Signal Compression (ECE 242) **March 14, 2008**
Gibson

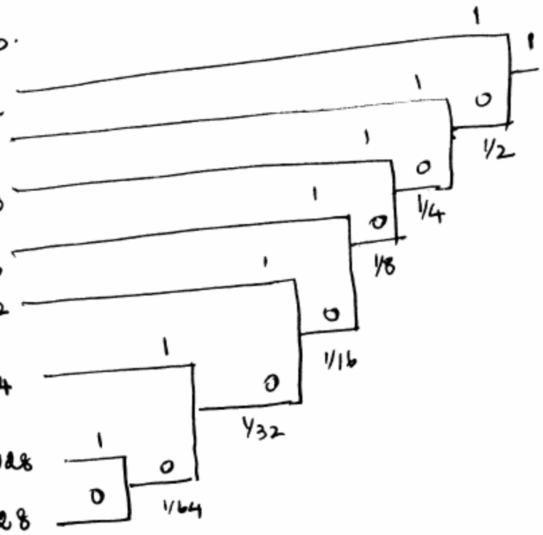
Homework No. 6 Solutions

9.1 1.) From (9.27), $H(X) \le \sum_a p(a) \log \frac{1}{q(a)}$

Let $q(a) = \frac{1}{K}$, so $H(X) \le \log K$

Problem 9.3

codeword	symbol	Prob.
1	a_1	1/2
01	a_2	1/4
001	a_3	1/8
0001	a_4	1/16
00001	a_5	1/32
000001	a_6	1/64
0000001	a_7	1/128
00000000	a_7	1/128



Average code word length

$$\bar{l} = 1(1/2) + 2(1/4) + 3(1/8) + 4(1/16) + 5(1/32) + 6(1/64) + 7(1/128) + 7(1/128)$$

$$\bar{l} = 1.984375$$

Entropy $H(x) = -\sum_i p(x_i) \log p(x_i)$

$$H(X) = 1.984375$$

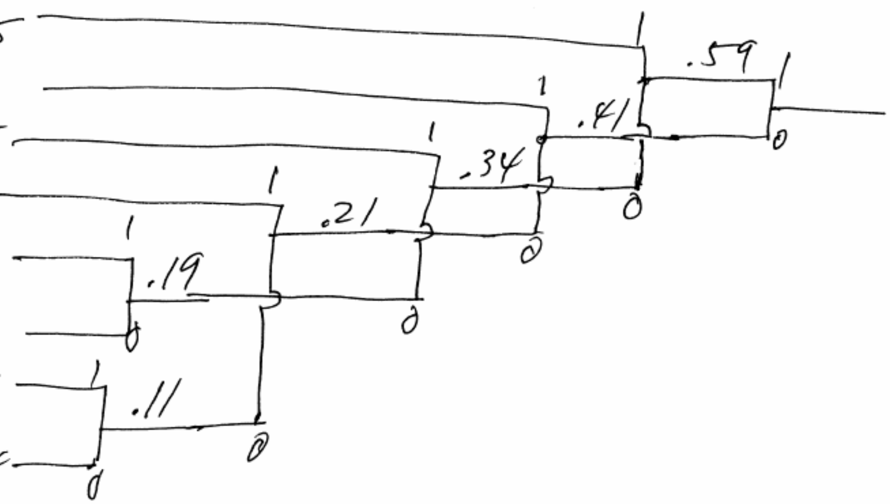
$$\Rightarrow \bar{l} = H(X)$$

Since $p_i = 2^{-l_i}$, we have $\bar{l} = H(X)$

Hence, entropy bound is achieved with equality.

9.11 $\boxed{5.1}$

Codeword	i	$p(i)$
11	1	0.25
01	2	.2
101	3	.15
001	4	.1
1001	5	.1
1000	6	.09
0001	7	.07
0000	8	.04



$$\bar{l}(\alpha) = 2(.25) + 2(.2) + 3(.15) + 3(.1) + 4(.1) + 4(.09) + 4(.07) + 4(.04)$$

$$= 2.85 \text{ bits/letter}$$

4.1 (a)

$$D(p\|q) = \sum_x p(x) \log \frac{p(x)}{q(x)}$$

$$\geq \sum_x p(x)\left[1 - \frac{q(x)}{p(x)}\right] = 0$$

(b)

$$-D(p\|q) = \sum p(x) \log \frac{q(x)}{p(x)}$$

$$\leq \log\left[\sum p(x) \cdot \frac{q(x)}{p(x)}\right] = 0$$

$\therefore D(p\|q) \geq 0.$