**HOMEWORK #1**

Due Friday, October 9, 2009 (5:00 p.m.)

**Reading:** Background, Chapters 1 (review) and 2 (2.1–2.7)

**Problems:**

1. Chapter 1: Problem 1
2. Chapter 1: Problem 6
3. Chapter 1: Problem 10
4. Chapter 1: Problem 15
5. Find $\phi_{xx}(k)$ for
   \[ \Phi_{xx}(z) = \frac{1}{(1 + 0.5z^{-1})(1 - 0.4z^{-1})(1 + 0.5)(1 - 0.4)}. \]
6. Consider a single-input, two-output system. Assume that the input $x(n)$ and the additive noise $v(n)$ affecting $y_1(n)$ are uncorrelated, and
   \[ \phi_{xx}(k) = 2\delta(k), \quad \phi_{vv}(k) = \frac{1}{2}\delta(k) \]
   \[ h_1(n) = \left(\frac{1}{4}\right)^n u(n), \quad h_2(n) = \left(\frac{1}{5}\right)^n u(n). \]

   (a) Find the noncausal Wiener filter for estimating $y_1(n)$ from $y_2(n)$.
   (b) Determine the minimum mean-square error (MSE) $\xi_{\text{min}}$.
   (c) What is $\xi_{\text{min}}$ without filtering $y_2(n)$ (i.e., the filter is zero)? Compare this result to that obtained in part (b) and comment.
7. Consider the MSE equation:

\[ \xi = \phi_{dd}(0) - 2 \sum_m h(m)\phi_{dx}(m) + \sum_m \sum_k h(m)h(k)\phi_{xx}(k - m) \]

where \( d(n) \) is the desired signal and \( x(n) \) is the filter input. Impose the constraint that \( \{ h(n) \} \) be the coefficients of an \( N \)-length tapped-delay line (i.e., FIR filter).

(a) Determine the corresponding Wiener-Hopf equation.

(b) Assume that \( x(n) \) is a white-noise sequence, and solve for the optimal filter coefficients. Compare your answer to the result obtained in class using the matrix/vector approach. In this case, what can you say about the correlation matrix \( R \)?

(c) Assume that \( x(n) \) is a general random sequence, and again solve for the optimal coefficients. How does your answer compare to that obtained in class? Are the solutions identical? (Hint: You must determine the appropriate range of values for \( k \) and \( m \) in the MSE equation, and for the time argument \( n \) of the filter impulse response.)