

13.9–23. With $R(0) = 1.0$, $R(1) = 0.866$, $R(2) = 0.554$, and $R(3) = 0.225$, we can use the recursion in Eqs. (13.5–15) through (13.5–19). Thus,

$$\begin{aligned} E^{(0)} &= R(0) = 1.0, \\ p_1 &= R(1)/R(0) = 0.866, \text{ so} \\ a_1^{(1)} &= p_1 = 0.866, \text{ and} \\ E^{(1)} &= (1 - (.866)^2)E^{(0)} = 0.25. \end{aligned}$$

Next,

$$p_2 = \frac{R(2) - a_1^{(1)}R(1)}{E^{(1)}} = \frac{0.554 - (0.866)^2}{0.25} = -0.7838,$$

hence

$$a_2^{(2)} = -0.7838$$

and

$$a_1^{(2)} = a_1^{(1)} - p_1 a_1^{(1)} = 0.866 - (-.7838)(.866) = 1.545.$$

So

$$E^{(2)} = (1 - p_2^2)E^{(1)} = (1 - (-.7838)^2)(0.25) = 0.0964.$$

Continuing

$$\begin{aligned} p_3 &= \frac{R(3) - a_1^{(2)}R(2) - a_2^{(2)}R(1)}{E^{(2)}} = 0.4969 = a_3^{(3)}. \\ a_1^{(3)} &= a_1^{(2)} - p_3 a_2^{(2)} = 1.545 - (.4969)(-.7838) = 1.9345, \\ a_2^{(3)} &= a_2^{(2)} - p_3 a_1^{(2)} = -0.7838 - (.4969)(1.545) = -1.552. \end{aligned}$$

The desired coefficients are thus

$$a_1 = 1.9345, \quad a_2 = -1.552, \quad a_3 = 0.4969.$$

13.9–24. $a_1 = 1.295$, $a_2 = -0.535$, $a_3 = 0.171$, $a_4 = -0.233$, and $R(0) = 1$.

(a) Using Eqs. (13.5–20) and (13.5–21),

$$\begin{aligned} p_4 &= a_4^{(4)} = -0.233 \\ a_1^{(3)} &= \frac{a_1^{(4)} + a_4^{(4)} a_3^{(4)}}{1 - p_4^2} \end{aligned}$$

$$\begin{aligned}
&= \frac{1.295 + (-.233)(.171)}{0.9457} \\
&= 1.327 \\
a_2^{(3)} &= \frac{a_2^{(4)} + a_4^{(4)} a_2^{(4)}}{1 - p_4^2} = -0.434 \\
a_3^{(3)} &= \frac{a_3^{(4)} + a_4^{(4)} a_1^{(4)}}{1 - p_4^2} = -0.138 = p_3 \\
a_1^{(2)} &= \frac{a_1^{(3)} + a_3^{(3)} a_2^{(3)}}{1 - p_3^2} = \frac{1.327 + (-.138)(-.434)}{0.981} \\
&= 1.414 \\
a_2^{(2)} &= \frac{a_2^{(3)} + a_3^{(3)} a_1^{(3)}}{1 - p_3^2} = -0.629 = p_2 \\
a_1^{(1)} &= \frac{a_1^{(2)} + a_2^{(2)} a_1^{(2)}}{1 - p_2^2} = \frac{1.414(.371)}{0.604} \\
&= 0.8685 = p_1.
\end{aligned}$$

(b)

$$\begin{aligned}
E^{(0)} &= R(0) = 1 \\
E^{(1)} &= 1 - p_1^2 = 0.2457 \\
E^{(2)} &= \prod_{i=1}^2 (1 - p_i^2) = 0.1484 \\
E^{(3)} &= \prod_{i=1}^3 (1 - p_i^2) = 0.1456 \\
E^{(4)} &= \prod_{i=1}^4 (1 - p_i^2) = 0.1377.
\end{aligned}$$

(c) Yes, since $|p_i| < 1$ for $i = 1, 2, 3,$ and 4 .

13.9–25. (a) Yes, since $|p_i| < 1$ for all i .

(b)
$$E^{(8)} = \prod_{i=1}^8 (1 - p_i^2) = 0.00641.$$

(c) Use Eqs. (13.5–17) and (13.5–18) starting with $i = 1$.

$$a_1^{(1)} = p_1 = -0.9454$$

$$a_2^{(2)} = p_2 = 0.92386$$

$$\begin{aligned} a_1^{(2)} &= a_1^{(1)} - p_2 a_1^{(1)} \\ &= -.9454 + (.92386)(.9454) \\ &= -0.072. \end{aligned}$$

$$a_3^{(3)} = p_3 = -0.56198$$

$$\begin{aligned} a_1^{(3)} &= a_1^{(2)} - p_3 a_2^{(2)} \\ &= (-.072) + (.56198)(.92386) \\ &= 0.444. \end{aligned}$$

$$\begin{aligned} a_2^{(3)} &= a_2^{(2)} - p_3 a_1^{(2)} \\ &= 0.92386 + (.56198)(-.072) \\ &= 0.8834. \end{aligned}$$

$$a_4^{(4)} = -0.09454$$

$$\begin{aligned} a_3^{(4)} &= a_3^{(3)} - p_4 a_1^{(3)} \\ &= -0.56198 + (.09454)(.444) \\ &= -0.52 \end{aligned}$$

$$\begin{aligned} a_2^{(4)} &= a_2^{(3)} - p_4 a_2^{(3)} \\ &= 0.8834 + (.09454)(.8834) \\ &= 0.9669 \end{aligned}$$

$$\begin{aligned} a_1^{(4)} &= a_1^{(3)} - p_4 a_3^{(3)} \\ &= 0.444 + (.09454)(-.56918) \\ &= 0.4073. \end{aligned}$$

13.9-26. From Eq. (13.5-1),

$$s(k) = \sum_{i=1}^4 a_i s(k-i) + Gw(k),$$

where $G = 2$ and $w(k) = 1$ for $k = 1$ and zero elsewhere. Thus,

$$s(k) = 1.793 s(k-1) - 1.401 s(k-2) + 0.566 s(k-3) - 0.147 s(k-4) + Gw(1),$$

k	1	2	3	4	5	6
$s(k)$	2	3.586	3.628	2.613	1.338	0.264
k	7	8	9	10		
$s(k)$	-0.4555	-0.8134	-0.8225	-0.6317		