Department of Electrical & Computer Engineering
University of California, Santa Barbara

ECE 245
Spring 2008
Shynk
H.O. #3

HOMEWORK #1

Due Thursday, April 10, 2008 (5:00 p.m.)

Reading: Background, Chapters 1 (review) and 2 (2.1–2.7)

Problems:

1. Chapter 1: Problem 3
2. Chapter 1: Problem 6
3. Chapter 1: Problem 7
4. Chapter 1: Problem 10
5. Find $\phi_{xx}(k)$ for

$$\Phi_{xx}(z) = \frac{1}{(1+0.3z^{-1})(1-0.4z^{-1})(1+0.3z)(1-0.4z)}.$$

6. Consider the following single-input, two-output system. Assume that $x(n)$ and $v(n)$ are uncorrelated, and

$$\phi_{xx}(k) = \delta(k), \quad \phi_{vv}(k) = \frac{1}{2}\delta(k)$$
$$h_1(n) = \left(\frac{1}{4}\right)^n u(n), \quad h_2(n) = \left(\frac{1}{3}\right)^n u(n).$$

   (a) Find the noncausal Wiener filter for estimating $y_1(n)$ from $y_2(n)$.
   (b) Determine the minimum mean-square error (MSE) $\xi_{\min}$.
   (c) What is $\xi_{\min}$ without filtering $y_1(n)$ (i.e., the filter is zero)? Compare this result to that obtained in part (b) and comment.

7. Consider the MSE equation:

$$\xi = \phi_{dd}(0) - 2\sum_m h(m)\phi_{dx}(m) + \sum_m \sum_k h(m)h(k)\phi_{xx}(k-m)$$

where $d(n)$ is the desired signal and $x(n)$ is the filter input. Impose the constraint that $\{h(n)\}$ be the coefficients of an N-length tapped-delay line (i.e., FIR filter).

(a) Determine the corresponding Wiener-Hopf equation.

(b) Assume that $x(n)$ is a white-noise sequence, and solve for the optimal filter coefficients. Compare your answer to the result obtained in class using the matrix/vector approach. In this case, what can you say about the correlation matrix R?

(c) Assume that $x(n)$ is a general random sequence, and again solve for the optimal coefficients. How does your answer compare to that obtained in class? Are the solutions identical? (Hint: You must determine the appropriate range of values for k and m in the MSE equation, and for the time argument n of the filter impulse response.)