ECE 242 | Winter Quarter 2008
Gibson | **02/14//08**

Midterm Exam

Instructions: Do all problems. Show all work. Problems are equally weighted.

1. Given the random vector X find an expression for the optimal estimate $\hat{Y}(X)$ that minimizes

$$\varepsilon^2(\hat{Y}) = E[(Y - \hat{Y})^t W (Y - \hat{Y})]$$

where W is a symmetric positive definite matrix that is not a function of X.

2. The Lloyd-Max quantizer for a Laplacian memoryless source and the squared error criterion at rate $R = 3$ bits/sample achieves an SNR of 12.61 dB and the corresponding entropy constrained quantizer (from Farvardin and Modestino) achieves an SNR of 17.20 dB.

(a) Compare the performance of these quantizers with the high resolution entropy constrained quantizer designed for a Laplacian source and the squared error criterion.

(b) Compare the performance of these quantizers to the best possible performance given by $D(R) = \frac{e}{\pi} 2^{-2R}$.

3. The *m*-law companding characteristic is given by

$$G(x) = [x_{\max}\left(1 - e^{-m|x|/x_{\max}}\right)/(1 - e^{-m})]\operatorname{sgn}(x)$$

(a) Find an expression for the output SNR if the input is a Laplacian source.

(b) What is the ratio $\Delta_{\max}/\Delta_{\min}$ for this characteristic?

(c) What is the point density function for this characteristic?

4. The *R*=3 bits/sample Lloyd-Max uniform quantizer for a Gaussian source and squared error criterion is in the attached table.

(a) What is the loading factor? Show how you got it.

(b) Compare the performance of this quantizer to the high rate result with the same loading factor. Assume the overload distortion is negligible.

(c) Write expressions for the overload and granular distortions for a unit variance input and simplify as much as possible.

5. A two-stage quantizer has zero mean input X, first stage output $\hat{X}$, second stage input $U = X - \hat{X}$, and second stage output $\hat{U}$. The overall system output is $\tilde{X} = \hat{X} + \hat{U}$.

(a) Evaluate the mean squared quantization error at each stage, including the overall error.

(b) What is $E[U\hat{U}]$?