Signal Compression (ECE 242) **January 22 2008**

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Homework 3

Solution

1. (a) Considering uniformly distributed case

$$f_X(x) = \begin{cases} \dfrac{1}{b-a} & a \leq x \leq b \\ 0 & elsewhere \end{cases}$$

We have

$$\sigma^2 = E[X^2] - E[X]^2 = \frac{(b-a)^2}{12}.$$

$$D_{min} = \frac{1}{6N^2}\left[\int_0^\infty \sqrt[3]{f_X(x)}dx\right]^3 = \frac{(b-a)^2}{6(128)^2}.$$

$$\begin{aligned} SNR &= 10\log \frac{\sigma^2}{D_{min}} \\ &= 10\log\frac{128^2}{2} \\ &= 39.1339dB. \end{aligned}$$

(b) Considering Laplacian input PDF

$$f_X(x) = \frac{\lambda}{2}e^{-\lambda|x|} \quad -\infty < x < \infty, \quad \lambda > 0.$$

We have $\sigma^2 = \frac{2}{\lambda^2}$.

$$\begin{aligned} D_{min} &= \frac{1}{6N^2}\left[\int_0^\infty \sqrt[3]{f_X(x)}dx\right]^3 \\ &= \frac{1}{6N^2}\left[\int_0^\infty \frac{\lambda^{\frac{1}{3}}}{2^{\frac{1}{3}}}e^{-\frac{\lambda}{3}x}dx\right]^3 \\ &= \frac{1}{6(128)^2}\frac{\lambda}{2}\frac{27}{\lambda^3} \\ &= \frac{9}{4.128^2\lambda^2} \end{aligned}$$

$$\begin{aligned} SNR &= 10\log\frac{\sigma^2}{D_{min}} \\ &= 10\log\frac{2^{17}}{9} \\ &= 41.6327dB. \end{aligned}$$

2.

$$D_{OL} = \int_{-\infty}^{-x_{max}}(x-y_1)^2 f_X(x)dx + \int_{x_{max}}^{\infty}(x-y_N)^2 f_X(x)dx$$

where $x_{max} = V;\ y_1 \simeq -V;\ y_N = \simeq\ V$

$$\therefore \quad D_{OL} = 2\int_{-\infty}^{-V}(x+V)^2 f_X(x)dx = 2\int_{-\infty}^{-\frac{\sigma_x}{\beta}}(x+\frac{\sigma_x}{\beta})^2 f_X(x)dx.$$

3. (a)

$$H(y) = -\sum_{i=1}^{N} p_i \log_2 p_i; \quad p_i = \int_{x_{i-1}}^{x_i} f_X(x)dx.$$

$$D = \sum_{i=1}^{N}\int_{x_{i-1}}^{x_i}(x-y_i)^2 f_X(x)dx.$$

For $N = 4$,

$$p_1 = \frac{1}{\sqrt{2\pi}} \int_0^{0.9957} e^{-\frac{x^2}{2}} dx = 0.341$$
$$p_2 = \frac{1}{\sqrt{2\pi}} \int_{0.9957}^{\infty} e^{-\frac{x^2}{2}} dx = 0.159$$
$$Entropy = [-p_1 \log \ p_1 - p_2 \log \ p_2] = 1.902.$$

Similarly do for other cases.

(b)

$$Distorsion \ D = \sum_{i=1}^{N} \int_{x_{i-1}}^{x_i} (x - y_i)^2 f_X(x) dx.$$

Compute using Matlab, for each case.

(c)

$$D(R) = \sigma^2 2^{-2R} = 2^{-2H(y)}; \quad \sigma^2 = 1$$

For uniform quantizer,

$$N = 4 \longrightarrow \quad D(R) = 0.0711$$
$$N = 8 \longrightarrow \quad D(R) = 0.0216$$
$$N = 16 \longrightarrow \quad D(R) = 0.0067$$

For non-uniform quantizer,

$$N = 4 \longrightarrow \quad D(R) = 0.0700$$
$$N = 8 \longrightarrow \quad D(R) = 0.0796$$
$$N = 16 \longrightarrow \quad D(R) = 0.0053$$

(d)

$$R(D) = \frac{1}{2} \log_2 \frac{\sigma^2}{D}.$$

(e)

$$D = \frac{2\pi}{12} e\ \sigma^2\ 2^{-2H(y)} = 1.423\ .\ 2^{-2H(y)}$$

Uniform MMSE Gaussian Quantizer

N	Output entropy	Distorsion	R(D)	D(R)	Hi.res.sca.quant....
4	1.904	0.1188	1.5367	0.0714	0.1016
8	2.761	0.0374	2.3696	0.0213	0.0310
16	3.602	0.0115	3.2186	0.0068	0.0097

Nonuniform MMSE Gaussian Quantizer

N	Output entropy	Distorsion	R(D)	D(R)	Hi.res.sca.quant....
4	1.911	0.1175	1.5446	0.0707	0.1006
8	2.825	0.03454	2.4278	0.0119	0.0283
16	3.7650	0.009497	3.3592	0.0054	0.0077

where Hi.res.sca.quant.... means High resolution scalar quantizer with entropy constraint.

(f) We get

Rates(bits)	Optimum	SNR
1	0.3436	4.64
2	0.0881	10.55
3	0.0221	16.56