Homework No. 1

Due: January 15, 2008

1. Given two independent random variables X and Y, form $Z=X+Y$.

(a) If X and Y are Gaussian with means μ_X and μ_Y and variances σ_X^2 and σ_Y^2, respectively, find the probability density function of Z.

(b) If X and Y are one-sided exponential random variables with parameter λ (see page 20 of Gersho and Gray), what is the pdf of Z?

2. A random variable X with alphabet $X=\{ x_1, x_{2,} x_3 \}$ has the probability mass function $p(x_1) = 0.7, p(x_2) = 0.15 = p(x_3)$. Calculate $H(X)$. Compare $H(X)$ to $H(Y)$ for a random variable Y with three equally likely values.

3. Given a random variable X, uniformly distributed over $\left[-\Delta/2, \Delta/2\right]$, calculate the variance of X.

4. A continuous time, weakly stationary random process (see p. 31 of the text) has the autocorrelation function

$$R_X(\tau) = e^{-|\tau|} \, for - \infty < \tau < \infty$$

Find the power spectral density.

5. A first order discrete-time autoregressive (AR(1)) process is given by $x_k = \alpha x_{k-1} + w_k$, with $\alpha > 0$, where the weakly stationary sequence w_k is white and zero mean with variance σ_w^2. Find the autocorrelation function of this AR(1) process.