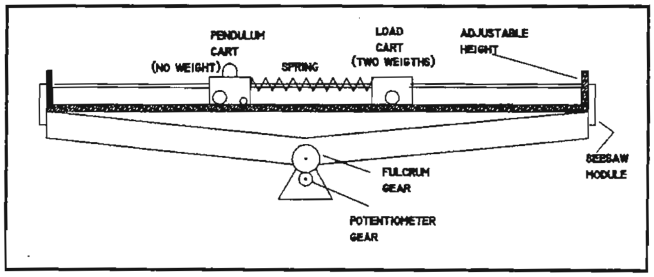
3.6.1 DESCRIPTION

This experiment is a variation on the Seesaw experiment. It combines the two masses and a spring with the seesaw experiment to obtain a sixth order system. The purpose of the experiment is to design a controller that balances the seesaw.

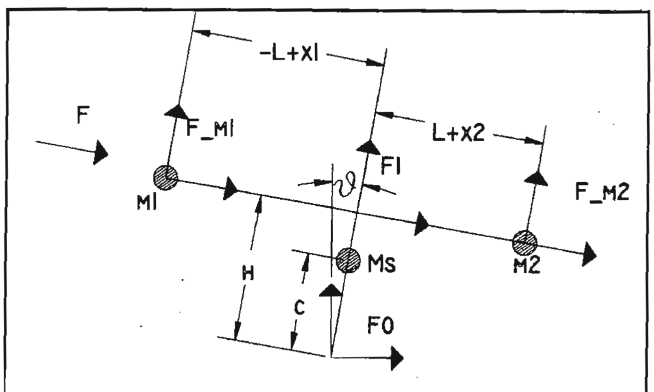
In order to assemble the system, you must first assemble the linear flexible joint as described in section 3.2. You then mount the structure on the seesaw as in the seesaw experiment to obtain the system shown in Figure LFJS1.



LFJS1 Assembly of Seesaw with flexible structure

3.6.2 MATHEMATICAL MODEL

Consider the simplified diagram of figure LFJS2 and the following definitions for the coordinate frames of interest.



LFJS2 Simplified model

The kinetic and potential energies of the moving elements are obtained as follows:

Consider the coordinate frames defined in Figure LFJS2. Using transformation matrices we have the following transformations:

$$T^{01} = \begin{bmatrix} \cos(\theta) & 0 & -\sin(\theta) & h\sin(\theta) \\ 0 & 1 & 0 & 0 \\ \sin(\theta) & 0 & \cos(\theta) & h\cos(\theta) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{1\,m1} = \begin{bmatrix} 1 & 0 & 0 & -\frac{L}{2}+x_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T^{1\,m2} = \begin{bmatrix} 1 & 0 & 0 & \frac{L}{2}+x_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The last column in each matrix represents the position of the frame relative to the previous frame.

The transformation $T^{m1} = T^{01}\, T^{1\,m1}$ represents the position and orientation of the motor cart relative to the base frame and $T^{m2} = T^{01}\, T^{1\,m2}$ is the position and orientation of the load cart attached relative to the base frame.

Defining

$$T^{01}\, T^{1\,m1} = \begin{bmatrix} & [R^{m1}] & & P_{m1}^{\ x} \\ & & & 0 \\ & & & P_{m1}^{\ z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(Note that there is no motion along the 'y' direction).

Then, the kinetic energy of the motor cart is given by:

$$KE_{m1} = 0.5\ M_1 \left(\left[\frac{\delta P_{m1}^{\ x}}{\delta t}\right]^2 + \left[\frac{\delta P_{m1}^{\ z}}{\delta t}\right]^2 \right)$$

and the potential energy of the motor cart is given by:

$$PE_{m1} = M_1 \; g \; P_{m1}^{\;z}$$

similarly for the load cart we have :

Defining

$$T^{01} \; T^{1\;m2} = \begin{bmatrix} & [R^{m2}] & & P_{m2}^{\;x} \\ & & & 0 \\ & & & P_{m2}^{\;z} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Then, the kinetic energy of the load cart is given by:

$$KE_{m2} = 0.5 \; M_2 \; \left(\left[\frac{\delta P_{m2}^{\;x}}{\delta t}\right]^2 + \left[\frac{\delta P_{m2}^{\;z}}{\delta t}\right]^2 \right)$$

and the potential energy of the motor cart is given by:

$$PE_{m2} = M_2 \; g \; P_{m2}^{\;z}$$

The potential energy in the spring

$$PE_{spring} = 0.5 \; K \; (x_1 - x_2)^2$$

The kinetic energy of the cart is given by:

$$KE_{seesaw} = 0.5 \; J_s \; \dot{\theta}^2$$

while the potential energy of the seesaw is:

$$PE_{seesaw} = 0.5 \; M_s \; g \; c \; \cos(\theta)$$

All the above equations are implemented in the MAPLE program SEEFLEX.MAP. The program computes the Lagrangian about each independent axis and derives the nonlinear differential equations. The nonlinear differential equations are written to disk. The program also linearizes the differential equations about the operating point (0,0,0). The linearized model results in the matrix equation:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ a_{41} & a_{42} & a_{43} & 0 & 0 & 0 \\ a_{51} & a_{52} & a_{53} & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ b_4 \\ b_5 \\ b_6 \end{bmatrix} F$$

The entries in the matrices are too complex to be included here but are derived in SEEFLEX.MAP and d written to the file SEEF_M.M which is later used in MATLAB to design the controller.

3.7.3 CONTROL SYSTEM DESIGN

Substituting system parameters into the matrix equation we obtain:

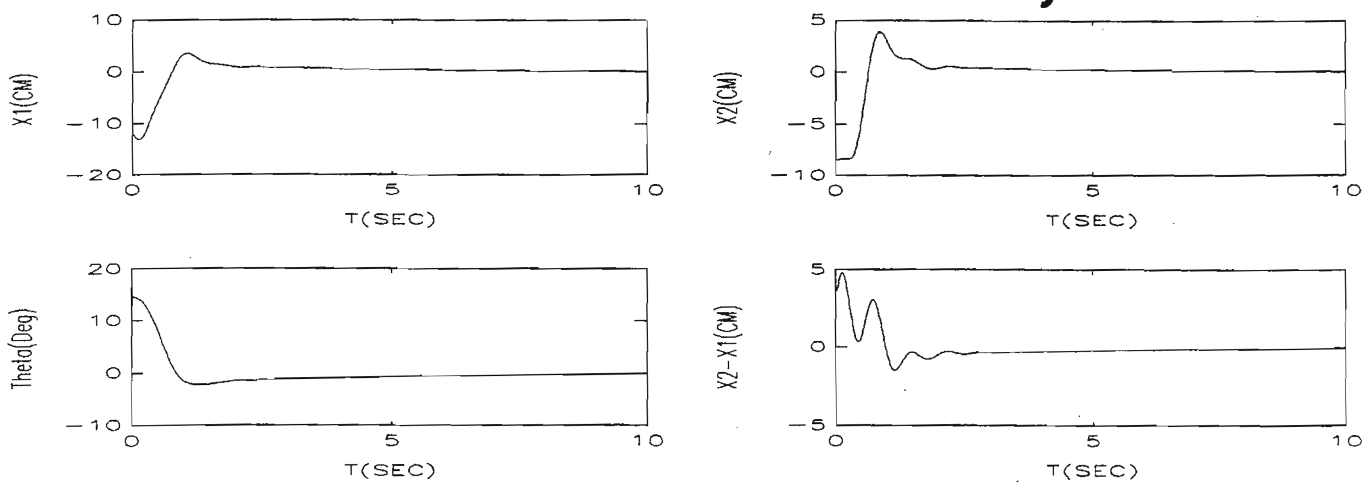
$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{\theta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -66.6 & 62.9 & 9.22 & 0 & 0 & 0 \\ 58.4 & -62.1 & 9.22 & 0 & 0 & 0 \\ 12.8 & 13.6 & 4.1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.73 \\ 0.04 \\ -0.3 \end{bmatrix} F$$

The force input must be converted to a voltage input since the motor is driven by a voltage (see section 3.1.2):

$$F = \frac{T}{r} = \frac{K_m K_g I_m}{r} = \frac{K_m K_g}{Rr} V - \frac{K_m^2 K_g^2}{Rr^2} \dot{x}$$

substituting parameter values into the matrix equation and introducing an integrator for θ results in:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{\theta} \\ \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{\theta} \\ \dot{\zeta} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ -66.6 & 62.9 & 9.22 & -13.5 & 0 & 0 & 0 \\ 58.4 & -62.1 & 9.22 & -0.33 & 0 & 0 & 0 \\ 12.8 & 13.6 & 4.1 & 2.4 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \theta \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{\theta} \\ \zeta \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 2.9 \\ 0.07 \\ -0.52 \\ 0 \end{bmatrix} V$$



LFJS4 Response of linear model to initial conditions [-12, -8.5, 14.4, 0, 0, 0]

The purpose for the integrator is the following: Since the two masses are not exactly equal and since you will be calibrating the zero for the two masses by hand, there will be constant disturbances in the system which will result in a steady state error in θ. The integrator for θ will eliminate the steady state error.

An LQR controller is designed using the program SEEFLEX.M. The Q and r weighting factors chosen are:

$$Q = diag\ (1000\ 1000\ 3000\ 0\ 0\ 0\ 300)$$
$$r = 20$$

resulting in the feedback gains:

$$K = [67\ \ 30\ \ 56\ \ 5.8\ \ 7.9\ \ 16.3\ \ 3.8]$$

for units in metres and radians.

$$K = [.67\ \ .30\ \ .98\ \ .06\ \ .08\ \ .28\ \ .07]$$

for units in cm and degrees

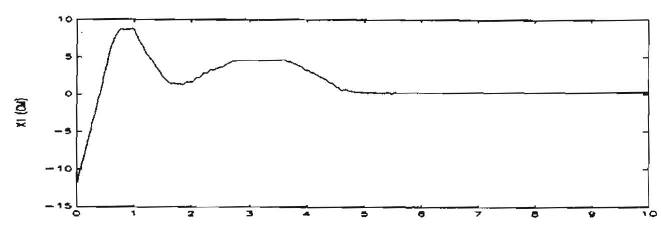
The closed loop eigenvalues for the above gain are:

$$[-1.8 \pm j\ 9.1],\ [-2.9 \pm j\ 3.3], -8.5, -4.4, -.26$$

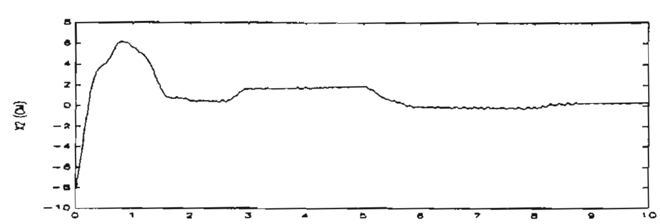
Figure LFJS4 shows the response of the simulated system to an initial condition Xi = [-12, -8.5, 14.4, 0, 0, 0] (cm. and Degrees).

3.7.4 RESULTS

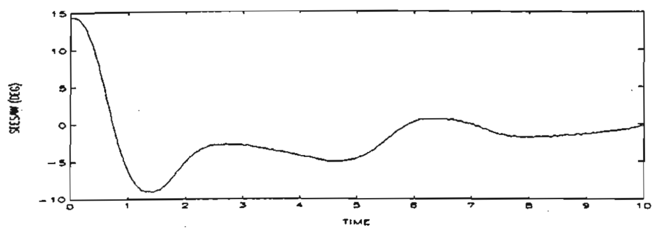
The above controller was implemented on the actual system and figures LFJS5, LFJS6 and LFJS7 show the response to the initial condition simulated using the linear model. The system is stable and behaves generally like the predicted model. There is however a limit cycle due to friction and a steady state error due to non-exact zero positions. The average steady state error is slowly reduced thanks to the integrator but the limit cycle cannot be completely eliminated. The amplitude of the limit cycle may be reduced by increasing the gain in the system but at the risk of introducing instability.



LFJS5 Motor cart position response to the initial condition described in the text.



LFJS6 Load cart position response to initial condition.



LFJS7 Seesaw angle response to initial condition

NOTES ON STARTING THE SYSTEM:

a) start the controller program with the motor turned off.
b) slide the masses such that the spring is approximately centered and hold the seesaw horizontal. Make sure the potentiometers do not reach physical limits in the range of motion of the carts
c) hit the letter 'z'. This takes the present measurements as zero.
d) hit 'o', this starts the controller.
e) when the system is balanced and the seesaw is at zero degrees approximately (the integrator should be turned on), hit the letter 'z' again. Since the seesaw angle is approximately zero, the positions x1 and x2 are now in the correct 'zero' position. Hitting 'z' also resets the integrator to zero.
f) to study the disturbance response, push the seesaw slowly to one end while the controller is running and let go. (This is how the initial conditions were obtained for the above plots.

Reference on transformation matrices

Richard P. Paul Robot manipulators: Mathematics, Programming and Control. The MIT Press, 1981.

SYSTEM PARAMETERS
5.2.6 LINEAR FLEXIBLE JOINT WITH SEESAW / SEEFLEX

The parameters for this experiment are the same as the Seesaw and the Linear flexible joint .

The equivalent circuit that is achieved using the Quick Connect Module (QCM) is shown in the figures for the Seesaw and the Linear Flexible Joint.

SYSTEM PARAMETERS
5.2.7 FLEXIBLE INVERTED PENDULUM

PARAMETER	SYMBOL	VALUE	UNITS
STFFNESS	K_s	1.5704	Nm/rad
BEAM LENGTH	Lb	0.41	m
BULB MASS	Mb	.05	Kg
LOAD MASSES (EACH)	Ml	0.1	Kg
CAMERA INERTIA	Jc	0.00032	Kgm2
ATTACHMENT HEIGHT	h	0 (adjustable)	m
CAMERA MASS	Mc	0.3	Kg

All other parameters are obtained from the inverted pendulum experiment and the flexible link FLEXCAM experiment.

This experiment is wired just like the inverted pendulum and the FLEXCAM experiment. The equivalent circuit that is achieved using the Quick Connect Module (QCM) is shown in the two respective wiring diagrams.