ECE 242 **Homework 3 Solutions**

1.

13.9–23. With $R(0) = 1.0$, $R(1) = 0.866$, $R(2) = 0.554$, and $R(3) = 0.225$, we can use the recursion in Eqs. (13.5–15) through (13.5–19). Thus,

$$E^{(0)} = R(0) = 1.0,$$

$$p_1 = R(1)/R(0) = 0.866, \text{ so}$$

$$a_1^{(1)} = p_1 = 0.866, \text{ and}$$

$$E^{(1)} = (1 - (.866)^2)E^{(0)} = 0.25.$$

Next,

$$p_2 = \frac{R(2) - a_1^{(1)}R(1)}{E^{(1)}} = \frac{0.554 - (0.866)^2}{0.25} = -0.7838,$$

hence

$$a_2^{(2)} = -0.7838$$

and

$$a_1^{(2)} = a_1^{(1)} - p_1 a_1^{(1)} = 0.866 - (-.7838)(.866) = 1.545.$$

So

$$E^{(2)} = (1 - p_2^2)E^{(1)} = (1 - (-.7838)^2)(0.25) = 0.0964.$$

Continuing

$$p_3 = \frac{R(3) - a_1^{(2)}R(2) - a_2^{(2)}R(1)}{E^{(2)}} = 0.4969 = a_3^{(3)}.$$

$$a_1^{(3)} = a_1^{(2)} - p_3 a_2^{(2)} = 1.545 - (.4969)(-.7838) = 1.9345,$$

$$a_2^{(3)} = a_2^{(2)} - k_3 a_1^{(2)} = -0.7838 - (.4969)(1.545) = -1.552.$$

The desired coefficients are thus

$$a_1 = 1.9345, \ a_2 = -1.552, \ a_3 = 0.4969.$$

2.

13.9–24. $a_1 = 1.295,\ a_2 = -0.535,\ a_3 = 0.171,\ a_4 = -0.233$, and $R(0) = 1$.

(a) Using Eqs. (13.5–20) and (13.5–21),

$$p_4 = a_4^{(4)} = -0.233$$

$$a_1^{(3)} = \frac{a_1^{(4)} + a_4^{(4)}a_3^{(4)}}{1 - p_4^2}$$

$$= \frac{1.295 + (-.233)(.171)}{0.9457}$$

$$= 1.327$$

$$a_2^{(3)} = \frac{a_2^{(4)} + a_4^{(4)}a_2^{(4)}}{1 - p_4^2} = -0.434$$

$$a_3^{(3)} = \frac{a_3^{(4)} + a_4^{(4)}a_1^{(4)}}{1 - p_4^2} = -0.138 = p_3$$

$$a_1^{(2)} = \frac{a_1^{(3)} + a_3^{(3)}a_2^{(3)}}{1 - p_3^2} = \frac{1.327 + (-.138)(-.434)}{0.981}$$

$$= 1.414$$

$$a_2^{(2)} = \frac{a_2^{(3)} + a_3^{(3)}a_1^{(3)}}{1 - p_3^2} = -0.629 = p_2$$

$$a_1^{(1)} = \frac{a_1^{(2)} + a_2^{(2)}a_1^{(2)}}{1 - p_2^2} = \frac{1.414(.371)}{0.604}$$

$$= 0.8685 = p_1.$$

(b)

$$E^{(0)} = R(0) = 1$$

$$E^{(1)} = 1 - p_1^2 = 0.2457$$

$$E^{(2)} = \prod_{i=1}^{2}(1 - p_i^2) = 0.1484$$

$$E^{(3)} = \prod_{i=1}^{3}(1 - p_i^2) = 0.1456$$

$$E^{(4)} = \prod_{i=1}^{4}(1 - p_i^2) = 0.1377.$$

(c) Yes, since $|p_i| < 1$ for $i = 1,\ 2,\ 3$, and 4.

3.

13.9–25. (a) Yes, since $|p_i| < 1$ for all i.

(b)
$$E^{(8)} = \prod_{i=1}^{8}(1 - p_i^2) = 0.00641.$$

(c) Use Eqs. (13.5–17) and (13.5–18) starting with $i = 1$.

$$a_1^{(1)} = p_1 = -0.9454$$

$$\begin{aligned}
a_2^{(2)} &= p_2 = 0.92386 \\
a_1^{(2)} &= a_1^{(1)} - p_2 a_1^{(1)} \\
&= -.9454 + (.92386)(.9454) \\
&= -0.072. \\
a_3^{(3)} &= p_3 = -0.56198 \\
a_1^{(3)} &= a_1^{(2)} - p_3 a_2^{(2)} \\
&= (-.072) + (.56198)(.92386) \\
&= 0.444. \\
a_2^{(3)} &= a_2^{(2)} - p_3 a_1^{(2)} \\
&= 0.92386 + (.56198)(-.072) \\
&= 0.8834. \\
a_4^{(4)} &= -0.09454 \\
a_3^{(4)} &= a_3^{(3)} - p_4 a_1^{(3)} \\
&= -0.56198 + (.09454)(.444) \\
&= -0.52 \\
a_2^{(4)} &= a_2^{(3)} - p_4 a_2^{(3)} \\
&= 0.8834 + (.09454)(.8834) \\
&= 0.9669 \\
a_1^{(4)} &= a_1^{(3)} - p_4 a_3^{(3)} \\
&= 0.444 + (.09454)(-.56918) \\
&= 0.4073.
\end{aligned}$$

4.

13.9–26. From Eq. (13.5–1),

$$s(k) = \sum_{i=1}^{4} a_i s(k-i) + Gw(k),$$

where $G = 2$ and $w(k) = 1$ for $k = 1$ and zero elsewhere. Thus,

$$s(k) = 1.793\ s(k-1) - 1.401\ s(k-2) + 0.566\ s(k-3) - 0.147\ s(k-4) + Gw(1),$$

k	1	2	3	4	5	6
$s(k)$	2	3.586	3.628	2.613	1.338	0.264

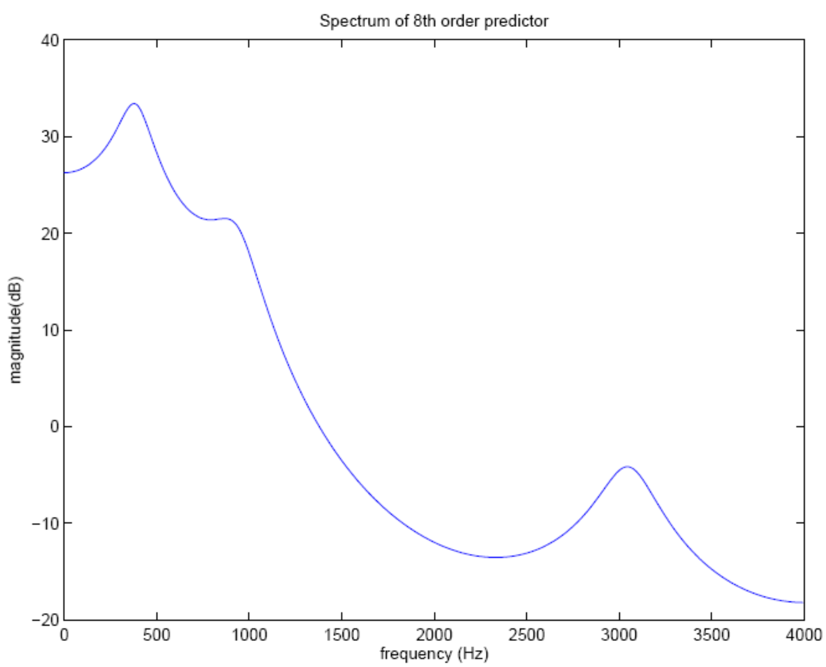
k	7	8	9	10
$s(k)$	−0.4555	−0.8134	−0.8225	−0.6317.

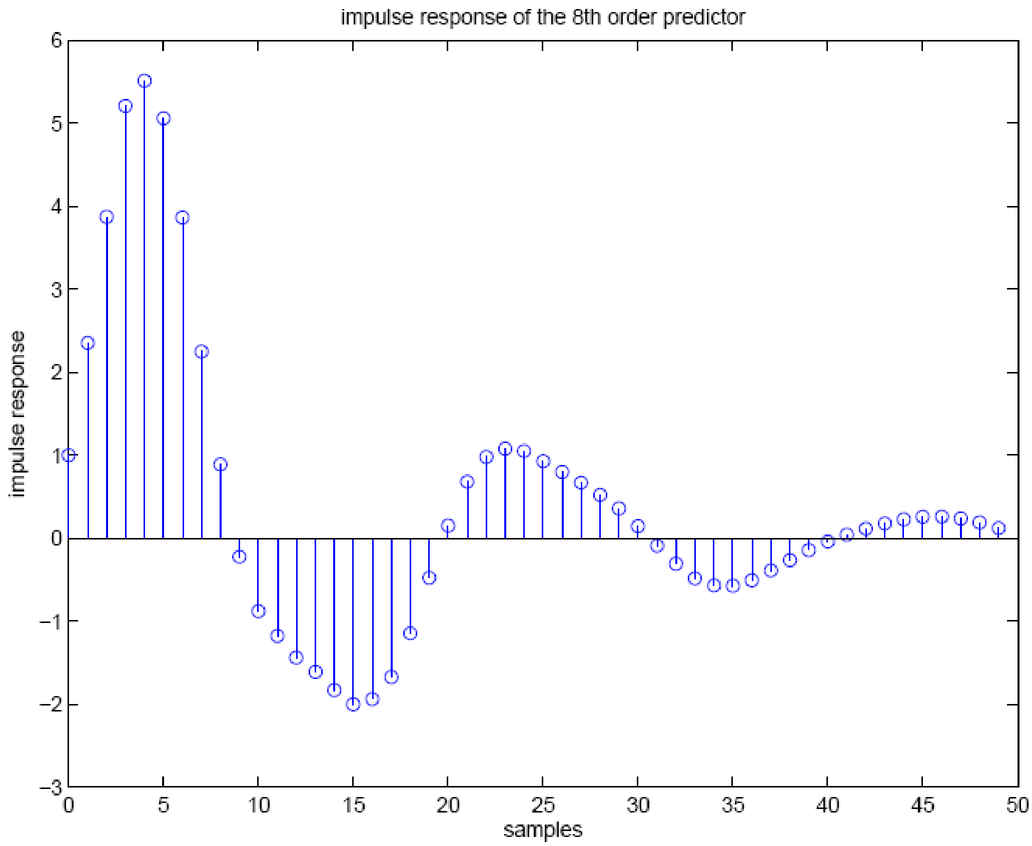
5.

Sign change on PARCORs.

PARCOR coefficients: $p_1 = 0.9454$, $p_2 = -0.92386$, $p_3 = 0.56198$, $p_4 = 0.09454$, $p_5 = -0.20218$, $p_6 = -0.53595$, $p_7 = 0.32922$, and $p_8 = 0.05899$.

The corresponding predictor coefficient for a 8th order coefficients: $a_1 = 2.3527$, $a_2 = -1.6605$, $a_3 = 0.0040$, $a_4 = -0.3229$, $a_5 = 1.4864$, $a_6 = -1.1565$, $a_7 = 0.1893$ and $a_8 = 0.0590$.




impulse response of the 8th order predictor
impulse response
samples
6
5
4
3
2
1
0
−1
−2
−3
0
5
10
15
20
25
30
35
40
45
50