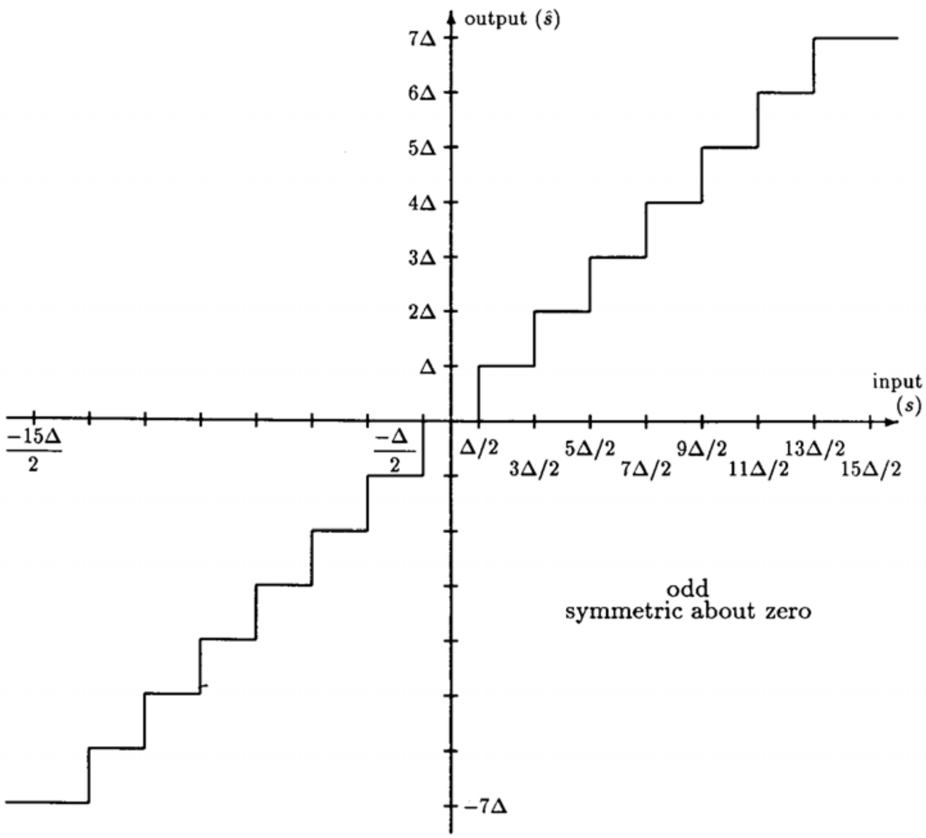
Signal Compression (ECE 242) **April 28, 2011**
Gibson

Homework No. 1 Solutions

1.



$\Delta = \frac{20 \text{ volts}}{15 \text{ levels}} = \frac{4}{3}$, $s = 1.2$ volts yields $\hat{s} = \frac{4}{3}$ volts. Thus, the quantization error (q) is $q = \hat{s} - s = \frac{4}{3} - \frac{6}{5} = \frac{20-18}{15} = \frac{2}{15}$ volts.

2.

$X \sim U(-V, V)$

$$f_X(x) = \begin{cases} \frac{1}{2V}, & \text{for } -V \le x \le V \\ 0, & \text{otherwise.} \end{cases}$$

$$Var[X] = E[X^2] - (E[X])^2$$

$$E[X] = \int_{-V}^{V} x f_X(x) dx = \frac{1}{2V} \int_{-V}^{V} x dx = 0$$

$$E[X^2] = \int_{-V}^{V} x^2 f_X(x) dx = \frac{1}{2V} \int_{-V}^{V} x^2 dx = \frac{V^2}{3}$$

$$Var[X] = E[X^2] - (E[X])^2 = \frac{V^2}{3}$$

3.

3.) The step size $\Delta = \frac{2X_{max}}{M}$, $M = 2^n$

As $\sigma_q^2 = \Delta^2/12$

But for the given input over $[-X_{max}/2, X_{max}/2]$,

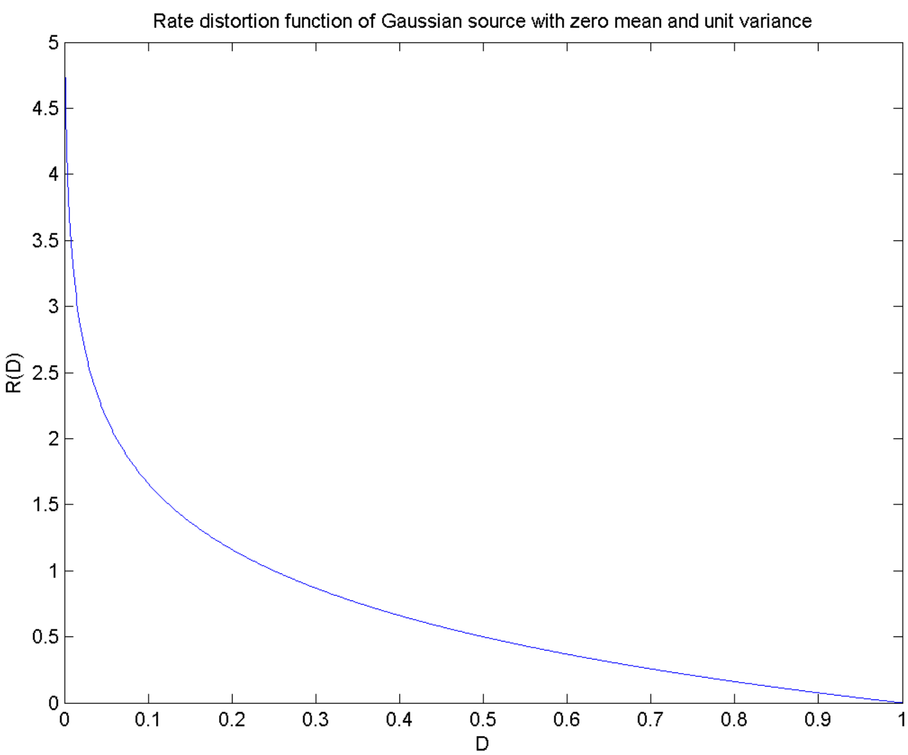
$\sigma_s^2 = X_{max}^2/12$.

Therefore, $SNR = 10\log_{10}\frac{\sigma_s^2}{\sigma_q^2} = 10\log_{10}\frac{\left(X_{max}^2/12\right)}{\left(\frac{4X_{max}^2}{12M^2}\right)}$

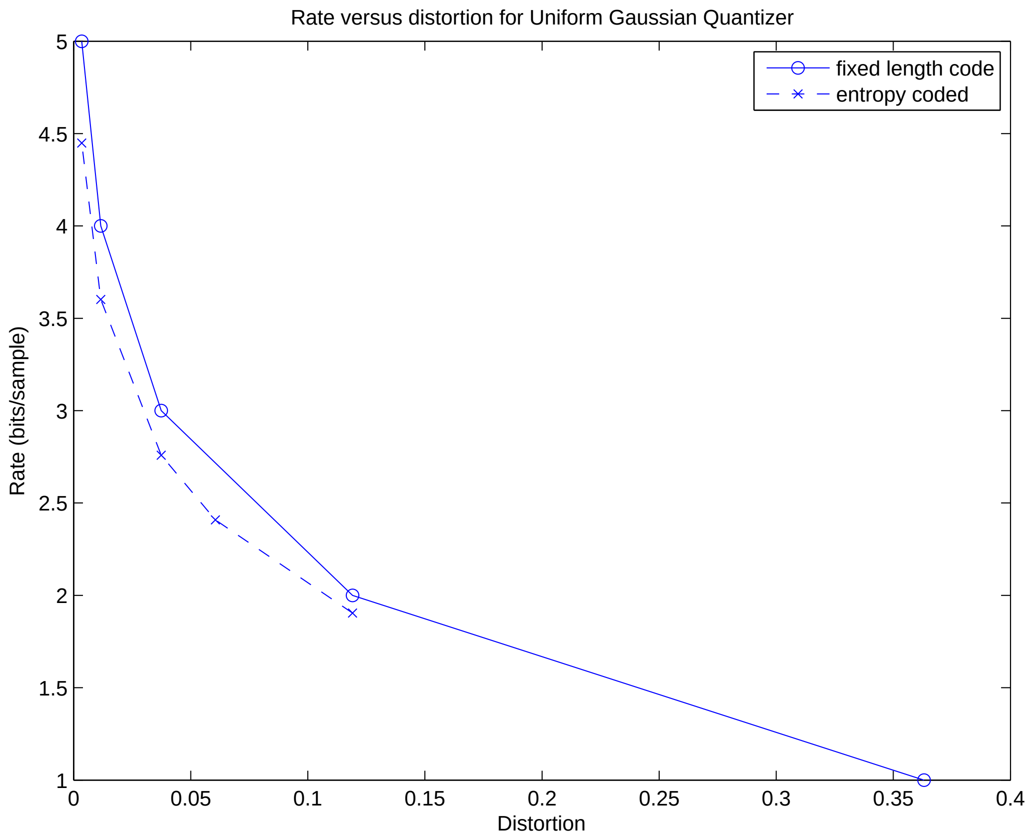
$= 10\log_{10}\frac{M^2}{4} = 20n\log_{10}2 - 10\log_{10}4$

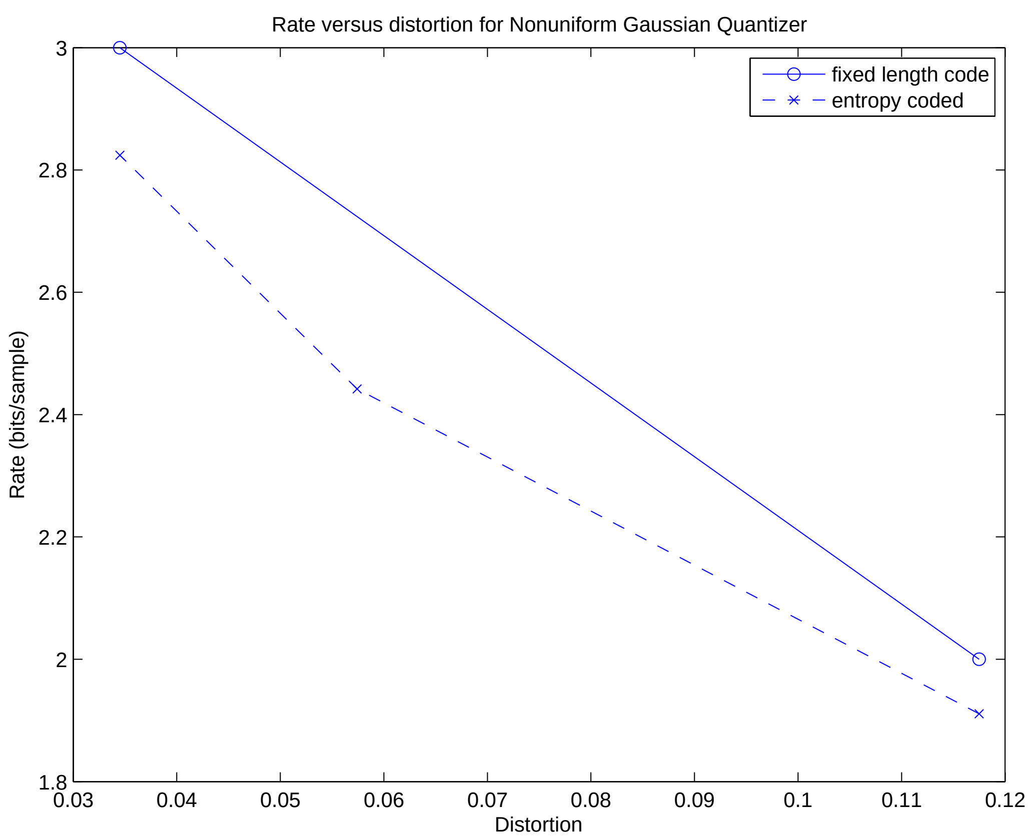
$= 6.02n - 6$ dB.

4.



5.


Rate versus distortion for Uniform Gaussian Quantizer
fixed length code
entropy coded
Rate (bits/sample)
5
4.5
4
3.5
3
2.5
2
1.5
1
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
Distortion


Rate versus distortion for Nonuniform Gaussian Quantizer
fixed length code
entropy coded
Rate (bits/sample)
3
2.8
2.6
2.4
2.2
2
1.8
0.03
0.04
0.05
0.06
0.07
0.08
0.09
0.1
0.11
0.12
Distortion

6.

$$f_X(x) = \frac{1}{\sqrt{2}} \exp(-\sqrt{2}|x|)$$

$$P_i = \int_{b_{i-1}}^{b_i} f_X(x) dx$$

$$P_1 = P_0 = \int_0^{0.5332} \frac{1}{\sqrt{2}} \exp(-\sqrt{2}|x|) dx = \frac{1}{2} - \frac{1}{2} \exp(-\sqrt{2} \cdot 0.5332) = 0.2648$$

$$P_2 = P_{-1} = \int_{0.5332}^{1.2527} \frac{1}{\sqrt{2}} \exp(-\sqrt{2}|x|) dx = \frac{1}{2} \exp(-\sqrt{2} \cdot 0.5332) - \frac{1}{2} \exp(-\sqrt{2} \cdot 1.2527) = 0.1502$$

$$P_3 = P_{-2} = \int_{1.2527}^{2.3796} \frac{1}{\sqrt{2}} \exp(-\sqrt{2}|x|) dx = \frac{1}{2} \exp(-\sqrt{2} \cdot 1.2527) - \frac{1}{2} \exp(-\sqrt{2} \cdot 2.3796) = 0.0678$$

$$P_4 = P_{-3} = \int_{2.3796}^{\infty} \frac{1}{\sqrt{2}} \exp(-\sqrt{2}|x|) dx = \frac{1}{2} \exp(-\sqrt{2} \cdot 2.3796) = 0.0173$$

$$H(Q) = -\sum_{i=-3}^{4} P_i \log_2 P_i = 2.5654$$