Department of Electrical & Computer Engineering
University of California, Santa Barbara

ECE 245
Spring 2011
Shynk
H.O. #14

HOMEWORK #5

Due Friday, May 6, 2011 (5:00 p.m.)

Reading: Chapter 8 (8.1–8.8)

Problems:

1. Chapter 6: Problem 1
2. Chapter 6: Problem 7
3. Verify equations (42) and (46) in the paper by Horowitz and Senne.
4. Consider the modified LMS algorithm

$$W(n+1) = W(n) + 2\alpha R^{-1} X(n) e(n)$$

where R is the autocorrelation matrix of the input signal $X(n)$.

(a) Find the range of values for α such that $W(n)$ converges in the mean.

(b) Specify the value of α which results in one-step convergence in the mean.

(c) Derive an equation for this algorithm similar to (37) in the paper mentioned above, and indicate how the new equation is different from (37).

5. Consider a system-identification application where the desired response $d(n)$ is obtained as follows (using mixed time-domain and z-domain notation):

$$d(n) = G(z)x(n) + v(n)$$

where the input signal $x(n)$ is a zero-mean random sequence with autocorrelation matrix R, and $G(z)$ is the system to be identified. The additive white-noise sequence $v(n)$ has zero mean and variance σ_v^2, and is uncorrelated with $x(n)$. Assume that the LMS algorithm with M coefficients is used to identify

$$G(z) = 1 - 0.5z^{-1} + 0.2z^{-2}$$

where

$$R = \begin{bmatrix} 1 & 0.3 & 0 \\ 0.3 & 1 & 0.3 \\ 0 & 0.3 & 1 \end{bmatrix},$$

$\sigma_v^2 = 0.02$, and $M = 3$.

(a) Determine $\xi_{\min}$ for the adaptive filter, and find the Wiener weight vector.

(b) For convergence in the mean, find the stable range of α.

(c) Repeat part (b) for convergence in the mean square (using the results of Horowitz and Senne).

(d) Plot $D(z)$ for real z (from the paper mentioned above) showing all M roots for a value of α that satisfies (b) but not (c).

6. **(a)** Verify that the following two LMS stability conditions are equivalent:

$$0 < \mu < 1/3\lambda_{\max} \qquad \text{and} \qquad \eta(\mu) < 1$$
$$0 < \mu < 1/2\lambda_{\max} \qquad \text{and} \qquad \eta(\mu) < 1$$

where

$$\eta(\mu) = \sum_{k=0}^{M-1} \frac{\mu\lambda_k}{1 - 2\mu\lambda_k}.$$

(b) Find examples for $M = 3$ where (i) $\eta(\mu) > 1$, $\mu < 1/2\lambda_{\max}$, and (ii) $\eta(\mu) < 1$, $\mu > 1/2\lambda_{\max}$.