**Signal Compression (ECE 242)**
**Gibson**

**January 24, 2008**
**Handout #6**

## Homework No. 3

**Due: January 31, 2008**

**1.** If the minimum distortion is given by

$$D_{\min} = \frac{2}{12N^2}\left[\int_0^\infty dx \sqrt[3]{f_X(x)}\right]^3 \text{ for large } N.$$

(a) Evaluate $D_{min}$ for a uniformly distributed input with $N=128$. What is the SNR in dB?

(b) Evaluate $D_{min}$ for a Laplacian source as shown on p. 20 of the text with $\sigma^2 = \frac{2}{\lambda^2}$ and $N=128$. What is the SNR in dB?

**2.** Derive an expression for $D_{ol}$ in terms of the loading fraction $\beta$ for a symmetric input pdf $f_X(x)$ with variance $\sigma_X^2$ and a symmetric quantizer.

**3.** Create a table showing:
(a) The output entropy and distortion for a uniform quantizer that minimizes MSE for 4, 8, and 16 level quantizers with a unit variance, zero mean Gaussian input,
(b) The output entropy and distortion for a nonuniform quantizer that minimizes MSE for 4, 8, and 16 level quantizers with a unit variance, zero mean Gaussian input,
(c) D(R) at these entropy values,
(d) R(D) at these distortion values,
(e) the corresponding high resolution scalar quantization with an entropy constraint results, and
(f) the optimum MSE subject to an entropy constraint result for rates of 1, 2, and 3 bits and a Gaussian source.

Parts (a) and (b) are from J. Max, "Quantizing for Minimum Disortion," IEEE Trans. on Information Theory, March 1960, pp. 7-12, and part (f) is from N. Farvardin and J. W. Modestino, "Optimum Quantizer Performance for a Class of Non-Gaussian Memoryless Sources," IEEE Trans. on Information Theory, May 1984, pp. 485-497.

See following pages for needed information.

## TABLE I
### Parameters for the Optimum Quantizer

| | $N = 1$ | | $N = 2$ | | $N = 3$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | — | 0.0 | 0.0 | 0.7980 | 0.0 | 1.224 |
| 2 | | | | | 0.6120 | |
| Error | 1.000 | | 0.3634 | | 0.1902 | |
| Entropy | 0.0 | | 1.000 | | 1.536 | |

| | $N = 4$ | | $N = 5$ | | $N = 6$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.0 | 0.4528 | 0.3823 | 0.0 | 0.0 | 0.3177 |
| 2 | 0.9816 | 1.510 | 1.244 | 0.7646 | 0.6589 | 1.000 |
| 3 | | | | 1.724 | 1.447 | 1.894 |
| Error | 0.1175 | | 0.07994 | | 0.05798 | |
| Entropy | 1.911 | | 2.203 | | 2.443 | |

| | $N = 7$ | | $N = 8$ | | $N = 9$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.2803 | 0.0 | 0.0 | 0.2451 | 0.2218 | 0.0 |
| 2 | 0.8744 | 0.5606 | 0.5006 | 0.7560 | 0.6812 | 0.4436 |
| 3 | 1.611 | 1.188 | 1.050 | 1.344 | 1.198 | 0.9188 |
| 4 | | 2.033 | 1.748 | 2.152 | 1.866 | 1.476 |
| | | | | | | 2.255 |
| Error | 0.04400 | | 0.03454 | | 0.02785 | |
| Entropy | 2.647 | | 2.825 | | 2.983 | |

| | $N = 10$ | | $N = 11$ | | $N = 12$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.0 | 0.1996 | 0.1837 | 0.0 | 0.0 | 0.1684 |
| 2 | 0.4047 | 0.6099 | 0.5599 | 0.3675 | 0.3401 | 0.5119 |
| 3 | 0.8339 | 1.058 | 0.9656 | 0.7524 | 0.6943 | 0.8768 |
| 4 | 1.325 | 1.591 | 1.436 | 1.179 | 1.081 | 1.286 |
| 5 | 1.968 | 2.345 | 2.059 | 1.693 | 1.534 | 1.783 |
| 6 | | | | 2.426 | 2.141 | 2.499 |
| Error | 0.02293 | | 0.01922 | | 0.01634 | |
| Entropy | 3.125 | | 3.253 | | 3.372 | |

| | $N = 13$ | | $N = 14$ | | $N = 15$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.1569 | 0.0 | 0.0 | 0.1457 | 0.1369 | 0.0 |
| 2 | 0.4760 | 0.3138 | 0.2935 | 0.4413 | 0.4143 | 0.2739 |
| 3 | 0.8126 | 0.6383 | 0.5959 | 0.7505 | 0.7030 | 0.5548 |
| 4 | 1.184 | 0.9870 | 0.9181 | 1.086 | 1.013 | 0.8512 |
| 5 | 1.623 | 1.381 | 1.277 | 1.468 | 1.361 | 1.175 |
| 6 | 2.215 | 1.865 | 1.703 | 1.939 | 1.776 | 1.546 |
| 7 | | 2.565 | 2.282 | 2.625 | 2.344 | 2.007 |
| 8 | | | | | | 2.681 |
| Error | 0.01406 | | 0.01223 | | 0.01073 | |
| Entropy | 3.481 | | 3.582 | | 3.677 | |

| | $N = 16$ | | $N = 17$ | | $N = 18$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.0 | 0.1284 | 0.1215 | 0.0 | 0.0 | 0.1148 |
| 2 | 0.2582 | 0.3881 | 0.3670 | 0.2430 | 0.2306 | 0.3464 |
| 3 | 0.5224 | 0.6568 | 0.6201 | 0.4909 | 0.4653 | 0.5843 |
| 4 | 0.7996 | 0.9424 | 0.8875 | 0.7493 | 0.7091 | 0.8339 |
| 5 | 1.099 | 1.256 | 1.178 | 1.026 | 0.9680 | 1.102 |
| 6 | 1.437 | 1.618 | 1.508 | 1.331 | 1.251 | 1.400 |
| 7 | 1.844 | 2.069 | 1.906 | 1.685 | 1.573 | 1.746 |
| 8 | 2.401 | 2.733 | 2.454 | 2.127 | 1.964 | 2.181 |
| | | | | 2.781 | 2.504 | 2.826 |
| Error | 0.009497 | | 0.008463 | | 0.007589 | |
| Entropy | 3.765 | | 3.849 | | 3.928 | |

| | $N = 19$ | | $N = 20$ | | $N = 21$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.1092 | 0.0 | 0.0 | 0.1038 | 0.09918 | 0.0 |
| 2 | 0.3294 | 0.2184 | 0.2083 | 0.3128 | 0.2989 | 0.1984 |
| 3 | 0.5551 | 0.4404 | 0.4197 | 0.5265 | 0.5027 | 0.3994 |
| 4 | 0.7908 | 0.6698 | 0.6375 | 0.7486 | 0.7137 | 0.6059 |
| 5 | 1.042 | 0.9117 | 0.8661 | 0.9837 | 0.9361 | 0.8215 |
| 6 | 1.318 | 1.173 | 1.111 | 1.239 | 1.175 | 1.051 |
| 7 | 1.634 | 1.464 | 1.381 | 1.524 | 1.440 | 1.300 |
| 8 | 2.018 | 1.803 | 1.690 | 1.857 | 1.743 | 1.579 |
| 9 | 2.550 | 2.232 | 2.068 | 2.279 | 2.116 | 1.908 |
| 10 | | 2.869 | 2.594 | 2.908 | 2.635 | 2.324 |
| 11 | | | | | | 2.946 |
| Error | 0.006844 | | 0.006203 | | 0.005648 | |
| Entropy | 4.002 | | 4.074 | | 4.141 | |

| | $N = 22$ | | $N = 23$ | | $N = 24$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.0 | 0.09469 | 0.09085 | 0.0 | 0.0 | 0.08708 |
| 2 | 0.1900 | 0.2852 | 0.2736 | 0.1817 | 0.1746 | 0.2621 |
| 3 | 0.3822 | 0.4793 | 0.4594 | 0.3654 | 0.3510 | 0.4399 |
| 4 | 0.5794 | 0.6795 | 0.6507 | 0.5534 | 0.5312 | 0.6224 |
| 5 | 0.7844 | 0.8893 | 0.8504 | 0.7481 | 0.7173 | 0.8122 |
| 6 | 1.001 | 1.113 | 1.062 | 0.9527 | 0.9122 | 1.012 |
| 7 | 1.235 | 1.357 | 1.291 | 1.172 | 1.119 | 1.227 |
| 8 | 1.495 | 1.632 | 1.546 | 1.411 | 1.344 | 1.462 |
| 9 | 1.793 | 1.955 | 1.841 | 1.681 | 1.595 | 1.728 |
| 10 | 2.160 | 2.366 | 2.203 | 2.000 | 1.885 | 2.042 |
| 11 | 2.674 | 2.982 | 2.711 | 2.406 | 2.243 | 2.444 |
| 12 | | | | 3.016 | 2.746 | 3.048 |
| Error | 0.005165 | | 0.004741 | | 0.004367 | |
| Entropy | 4.206 | | 4.268 | | 4.327 | |

| | $N = 25$ | | $N = 26$ | | $N = 27$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.08381 | 0.0 | 0.0 | 0.08060 | 0.07779 | 0.0 |
| 2 | 0.2522 | 0.1676 | 0.1616 | 0.2425 | 0.2340 | 0.1556 |
| 3 | 0.4231 | 0.3368 | 0.3245 | 0.4066 | 0.3921 | 0.3124 |
| 4 | 0.5982 | 0.5093 | 0.4905 | 0.5743 | 0.5537 | 0.4719 |
| 5 | 0.7797 | 0.6870 | 0.6610 | 0.7477 | 0.7202 | 0.6354 |
| 6 | 0.9702 | 0.8723 | 0.8383 | 0.9289 | 0.8936 | 0.8049 |
| 7 | 1.173 | 1.068 | 1.025 | 1.121 | 1.077 | 0.9824 |
| 8 | 1.394 | 1.279 | 1.224 | 1.328 | 1.273 | 1.171 |
| 9 | 1.641 | 1.510 | 1.442 | 1.556 | 1.487 | 1.374 |
| 10 | 1.927 | 1.772 | 1.685 | 1.814 | 1.727 | 1.599 |
| 11 | 2.281 | 2.083 | 1.968 | 2.121 | 2.006 | 1.854 |
| 12 | 2.779 | 2.480 | 2.318 | 2.514 | 2.352 | 2.158 |
| 13 | | 3.079 | 2.811 | 3.109 | 2.842 | 2.547 |
| 14 | | | | | | 3.137 |
| Error | 0.004036 | | 0.003741 | | 0.003477 | |
| Entropy | 4.384 | | 4.439 | | 4.491 | |

| | $N = 28$ | | $N = 29$ | | $N = 30$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.0 | 0.07502 | 0.07257 | 0.0 | 0.0 | 0.07016 |
| 2 | 0.1503 | 0.2256 | 0.2182 | 0.1451 | 0.1406 | 0.2110 |
| 3 | 0.3018 | 0.3780 | 0.3655 | 0.2913 | 0.2821 | 0.3532 |
| 4 | 0.4556 | 0.5333 | 0.5154 | 0.4396 | 0.4255 | 0.4978 |
| 5 | 0.6132 | 0.6930 | 0.6693 | 0.5912 | 0.5719 | 0.6460 |
| 6 | 0.7760 | 0.8589 | 0.8287 | 0.7475 | 0.7225 | 0.7990 |
| 7 | 0.9460 | 1.033 | 0.9956 | 0.9100 | 0.8788 | 0.9586 |
| 8 | 1.126 | 1.218 | 1.172 | 1.081 | 1.043 | 1.127 |
| 9 | 1.319 | 1.419 | 1.362 | 1.263 | 1.217 | 1.306 |
| 10 | 1.529 | 1.640 | 1.570 | 1.461 | 1.404 | 1.501 |
| 11 | 1.766 | 1.892 | 1.804 | 1.680 | 1.609 | 1.717 |
| 12 | 2.042 | 2.193 | 2.077 | 1.929 | 1.840 | 1.964 |
| 13 | 2.385 | 2.578 | 2.417 | 2.226 | 2.111 | 2.258 |
| 14 | 2.871 | 3.164 | 2.899 | 2.609 | 2.448 | 2.638 |
| 15 | | | | 3.190 | 2.926 | 3.215 |
| Error | 0.003240 | | 0.003027 | | 0.002834 | |
| Entropy | 4.542 | | 4.591 | | 4.639 | |

*Cont'd next page*

TABLE I, *Cont'd*

| | $N = 31$ | | $N = 32$ | | $N = 33$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.06802 | 0.0 | 0.0 | 0.06590 | 0.06400 | 0.0 |
| 2 | 0.2045 | 0.1360 | 0.1320 | 0.1981 | 0.1924 | 0.1280 |
| 3 | 0.3422 | 0.2729 | 0.2648 | 0.3314 | 0.3218 | 0.2567 |
| 4 | 0.4822 | 0.4115 | 0.3991 | 0.4668 | 0.4530 | 0.3868 |
| 5 | 0.6254 | 0.5528 | 0.5359 | 0.6050 | 0.5869 | 0.5192 |
| 6 | 0.7730 | 0.6979 | 0.6761 | 0.7473 | 0.7245 | 0.6547 |
| 7 | 0.9265 | 0.8481 | 0.8210 | 0.8947 | 0.8667 | 0.7943 |
| 8 | 1.088 | 1.005 | 0.9718 | 1.049 | 1.015 | 0.9392 |
| 9 | 1.259 | 1.170 | 1.130 | 1.212 | 1.171 | 1.091 |
| 10 | 1.444 | 1.347 | 1.299 | 1.387 | 1.338 | 1.252 |
| 11 | 1.646 | 1.540 | 1.482 | 1.577 | 1.518 | 1.424 |
| 12 | 1.875 | 1.753 | 1.682 | 1.788 | 1.716 | 1.612 |
| 13 | 2.143 | 1.997 | 1.908 | 2.029 | 1.940 | 1.821 |
| 14 | 2.477 | 2.289 | 2.174 | 2.319 | 2.204 | 2.060 |
| 15 | 2.952 | 2.665 | 2.505 | 2.692 | 2.533 | 2.347 |
| 16 | | 3.239 | 2.977 | 3.263 | 3.002 | 2.718 |
| 17 | | | | | | 3.285 |
| Error | 0.002658 | | 0.002499 | | 0.002354 | |
| Entropy | 4.685 | | 4.730 | | 4.773 | |

| | $N = 34$ | | $N = 35$ | | $N = 36$ | |
|---|---|---|---|---|---|---|
| | $x_i$ | $y_i$ | $x_i$ | $y_i$ | $x_i$ | $y_i$ |
| $j = 1$ | 0.0 | 0.06212 | 0.06043 | 0.0 | 0.0 | 0.05876 |
| 2 | 0.1244 | 0.1867 | 0.1816 | 0.1209 | 0.1177 | 0.1765 |
| 3 | 0.2495 | 0.3122 | 0.3036 | 0.2423 | 0.2359 | 0.2952 |
| 4 | 0.3758 | 0.4394 | 0.4272 | 0.3650 | 0.3552 | 0.4152 |
| 5 | 0.5043 | 0.5691 | 0.5530 | 0.4895 | 0.4762 | 0.5372 |
| 6 | 0.6355 | 0.7020 | 0.6819 | 0.6166 | 0.5996 | 0.6620 |
| 7 | 0.7705 | 0.8391 | 0.8146 | 0.7471 | 0.7261 | 0.7903 |
| 8 | 0.9104 | 0.9818 | 0.9523 | 0.8820 | 0.8567 | 0.9231 |
| 9 | 1.057 | 1.131 | 1.096 | 1.023 | 0.9923 | 1.062 |
| 10 | 1.211 | 1.290 | 1.248 | 1.170 | 1.134 | 1.207 |
| 11 | 1.375 | 1.460 | 1.411 | 1.327 | 1.285 | 1.362 |
| 12 | 1.553 | 1.646 | 1.587 | 1.495 | 1.445 | 1.528 |
| 13 | 1.749 | 1.853 | 1.781 | 1.679 | 1.619 | 1.710 |
| 14 | 1.971 | 2.090 | 2.001 | 1.883 | 1.812 | 1.913 |
| 15 | 2.232 | 2.375 | 2.260 | 2.119 | 2.030 | 2.146 |
| 16 | 2.559 | 2.743 | 2.584 | 2.401 | 2.287 | 2.427 |
| 17 | 3.025 | 3.307 | 3.048 | 2.767 | 2.609 | 2.791 |
| 18 | | | | 3.328 | 3.070 | 3.349 |
| Error | 0.002220 | | 0.002097 | | 0.001985 | |
| Entropy | 4.815 | | 4.856 | | 4.895 | |

TABLE II

PARAMETERS FOR THE OPTIMUM EQUALLY SPACED LEVEL QUANTIZER

| Number Output Levels | Output Level Spacing | Mean Squared Error | Informational Entropy |
|---|---|---|---|
| 1 | — | 1.000 | 0.0 |
| 2 | 1.596 | 0.3634 | 1.000 |
| 3 | 1.224 | 0.1902 | 1.536 |
| 4 | 0.9957 | 0.1188 | 1.904 |
| 5 | 0.8430 | 0.08218 | 2.183 |
| 6 | 0.7334 | 0.06065 | 2.409 |
| 7 | 0.6508 | 0.04686 | 2.598 |
| 8 | 0.5860 | 0.03744 | 2.761 |
| 9 | 0.5338 | 0.03069 | 2.904 |
| 10 | 0.4908 | 0.02568 | 3.032 |
| 11 | 0.4546 | 0.02185 | 3.148 |
| 12 | 0.4238 | 0.01885 | 3.253 |
| 13 | 0.3972 | 0.01645 | 3.350 |
| 14 | 0.3739 | 0.01450 | 3.440 |
| 15 | 0.3534 | 0.01289 | 3.524 |
| 16 | 0.3352 | 0.01154 | 3.602 |
| 17 | 0.3189 | 0.01040 | 3.676 |
| 18 | 0.3042 | 0.009430 | 3.746 |
| 19 | 0.2909 | 0.008594 | 3.811 |
| 20 | 0.2788 | 0.007869 | 3.874 |
| 21 | 0.2678 | 0.007235 | 3.933 |
| 22 | 0.2576 | 0.006678 | 3.990 |
| 23 | 0.2482 | 0.006185 | 4.045 |
| 24 | 0.2396 | 0.005747 | 4.097 |
| 25 | 0.2315 | 0.005355 | 4.146 |
| 26 | 0.2240 | 0.005004 | 4.194 |
| 27 | 0.2171 | 0.004687 | 4.241 |
| 28 | 0.2105 | 0.004401 | 4.285 |
| 29 | 0.2044 | 0.004141 | 4.328 |
| 30 | 0.1987 | 0.003905 | 4.370 |
| 31 | 0.1932 | 0.003688 | 4.410 |
| 32 | 0.1881 | 0.003490 | 4.449 |
| 33 | 0.1833 | 0.003308 | 4.487 |
| 34 | 0.1787 | 0.003141 | 4.524 |
| 35 | 0.1744 | 0.002986 | 4.560 |
| 36 | 0.1703 | 0.002843 | 4.594 |

TABLE I
COMPARISON OF THE SNR (IN DB) OF THE OPTIMUM AND LLOYD–MAX QUANTIZER VERSUS RATE DISTORTION BOUND FOR GAUSSIAN AND LAPLACIAN SOURCES

| Rate | Gaussian Source | | | Laplacian Source | | |
|---|---|---|---|---|---|---|
| (bits/sample) | $R(D)$ | Optimum† | Lloyd–Max | $R(D)$ | Optimum† | Lloyd–Max |
| 1 | 6.02 | 4.64 (4.40) | 4.40 | 6.62 | 5.76 (3.01) | 3.01 |
| 2 | 12.04 | 10.55 (10.51) | 9.30 | 12.66 | 11.31 (10.85) | 7.53 |
| 3 | 18.06 | 16.56 (16.53) | 14.62 | 18.68 | 17.20 (17.09) | 12.61 |

†Quantities in parenthesis are taken from [10].

zation levels is omitted, however, the optimum quantizer will be only negligibly superior to the uniform threshold quantizer. In light of this fact, therefore, one could forego the formidable task of designing optimum quantizers by using a uniform threshold quantization scheme (probably with a larger number of levels) without losing much in performance. Furthermore, we have demonstrated that the difference between the performance curves of the optimum quantizer and the rate-distortion function lower bound is only 0.255 bits/sample at high rates and becomes even smaller at low rates, except for nearly uniform distributions. This is in marked contrast with previously reported results [10].

For the three classes of parametric non-Gaussian sources, we have obtained explicit formulas for the source differential entropy which, together with the Gish–Pierce asymptotic formula, describe the rate-distortion performance of the optimum quantizer at low distortions. It has been shown that for most cases this asymptotic relationship provides an upper bound to the performance of optimum zero-memory quantizers even at low rates.

The gap between the performance of the optimum quantizers and that of the rate-distortion bound is the penalty one has to pay by using a zero-memory quantization scheme. This gap, of course, can be narrowed by using a multidimensional scheme. The question regarding the rate of improvement as a function of the dimensionality of the scheme as well as the complexity associated with it, is yet to be completely developed [19].

Our algorithmic methods are based on the necessary conditions for optimality. A more rigorous treatment of this issue, including the sufficiency and uniqueness conditions seems to be required. Moreover, in regards to uniform threshold quantizers, it would be interesting to consider the possibility of asymmetric quantizers and find examples for which the optimum uniform threshold quantizers with entropy constraint are indeed asymmetric.

## APPENDIX A

### *A Bound on the Output Entropy of Symmetric Quantizers*

A useful bound on the output entropy of a symmetric quantizer is provided by the following theorem.

*Theorem:* If $p(x)$, the source pdf, is symmetric and the number of levels $N$ is even, the output entropy of a symmetric quantizer is no smaller than 1 bit/sample.

*Proof:* Since both the quantizer and the pdf are symmetric, we have

$$P_i = P_{N+1-i}, \qquad i = 1,2,\cdots,N/2, \tag{A1}$$

and hence

$$H = -2\sum_{i=1}^{N/2} P_i \log_2 P_i \qquad \text{bits/sample.} \tag{A2}$$

Let

$$\begin{aligned} q_1 &= P_1 + P_2, \\ q_2 &= q_1 + P_3, \\ &\vdots \\ q_{N/2-1} &= q_{N/2-2} + P_{N/2}, \end{aligned} \tag{A3}$$

and note that

$$\begin{aligned} P_1 \log_2 \frac{1}{P_1} + P_2 \log_2 \frac{1}{P_2} &\geq q_1 \log_2 \frac{1}{q_1}, \\ q_1 \log_2 \frac{1}{q_1} + P_3 \log_2 \frac{1}{P_3} &\geq q_2 \log_2 \frac{1}{q_2}, \\ &\vdots \\ q_{N/2-2} \log_2 \frac{1}{q_{N/2-2}} + P_{N/2} \log_2 \frac{1}{P_{N/2}} &\geq q_{N/2-1} \log_2 \frac{1}{q_{N/2-1}}. \end{aligned} \tag{A4}$$

Adding inequalities (A4) leads to

$$\sum_{i=1}^{N/2} P_i \log_2 \frac{1}{P_i} \geq q_{N/2-1} \log_2 \frac{1}{q_{N/2-1}} = 0.5. \tag{A5}$$

Using (A5) in (A2) results in

$$H \geq 1 \qquad \text{bit/sample.}$$

## APPENDIX B

### *Evaluation of the Entropy-Distortion Performance of the Optimum Quantizer with Entropy Constraint for a Source with Uniform Distribution*

In this section we evaluate the minimum distortion incurred by an $N$-level optimum quantizer for a fixed output entropy $H_0$ and a source with uniform distribution. The problem is analytically simplified to a point where a simple algorithmic solution becomes possible.

We shall assume that the source pdf is given by

$$p(x) = \begin{cases} 1/2a, & |x| \leq a, \\ 0, & \text{elsewhere,} \end{cases} \tag{A6}$$