Department of Electrical & Computer Engineering
University of California, Santa Barbara

ECE 245
Spring 2008
Shynk
H.O. #11

PROGRAMMING ASSIGNMENT

Due Thursday, June 5, 2008 (5:00 p.m.)

A second-order autoregressive (AR) process $\{u(n)\}$ is described by the following difference equation:

$$u(n) + a_1 u(n-1) + a_2 u(n-2) = v(n)$$

where $v(n)$ is a white-noise process with zero mean and variance σ_v^2. The parameters of this process are assigned one of the following three sets of values:

(i) $a_1 = -0.1, \quad a_2 = -0.8, \quad \sigma_v^2 = 0.27$

(ii) $a_1 = -0.1636, \quad a_2 = -0.8, \quad \sigma_v^2 = 0.119$

(iii) $a_1 = -0.196, \quad a_2 = -0.8, \quad \sigma_v^2 = 0.014$

The AR process is applied to a two-weight *linear predictor*. Repeat Problems 1 – 4 below for each set of parameters. Note that only one computer program should be written; use a separate subroutine for each algorithm.

1. **Steepest-Descent (SD) Algorithm**:

   (a) Calculate the eigenvalues of the 2×2 input autocorrelation matrix R. Determine the corresponding stable range for the step-size parameter α.

   (b) Using two elements of the weight-error vector (i.e., $v_1(n)$ and $v_2(n)$) as the variables in the SD algorithm, construct loci for constant values of the mean-square error (MSE). Superimpose the trajectory that describes the change in the coordinates $v_1(n)$ and $v_2(n)$ with time n, assuming that $\alpha = \alpha_{\max}/10$ and the initial values of the predictor weights are zero.

   (c) Repeat the computations in part (b) using the original weights $w_1(n)$ and $w_2(n)$ as the variables in the SD algorithm.

   (d) Plot the learning curves for the following three step-size parameter values: $\alpha = \alpha_{\max}/100$, $\alpha = \alpha_{\max}/10$, and $\alpha = \alpha_{\max}/5$.

2. **Least-Mean-Square (LMS) Algorithm**:

   (a) What is the stable range of values for the step-size parameter α for convergence in the mean?

   (b) What is the stable range of values for the step-size parameter α for convergence in the mean square?

   (c) Generate a 1000-sample sequence representing the AR process $\{u(n)\}$. By averaging over 200 independent trials, plot the MSE learning curves of the LMS algorithm for the following three step-size parameter values: $\alpha = \alpha_{\max}/100$, $\alpha = \alpha_{\max}/10$, and $\alpha = \alpha_{\max}/5$.

   (d) Estimate the mean values of $w_1(\infty)$ and $w_2(\infty)$ for the three step-size parameter values in part (c). You may do this by averaging the steady-state values of the weights (obtained from the last iteration) over 200 independent trials of the experiment. Compare your results with the theoretical values.

   (e) Repeat parts (a) – (d) using the sign-data LMS algorithm:

   $$W(n+1) = W(n) + \alpha\ e(n)\mathrm{sgn}[U(n)]$$

   where the signum function retains the sign of each component of $U(n)$.

   (f) Repeat parts (c) and (d) using the sign-error LMS algorithm:

   $$W(n+1) = W(n) + \alpha\ \mathrm{sgn}[e(n)]U(n).$$

3. **Recursive-Least-Squares (RLS) Algorithm**:

   (a) Repeat parts (c) and (d) of Problem 2 using the RLS algorithm.

   (b) Suppose that the AR parameter a_1 is made positive; otherwise, its magnitude is left as specified for each of the three cases. How would you assess the effect of this change on the performance of the RLS algorithm?

4. **Least-Squares-Lattice (LSL) Algorithm:**

   (a) Repeat part (c) of Problem 2 for a two-stage lattice predictor.

   (b) By averaging the steady-state values of the two reflection coefficients over 200 independent trials, compute their mean values and compare them with the theoretical results.

   (c) Repeat parts (a) and (b) of this problem for a three-stage lattice predictor.

5. **Discussion**: Compare and discuss your results. What can you say about the computational complexity and speed of convergence for each algorithm?