial disturbance is about 2 degrees but the pendulum swings to 10 degrees to recover.

CAUTION

To zero the angle measurement you should hold the pendulum vertical (with the motor turned off) and hit the letter 'z' from the main menu of the controller. This takes the present measurement as zero. Do this before you start the controller. Always start the controller with the pendulum held vertical!

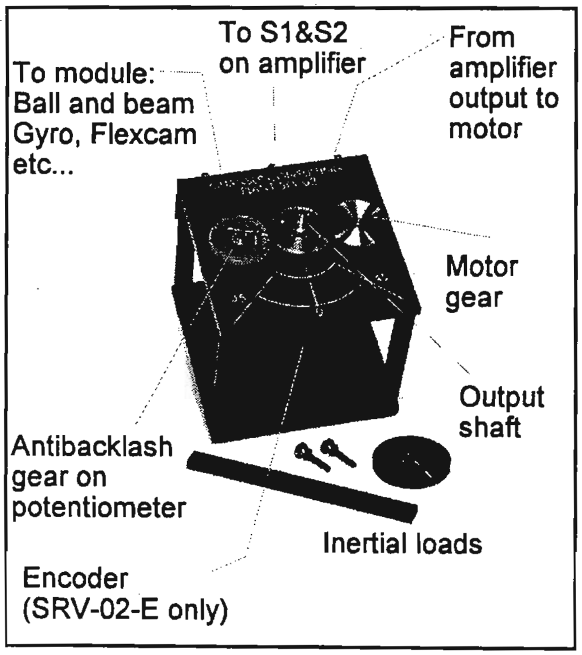
2.3 Actuator modules

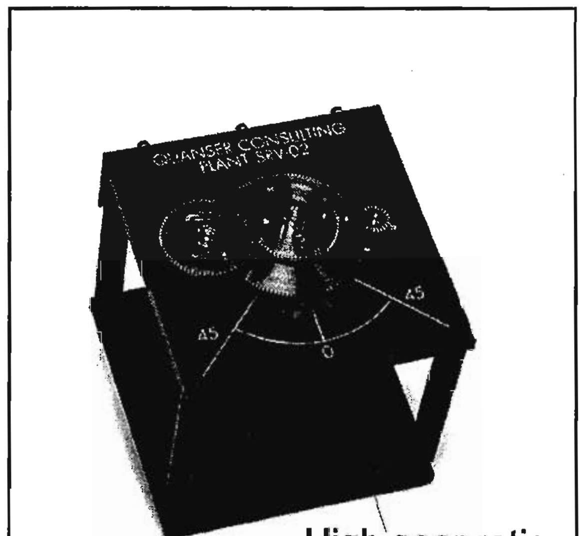
2.3.1 Rotary Motion SRV02 & SRV02-E

The SRV02 and SRV02-E are essentially the same except that the SRV02-E is furthermore equipped with an encoder coupled directly to the output shaft.

The SRV02(-E) can be converted to the "high gear ratio" configuration shown on the right. In order to do this you must remove all the gears from the low gear ratio configuration and reassemble the system as shown above.

In order to calibrate the potentiometer measurement to the "zero" of the system, attach the 6 pin mini din cable from the Servo module (either connector) to the S1&S2 connector on the amplifier. Measure the voltage at the terminal labelled S1 on the amplifier with respect to the ground terminal. Loosen the set screw on the antibacklash gear and rotate the potentiometer shaft while holding the central gear at the zero position.Tighten the screw when the measurement is within 50 millivolts of zero volts.





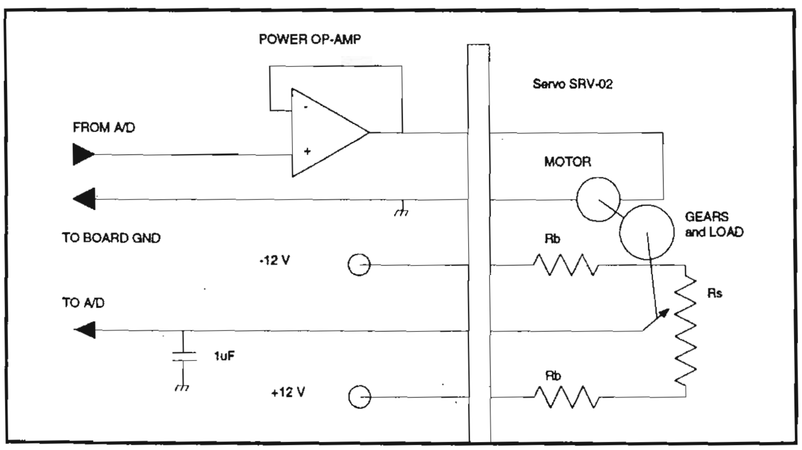
High gear ratio configuration

Antibacklash gears: These gears have two surfaces coupled via a spring. In order to couple them properly to another gear, you rotate the two surfaces against the internal spring and insert the gear. You then tighten the screw. In this manner, the teeth of either side are in contact with the other gear.

5.1.1 ROTARY SERVO SRV-02

PARAMETER	SYMBOL	VALUE	UNITS	PARAMETER	SYMBOL	VALUE	UNITS
Motor Torque Constant Back EMF constant	Kt,Kb,Km	0.00767	Nm/amp V/(rad/sec)	120 tooth gear inertia	J120	$2.27e^{-5}$	Kgm^2
Efficiency	Eff	0.9		72 tooth gear inertia	J72	$1.4e^{-6}$	Kgm^2
Armature resistance	Rm	2.6	Ω	24 tooth gear inertia	J24	$1.0e^{-7}$	Kgm^2
Armature inductance	Lm	0.18	mHenry	Poten-tiometer Resistance	Rs	10	KΩ
Maximum voltage	Vmax	6.0	Volts	Bias voltage	Vb	+/- 12	Volts
Internal Gear Ratio	Kgi	14:1		Bias resistors	Rb	7.15	KΩ
External Gear ratio (low) External Gear ratio (high)	Kge	1:1 5:1		Range	N/A	+/- 176	Deg
Gear ratio	Kg = Kgi*Kge	14:1 70:1		Sensitivity	S	0.0284	V/deg
Load inertia Disc Arm at centre Arm at end	Jl	0.00003 0.000275 0.001	Kgm^2	Armature Inertia	Jm	$3.87e^{-7}$	Kgm^2

The equivalent circuit that is achieved using the Quick Connect Module (QCM) is shown below in figure WI 1.



WI- 1 Wiring the SRV-02 servo motor & potentiometer

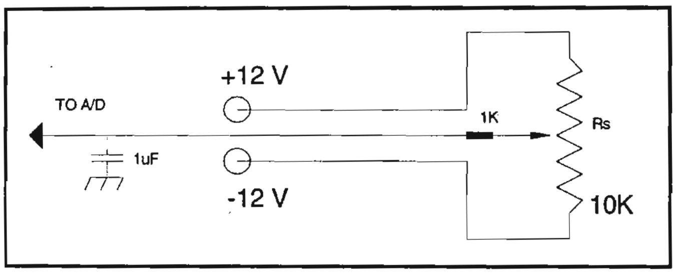
CAUTION

High frequency voltage applied to a (any) motor will eventually damage the gearbox or the brushes. The most likely source for high frequency noise is derivative feedback. If the derivative feedback gain is too high, a noisy voltage will be fed to the motor. You should have a band-limited differentiator rather than a pure differentiator running in the feedback loop (See section 4.5). If you hear a "buzz" in the motor you are feeding high frequency noise to the motor and will damage it. Turn the motor off immediately and reconsider your design! Select a low pass filter frequency that eliminates the "buzz" or reduce the derivative gain. Always have an anti-aliasing filter connected to the input of the A/D. This could simply be a capacitor as shown in the wiring diagrams. The capacitor will filter out high frequency noise before it is processed.

5.1.3 ROTARY INVERTED PENDULUM

PARAMETER	SYMBOL	VALUE	UNITS
ARM LENGTH	r	0.145*	m
ARM INERTIA	J_b	0.0044*	Kgm^2
PENDULUM LENGTH	$L_p=2l_p$	0.61	m
PENDULUM MASS	m_p	0.210	Kg
ANGLE SENSOR RESISTANCE	Rs	10	KΩ
BIAS RESISTORS	Rb	0	Ω
BIAS VOLTAGE	Vb	+/- 12	Volts
SENSISTIVITY	S	0.068	V/deg
RANGE		+/- 26	Deg

The equivalent circuit that is achieved using the Quick Connect Module (QCM) is shown below in figure WI 3.



WI- 3 Wiring diagram for the Rotary Inverted Pendulum sensor

CAUTION

To zero the angle measurement you should hold the pendulum vertical(with the motor turned off) and hit the letter 'z' from the main menu of the controller. This takes the present measurement as zero. Do this before you start the controller.

Always start the controller with the pendulum held vertical!