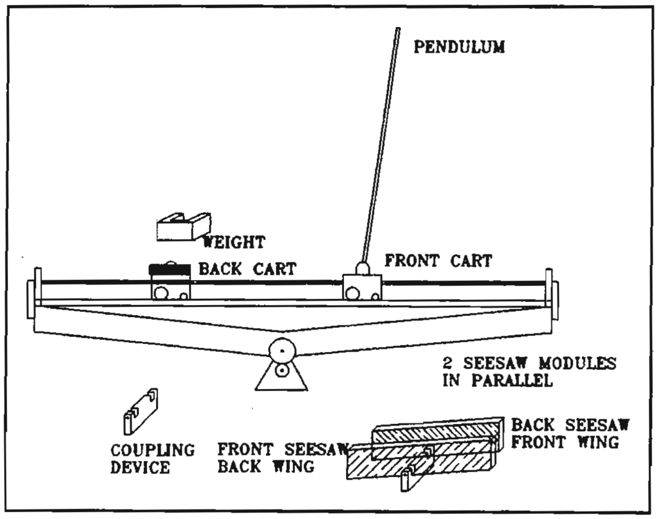
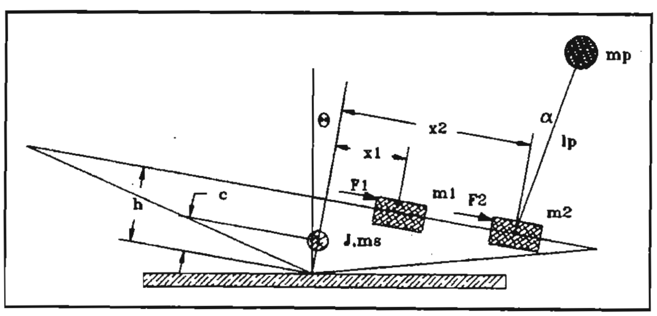
3.5.1 DESCRIPTION

For this experiment, you need two seesaw experiments. Place the two seesaws one behind the other, with the gears facing you and place on each Seesaw an Inverted Pendulum track with cart. Align the two seesaw axes and couple the two seesaws using the coupling fixture supplied as shown in Figure SSP1. The front cart carries a pendulum while the back cart does not. The back cart carries the extra weight supplied. The entire combination (2 Seesaw + 2 track) is referred to as "the seesaw". The front cart is referred to as the "pendulum cart" and the back cart is referred to as the "weight cart".



SSP 1 Seesaw Pendulum Experimental Setup.

3.5.2 MATHEMATICAL MODEL



SSP2 Simplified model for Seesaw/Pendulum experiment

The kinetic (KE) and potential (PE) energies of each moving element in the system are given below. The derivation of these can be clarified by examining Figure SSP2.

The defined variables are:

- m_1: mass of cart with weight
- m_2: mass of cart with pendulum
- m_p: mass of pendulum
- m_s: mass of seesaw (meaning 2 Seesaw + 2 Track)
- J : Moment of inertia of two seesaws about compound centre of mass (with both tracks)
- l_p: centre of mass of pendulum (half of full length)
- h : height of track from pivot point
- c : centre of mass of seesaw (height from pivot point)
- x_1: translation of m_1 from centre of track (+ve to the right)
- x_2: translation of m_2 from centre of track (+ve to the right)
- θ: angle of seesaw with vertical(+ve to the right)
- α: angle of pendulum with normal to track(+ve to the right)
- F_1: force applied to cart 1(+ve to the right)
- F_2: force applied to cart 2 (+ve to the right)

$$PE_p = m_p g\ (h\cos(\theta) - x_2\sin(\theta) + l_p\cos(\theta+\alpha))$$

$$PE_{m2} = m_2 g\ (h\cos(\theta) - x_2\sin(\theta))$$

$$PE_{m1} = m_1 g\ (h\cos(\theta) - x_1\sin(\theta))$$

$$PE_s = m_s\ g\ c\ \cos(\theta)$$

$$KE_p = 0.5 m_p\ \big(((h\cos(\theta) - x_2\sin(\theta) + l_p\cos(\theta+\alpha))\dot{\theta} + \dot{x}_2\cos(\theta) + l_p\cos(\theta+\alpha)\dot{\alpha})^2 + ((-h\sin(\theta) - x_2\cos(\theta) - l_p\sin(\theta+\alpha))\dot{\theta} - \dot{x}_2\sin(\theta) - l_p\sin(\theta+\alpha)\dot{\alpha})^2\ \big)$$

$$KE_{m1} = 0.5 m_1\ ((\dot{x}_1 + h\dot{\theta})^2 + (x_1\dot{\theta})^2);$$

$$KE_{m2} = 0.5 m_2\ ((\dot{x}_2 + h\dot{\theta})^2 + (x_2\dot{\theta})^2)$$

$$KE_s = 0.5\ J(\dot{\theta})^2$$

The total kinetic energy and potential energies are:

$$T = KE_{m1} + KE_{m2} + KE_s + KE_p$$

$$L = PE_{m1} + PE_{m2} + PE_s + PE_p$$

and the Lagrangian is given by:

$$L = T - V$$

we can now proceed to develop the equations of motion using the generalized formulation:

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{x}_1} - \frac{\partial L}{\partial x_1} = F_1$$

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{x}_2} - \frac{\partial L}{\partial x_2} = F_2$$

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{\theta}} - \frac{\partial L}{\partial \theta} = 0$$

$$\frac{\partial}{\partial t}\frac{\partial L}{\partial \dot{\alpha}} - \frac{\partial L}{\partial \alpha} = 0$$

proceeding with the tedious differentiations, we obtain four coupled nonlinear differential equations of the form:

$$G_1(x_1,x_2,\theta,\alpha,\dot{x}_1,\dot{x}_2,\dot{\theta},\dot{\alpha},\ddot{x}_1,\ddot{x}_2,\ddot{\alpha},\ddot{\theta}) = F_1$$

$$G_2(x_1,x_2,\theta,\alpha,\dot{x}_1,\dot{x}_2,\dot{\theta},\dot{\alpha},\ddot{x}_1,\ddot{x}_2,\ddot{\alpha},\ddot{\theta}) = F_2$$

$$G_3(x_1,x_2,\theta,\alpha,\dot{x}_1,\dot{x}_2,\dot{\theta},\dot{\alpha},\ddot{x}_1,\ddot{x}_2,\ddot{\alpha},\ddot{\theta}) = 0$$

$$G_4(x_1,x_2,\theta,\alpha,\dot{x}_1,\dot{x}_2,\dot{\theta},\dot{\alpha},\ddot{x}_1,\ddot{x}_2,\ddot{\alpha},\ddot{\theta}) = 0$$

The *dreadful* part is to solve these to obtain the closed form expressions (See MAPLE program SEEPEN1.MAP and SEEPEN2.MAP):

$$\ddot{x}_1 = D_1(x_1,x_2,\theta,\alpha,\dot{x}_1,\dot{x}_2,\dot{\theta},\dot{\alpha},F_1,F_2)$$

$$\ddot{x}_2 = D_2(x_1,x_2,\theta,\alpha,\dot{x}_1,\dot{x}_2,\dot{\theta},\dot{\alpha},F_1,F_2)$$

$$\ddot{\theta} = D_3(x_1,x_2,\theta,\alpha,\dot{x}_1,\dot{x}_2,\dot{\theta},\dot{\alpha},F_1,F_2)$$

$$\ddot{\alpha} = D_4(x_1,x_2,\theta,\alpha,\dot{x}_1,\dot{x}_2,\dot{\theta},\dot{\alpha},F_1,F_2)$$

Now we can linearize these equations about the quiescent point 0 (all states and forces equal to zero)

This results in the following state space representation:

$$\begin{bmatrix}\dot{x}_1\\ \dot{\theta}\\ \dot{x}_2\\ \dot{\alpha}\\ \ddot{x}_1\\ \ddot{\theta}\\ \ddot{x}_2\\ \ddot{\alpha}\end{bmatrix} = \begin{bmatrix}A_{11} & A_{12}\\ A_{21} & A_{22}\end{bmatrix}\begin{bmatrix}x_1\\ \theta\\ x_2\\ \alpha\\ \dot{x}_1\\ \dot{\theta}\\ \dot{x}_2\\ \dot{\alpha}\end{bmatrix} + \begin{bmatrix}B_1\\ B_2\end{bmatrix}\begin{bmatrix}F_1\\ F_2\end{bmatrix}$$

where

$$A_{12} = I^{4x4}$$

$$A_{11} = A_{22} = 0^{4x4}$$

$$A_{21} = \begin{bmatrix} -\frac{m_1hg}{J} & -g\frac{hm_sc-J}{J} & -gh\frac{m_2+m_p}{J} & 0 \\ g\frac{m_1}{J} & gc\frac{m_s}{J} & g\frac{m_2+m_p}{J} & 0 \\ -\frac{m_1hg}{J} & -g\frac{hm_sc-J}{J} & -gh\frac{m_2+m_p}{J} & -m_p\frac{g}{m_2} \\ -m_1\frac{g}{J} & -m_sg\frac{c}{J} & -g\frac{m_2+m_p}{J} & g\frac{m_2+m_p}{m_2\, l_p} \end{bmatrix}$$

$$B_1 = 0^{4x2}$$

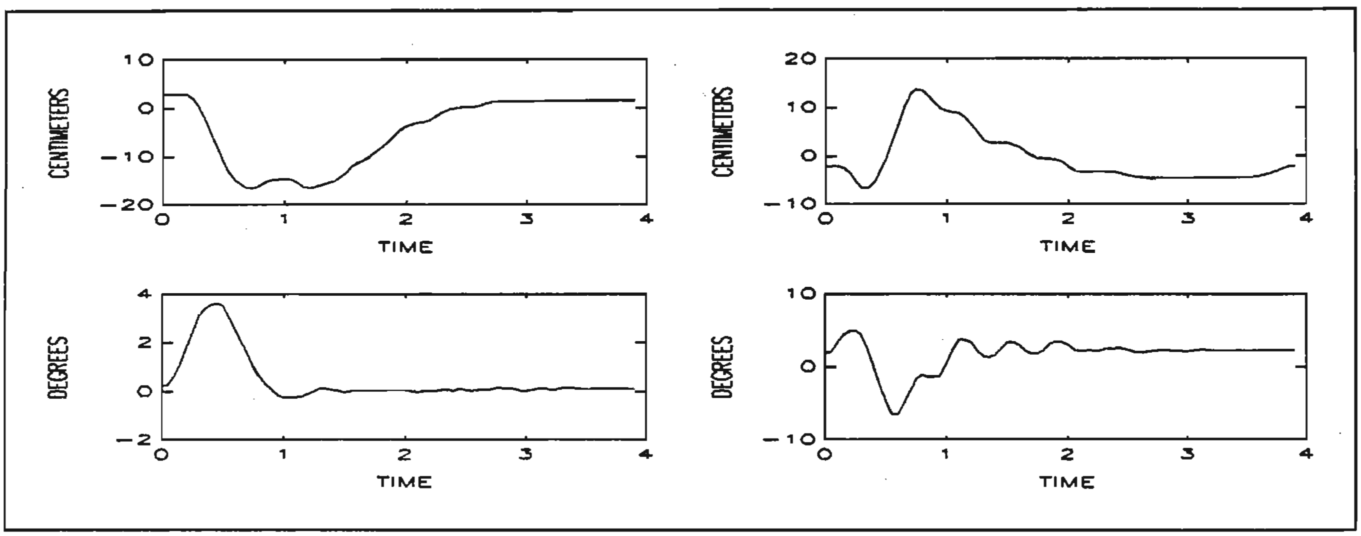
$$B_2 = \begin{bmatrix} \frac{J+m_1h^2}{Jm_1} & \frac{h^2}{J} \\ -\frac{h}{J} & -\frac{h}{J} \\ \frac{h^2}{J} & \frac{J+m_2h^2}{Jm_2} \\ \frac{h}{J} & \frac{l_phm_2-J}{Jm_2l_p} \end{bmatrix}$$

3.5.3 CONTROL SYSTEM DESIGN

The MATLAB program SEEPEN.M performs the appropriate design. The design is performed using LQR. The Q and R matrices are chosen through trial and error and the optimal feedback gain is computed. The Q and R matrices are difficult to obtain due to the coupled nature of the problem. To start with, use the Q and r matrices used in the program SEEPEN.M. Once you have the system working with these gains, you may vary the entries in the Q and r matrices to see the effects they have on closed loop behaviour. Large variations from the gains will probably result in an unstable system!

3.5.4 RESULTS

Figure SSP3 shows the disturbance response of the stabilized system. The top two traces are x_1 (left) and x_2 (right) respectively while the bottom two traces are θ(left) and α(right). The disturbance is a tap to seesaw that results in the seesaw tipping to about +4 degrees. The two carts move in opposite directions to stabilize the system within 4 seconds. Note however that the pendulum cart (x_2) first moves to the left for about 5 cm (non minimum phase behaviour) causing the inverted pendulum to tip to the right. As a result, the cart (x_2) has to move in the positive direction in order to stabilize the inverted pendulum.



SSP3 Disturbance response of Seesaw/Pendulum experiment

CAUTION

To zero the measurement you should hold the pendulum vertical and the seesaw horizontal and hit the letter 'z' from the main menu of the controller. This takes the present measurement as zero. Do this before you turn on the motors and while the carts placed in the middle.

Always start the controller with the pendulum held vertical and the seesaw horizontal!

THIS EXPERIMENT IS BEST PERFORMED WITH TWO INDIVIDUALS PRESENT

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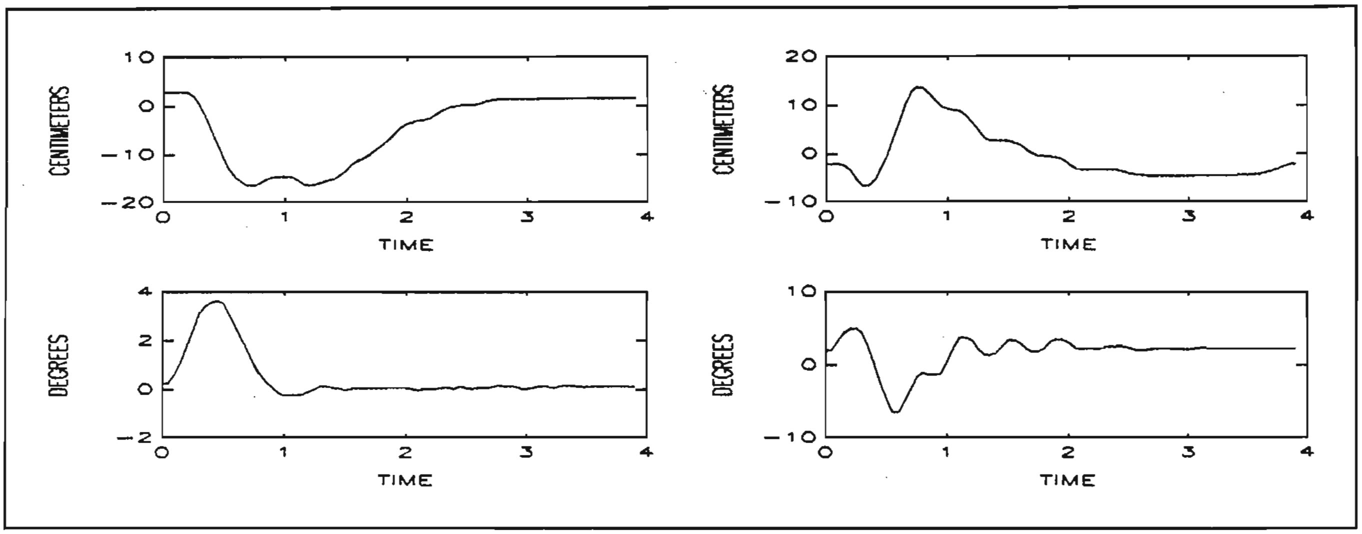

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5.2.5 SEESAW/PENDULUM

PARAMETER	SYMBOL	VALUE	UNITS
EXTRA MASS FOR CART WITH NO PENDULUM		0.39	Kg
2 SEESAW + 2 TRACK MASS	m	6.6	Kg
2 SEESAW + 2 TRACK INERTIA	J*	0.84	Kgm^2
CENTER OF GRAVITY	c*	0.058	m
HEIGHT OF TRACK	h*	0.14	m

* at nominal height / factory setting. Values change with height h
All other parameters are the same as Seesaw, Inverted Pendulum and Linear cart

The equivalent circuits that are achieved using the Quick Connect Module (QCM) is shown the in figures referenced below:

The system is wired as shown in Figures WI-7and WI-9 for the pendulum cart and WI-7 for the load cart. One of the seesaws is wired as shown in figure WI-10. You need 2 Seesaws, 2 Carts and 1 Pendulum.

CAUTION

To zero the measurement you should hold the pendulum vertical and the seesaw horizontal and hit the letter 'z' from the main menu of the controller. This takes the present measurement as zero. Do this before you turn on the motors and while the carts placed in the middle.

Always start the controller with the pendulum held vertical and the seesaw horizontal!

THIS EXPERIMENT IS BEST PERFORMED WITH TWO INDIVIDUALS PRESENT