ECE 241 **Homework No. 5 Solutions** **Spring 2007**
Gibson

Problem 1.

Mimimize $D = \sum_{i=1}^{M} w_i \sigma_i^2 2^{-2R_i}$ subject to the constraint $R = \frac{1}{M}\sum_{i=1}^{M} R_i$.

We form the functional

$$J = \sum_{i=1}^{M} w_i \sigma_i^2 2^{-2R_i} + \lambda\left(\sum_{i=1}^{M} R_i - R\right)$$ and take partials with respect to the R_j and λ,

so we have

$$\frac{\partial J}{\partial R_j} = 0 \Rightarrow 2^{-2R_j} = \frac{-\lambda}{2 w_j \sigma_j^2 \ln 2}$$ so letting $\Lambda = \frac{-\lambda}{2\ln 2}$,

we get

$$R_j = \frac{1}{2}\log\frac{w_j \sigma_j^2}{\Lambda}$$ which when substituted into the constraint equation

yields

$$R = \frac{1}{M}\sum_{i=1}^{M}\frac{1}{2}\log\frac{w_i\sigma_i^2}{\Lambda} = \frac{1}{2}\log\frac{\left(\prod_{i=1}^{M} w_i\sigma_i^2\right)^{1/M}}{\Lambda}$$

so

$$\Lambda = 2^{-2R}\left(\prod_{i=1}^{M} w_i\sigma_i^2\right)^{1/M}$$ and $$R_j = R + \frac{1}{2}\log\frac{w_j\sigma_j^2}{\left(\prod_{i=1}^{M} w_i\sigma_i^2\right)^{1/M}}$$

For audio coding, choose the weighting functions, w_i, to be the inverse of the masking thresholds for the band. This will minimize the noise to mask ratio.