Signal Compression (ECE 242) **January 29 2008**

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Homework 4

Solution

1. Theorem 4.2.3

   Let C and d be any matrix and vector of suitable dimensions.

   To show:

$$E\left[||Y-(CX+d)||^2\right] \geq E\left[||Y-(AX+b)||^2\right].$$

   We have

$$\begin{aligned}
E[||Y-(CX+d)||^2] &= E[||Y-E[Y]-C(X-E[X])+E[Y]-CE[X]-d||^2] && (1)\\
&= E[||Y-E[Y]-C(X-E[X])||^2] && (2)\\
&+ E[||E[Y]-CE[X]-d||^2] && (3)\\
&+ 2E[[Y-E[Y]-C(X-E[X])]^t[E[Y]-CE[X]-d]] && (4)\\
&= (1)+(2)+(3) && (5)
\end{aligned}$$

   where

$$(1) = E[||Y-E[Y]-C(X-E[X])||^2]$$

$$(2) = E[||E[Y]-CE[X]-d||^2]$$

$$(3) = 2E[[Y-E[Y]-C(X-E[X])]^t[E[Y]-CE[X]-d]]$$

   Taking $b = E[Y]-AE[X], \quad C = A, \quad d = b$, we have

$$\begin{aligned}
(2) &= E[||E[Y]-CE[X]-d||^2] && (6)\\
&= E[||b-b||^2] = 0 && (7)
\end{aligned}$$

$$\begin{aligned}
(3) &= 2E[[Y-E[Y]-C(X-E[X])]^t[E[Y]-CE[X]-d]] && (8)\\
&= 2E[[Y-E[Y]-C(X-E[X])]^t[E[Y]-AE[X]-b]] && (9)\\
&= 2E[[Y-E[Y]-C(X-E[X])]^t[0]] && (10)\\
&= 2E[0] = 0 && (11)
\end{aligned}$$

Now, (1) is minimized by choosing $C = A$ and we get

$$(1) = E[||Y - E[Y] - C(X - E[X])||^2] \geq E\left[||Y - (AX + b)||^2\right] \qquad (12)$$

From equations [1] *to* [12] we get

$$E\left[||Y - (CX + d)||^2\right] \geq E\left[||Y - (AX + b)||^2\right].$$

2.

$$D_{av} = \frac{1}{100} \sum_{i=1}^{100} d(x_i, Q(x_i))$$

$a = [-0.1996, -0.6099, -1.058, -1.591, -2.34, 0.1996, 0.6099, 1.058, 1.591, 2.34]$

$g = randn(100);$

$S = 0;$

for $k = 1 : 100$

for $i = 1 : 10$

$S(i) = g(k) - a(i);$

end

$d(k) = min(S)^2;$

$S = S + d(k);$

end

disp $\left(\frac{s}{100}\right);$

$D = 0.017.$

3. (a)

$$E[X] = E[Y] = -2(0.5) + 1(0.3) + 3(0.2) = -0.1$$

(b)

$$E[(X - Q(X))Q(X)] = 0;\;\; E[XQ(X)] = E[Q(X)^2] = (-2)^2(0.5) + (0.3)1^1 + (0.2)3^2 = 4.1.$$

(c)

$$0.2 = \sigma_X^2 - \sigma_{Q(X)}^2 = \sigma_X^2 - E[X^2] + (E[X])^2$$

$$\therefore \sigma_X^2 = 0.2 + 4.1 - (0.1)^2 = 4.29$$