Department of Electrical & Computer Engineering
University of California, Santa Barbara

ECE 245
Spring 2008
Shynk
H.O. #7

HOMEWORK #3

Due Thursday, April 24, 2008 (5:00 p.m.)

Reading: Chapter 5 (5.1–5.7)

Problems:

1. Chapter 2: Problem 15
2. Chapter 4: Problem 5
3. Chapter 4: Problem 12
4. Suppose that we have a tapped-delay line with mean-square error (MSE)

$$\xi = 2w_0^2 + 2w_1^2 + 2w_0w_1 - 14w_0 - 16w_1 + 42$$

where $\phi_{xx}(0) = 2$ and $\phi_{xx}(1) = 1$.

(a) Assume that the gradient estimate is based on 100 observations of the error at each perturbed weight setting. Find the covariance matrix of the gradient estimate, assuming that $e(n)$ has a Gaussian distribution with zero mean and variance σ^2.

(b) Determine the weight-vector covariance matrix. Assume that the steepest-descent (SD) algorithm is used with step-size parameter α equal to one half its maximum stable value, and that there are 100 error observations for each weight setting. Find the excess mean-square error and the misadjustment M.

5. Consider using the SD algorithm to solve the Wiener-Hopf equation.

(a) Show that the kth component of the weight vector can be written in rotated (but not translated) coordinates as follows:

$$W'_k(n+1) = (1 - 2\alpha\lambda_k)W'_k(n) + 2\alpha P'_k$$

where P' is the rotated cross-correlation vector.

(b) Using the one-sided z-transform, find a closed-form expression for $W'_k(n)$ in terms of the initial weight $W'_k(0)$.

(c) Using the result in part (b), find the limit as $n \to \infty$. Is your answer consistent with that obtained using the rotated and translated weight vector V'?

6. Derive a bound on α for the SD algorithm such that the rotated and translated weights $V'(n)$ converge in a monotonic manner to their optimal values (i.e., there is no overshoot or oscillation as the uncoupled weights evolve). How does your answer compare to that obtained for Newton's method (NM)?

7. Suppose that $e(n)$ is a Gaussian random process with zero mean and variance σ^2. Let

$$\hat{\xi} = \frac{1}{N} \sum_{k=0}^{N-1} e^2(k)$$

and assume that the $\{e(k)\}$ are independent. Note that $E[\hat{\xi}] = \sigma^2 = \xi$, i.e., $\hat{\xi}$ is an unbiased estimate of the MSE. Derive an expression for the variance of $\hat{\xi}$.