Homework 4

Solution

1. Theorem 4.2.3

Let $C$ and $d$ be any matrix and vector of suitable dimensions.

To show:

$$E \left[ ||Y - (CX + d)||^2 \right] \geq E \left[ ||Y - (AX + b)||^2 \right].$$

We have


$$= E[||Y - E[Y] - C(X - E[X])||^2]$$  

$$+ E[||E[Y] - CE[X] - d||^2]$$  

$$+ 2E[(Y - E[Y] - C(X - E[X]))^t(E[Y] - CE[X] - d)]$$  

$$= (1) + (2) + (3)$$  

where

$$1 = E[||Y - E[Y] - C(X - E[X])||^2]$$

$$2 = E[||E[Y] - CE[X] - d||^2]$$

$$3 = 2E[(Y - E[Y] - C(X - E[X]))^t(E[Y] - CE[X] - d)]$$

Taking $b = E[Y] - AE[X]$, $C = A$, $d = b$, we have

$$2 = E[||E[Y] - CE[X] - d||^2]$$

$$= E[||b - b||^2] = 0$$

$$3 = 2E[(Y - E[Y] - C(X - E[X]))^t(E[Y] - CE[X] - d)]$$

$$= 2E[(Y - E[Y] - C(X - E[X]))^t(E[Y] - AE[X] - b)]$$

$$= 2E[(Y - E[Y] - C(X - E[X]))^t(0)]$$

$$= 2E[0] = 0$$
Now, (1) is minimized by choosing \( C = A \) and we get

\[
(1) = E[||Y - E[Y] - C(X - E[X])||^2] \geq E[||Y - (AX + b)||^2]
\]  

(12)

From equations [1] to [12] we get

\[
E[||Y - (CX + d)||^2] \geq E[||Y - (AX + b)||^2].
\]

2.

\[
D_{av} = \frac{1}{100} \sum_{i=1}^{100} d(x_i, Q(x_i))
\]

\[
a = [-0.1996, -0.6099, -1.058, -1.591, -2.34, 0.1996, 0.6099, 1.058, 1.591, 2.34]
\]

\[
g = \text{randn}(100);
\]

\[
S = 0;
\]

for \( k = 1 : 100 \)

for \( i = 1 : 10 \)

\[
S(i) = g(k) - a(i);
\]

end

\[
d(k) = \text{min}(S)^2;
\]

\[
S = S + d(k);
\]

end

\[
disp\left(\frac{s}{100}\right);
\]

\[
D = 0.017.
\]

3. (a)

\[
E[X] = E[Y] = -2(0.5) + 1(0.3) + 3(0.2) = -0.1
\]

(b)

\[
E[(X - Q(X))Q(X)] = 0; \quad E[XQ(X)] = E[Q(X)^2] = (-2)^2(0.5) + (0.3)^1 + (0.2)^3 = 4.1.
\]

(c)

\[
0.2 = \sigma_X^2 - \sigma_Q^2 = \sigma_X^2 - E[X^2] + (E[X])^2
\]

\[
\therefore \sigma_X^2 = 0.2 + 4.1 - (0.1)^2 = 4.29
\]