Signal Compression (ECE 242) **April 28, 2011**
Gibson

Homework No. 2 Solutions

1. Straightforward substitution.

2.

Problem 1. Leibnitz rule is given by

$$\frac{\delta}{\delta t}\int_{a(t)}^{b(t)} f(x,t)dx = \int_{a(t)}^{b(t)} \frac{\delta}{\delta t} f(x,t)dx + f(b(t),t)\frac{\delta}{\delta t}b(t) - f(a(t),t)\frac{\delta}{\delta t}a(t)$$

where $a(t)$ and $b(t)$ are monotonic functions of t.

The quantization noise power is given by:

$$\sigma_q^2 = 2\sum_{i=1}^{\frac{M}{2}-1}\int_{(i-1)\Delta}^{i\Delta}\left(x-\frac{2i-1}{2}\Delta\right)^2 f_X(x)dx + 2\int_{(\frac{M}{2}-1)\Delta}^{\infty}\left(x-\frac{M-1}{2}\Delta\right)^2 f_X(x)dx$$

Taking a derivative with respect to Δ and using Leibnitz rule we obtain:

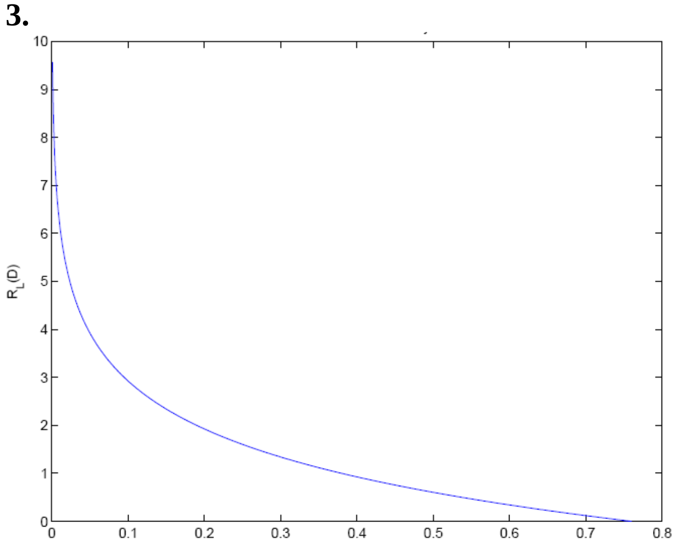
$$\begin{aligned}\frac{\delta}{\delta t}\sigma_q^2 &= 2\sum_{i=1}^{\frac{M}{2}-1}\left[2(-\frac{(2i-1)}{2})\int_{(i-1)\Delta}^{i\Delta}\left(x-\frac{2i-1}{2}\Delta\right)f_X(x)dx + \frac{i\Delta^2}{4}f_X(i\Delta) - \frac{(i-1)\Delta^2}{4}f_X((i-1)\Delta)\right] \\ &+ \left[2(-\frac{(M-1)}{2})\int_{(\frac{M}{2}-1)\Delta}^{\infty}\left(x-\frac{M-1}{2}\Delta\right)f_X(x)dx - \frac{(\frac{M}{2}-1)\Delta^2}{4}f_X((\frac{M}{2}-1)\Delta)\right]\end{aligned}$$

Setting this to zero and noticing that

$$\sum_{i=1}^{\frac{M}{2}-1}\left[\frac{i\Delta^2}{4}f_X(i\Delta) - \frac{(i-1)\Delta^2}{4}f_X((i-1)\Delta)\right] = \frac{(\frac{M}{2}-1)\Delta^2}{4}f_X((\frac{M}{2}-1)\Delta)$$

we obtain the desired result.

3.



4.

9.10–22. Encoding from Table 9.6–1 can be expedited by the following definitions.

Let $x \triangleq$ magnitude of the sample to be encoded

$x_L \triangleq$ lowest value in the quantization segment input amplitude range

$\Delta \triangleq$ step size for the particular quantization segment of interest.

Then the quantizer step s in the particular segement is

$$s = \left\lceil \frac{x - x_L}{\Delta} \right\rceil = \begin{array}{c}\text{least integer greater}\\ \text{than } (x - x_L)/\Delta.\end{array}$$

Example:

$$x = 291, \; s = \left\lceil \frac{291 - 223}{16} \right\rceil = \lceil 4.25 \rceil = 5.$$

The 5th step has the code 1011. The TDMed binary sequence with m_1 first is:

1111101111110001110010111101101110111111

1100101111010110101111110111100111001011

0101000111011011001111101110101001001101

11111100

5.

Input:	−2	−1.45	−0.2	0.15	0.24	0.68	2.2	2.9	3.6	3.9	4.95
Normalized Input:	−1	−.725	−.1	.075	.12	.34	1.1	1.45	1.8	1.95	2.475
Binary Sequence:	0011	0010	0000	1000	1000	1001	1011	1101	1101	1110	111
Output (normalized)	−.9424	−.6568	−.1284	.1284	.1284	.3881	.9424	1.618	1.618	2.069	2.733
Output:	−1.885	−1.314	−.2568	.2568	.2568	.7762	1.8848	3.236	3.236	4.138	5.466

6.

1.) $\sigma_q^2 = \sum_{k=1}^{L} \int_{z_k}^{z_{k+1}} (z - w_k)^2 p_z(z)\, dz$

$$\frac{\partial \sigma_q^2}{\partial w_j} = 0 \Rightarrow w_j = \frac{\int_{z_j}^{z_{j+1}} z\, f_z(z)\, dz}{\int_{z_j}^{z_{j+1}} f_z(z)\, dz}$$

and $\dfrac{\partial \sigma_q^2}{\partial z_j} = 0 \Rightarrow z_j = \dfrac{w_j + w_{j-1}}{2}$.

Working with w_j, make the change of variables $X = Z/\sigma$, so the numerator is

$$\int_{z_j}^{z_{j+1}} z f_Z(z)\,dz = \int_{z_j/\sigma}^{z_{j+1}/\sigma} \sigma x f_Z(\sigma x)\,\sigma\,dx$$

but $f_X(x) = \left.\dfrac{f_Z(z)}{1/\sigma}\right|_{z=\sigma x}$ so the above gives

$$\sigma \int_{z_j/\sigma}^{z_{j+1}/\sigma} x f_X(x)\,dx$$

, and for the denominator,

$$\int_{z_j}^{z_{j+1}} f_Z(z)\,dz = \sigma \int_{z_j/\sigma}^{z_{j+1}/\sigma} f_Z(\sigma x)\,dx = \int_{z_j/\sigma}^{z_{j+1}/\sigma} f_X(x)\,dx$$

Therefore,

$$w_j = \sigma \frac{\int_{x_j}^{x_{j+1}} x f_X(x)\,dx}{\int_{x_j}^{x_{j+1}} f_X(x)\,dx} = \sigma y_j$$

and also $z_j = \dfrac{w_j + w_{j-1}}{2} = \sigma\left[\dfrac{y_j + y_{j-1}}{2}\right]$

$= \sigma x_j$