Department of Electrical & Computer Engineering
University of California, Santa Barbara

ECE 245
Spring 2011
Shynk
H.O. #11

PROGRAMMING ASSIGNMENT

Due Thursday, June 2, 2011 (4:00 p.m.)

A second-order autoregressive (AR) process $\{u(n)\}$ is described by the following difference equation:

$$u(n) + a_1 u(n-1) + a_2 u(n-2) = v(n)$$

where $v(n)$ is a white-noise process with zero mean and variance σ_v^2. The parameters of this process are assigned one of the following three sets of values:

(i) $a_1 = -0.1, \quad a_2 = -0.8, \quad \sigma_v^2 = 0.27$

(ii) $a_1 = -0.1636, \quad a_2 = -0.8, \quad \sigma_v^2 = 0.119$

(iii) $a_1 = -0.196, \quad a_2 = -0.8, \quad \sigma_v^2 = 0.014$

The AR process is applied to a two-weight *linear predictor*. Repeat Problems 1 – 3 below for each set of parameters. Note that only one computer program should be written; use a separate subroutine for each algorithm.

1. **Steepest-Descent (SD) Algorithm**:

   (a) Calculate the eigenvalues of the 2×2 input autocorrelation matrix R. Determine the corresponding stable range for the step-size parameter α.

   (b) Using two elements of the weight-error vector (i.e., $v_1(n)$ and $v_2(n)$) as the variables in the SD algorithm, construct loci for constant values of the mean-square error (MSE). Superimpose the trajectory that describes the change in the coordinates $v_1(n)$ and $v_2(n)$ with time n, assuming that $\alpha = \alpha_{\max}/10$ and the initial values of the predictor weights are zero.

   (c) Repeat the computations in part (b) using the original weights $w_1(n)$ and $w_2(n)$ as the variables in the SD algorithm.

   (d) Plot the learning curves for the following three step-size parameter values: $\alpha = \alpha_{\max}/100$, $\alpha = \alpha_{\max}/50$, and $\alpha = \alpha_{\max}/10$.

2. **Least-Mean-Square (LMS) Algorithm**:

   (a) What is the stable range of values for the step-size parameter α for convergence in the mean?

   (b) What is the stable range of values for the step-size parameter α for convergence in the mean square?

   (c) Generate a 1000-sample sequence representing the AR process $\{u(n)\}$. By averaging over 200 independent trials, plot the MSE learning curves of the LMS algorithm for the following three step-size parameter values: $\alpha = \alpha_{\max}/1000$, $\alpha = \alpha_{\max}/100$, and $\alpha = \alpha_{\max}/10$.

   (d) Estimate the mean values of $w_1(\infty)$ and $w_2(\infty)$ for the three step-size parameter values in part (c). You may do this by averaging the steady-state values of the weights (obtained from the last iteration) over 200 independent trials of the experiment. Compare your results with the theoretical values.

   (e) Repeat parts (a) – (d) using the sign-data LMS algorithm:

   $$W(n+1) = W(n) + \alpha\ e(n)\text{sgn}[U(n)]$$

   where the signum function retains the sign of each component of $U(n)$.

   (f) Repeat parts (c) and (d) using the sign-error LMS algorithm:

   $$W(n+1) = W(n) + \alpha\ \text{sgn}[e(n)]U(n).$$

   (g) Repeat parts (c) and (d) using the normalized LMS algorithm:

   $$W(n+1) = W(n) + \alpha\ \frac{e(n)U(n)}{U^T(n)U(n)}.$$

3. **Recursive-Least-Squares (RLS) Algorithm**:

   (a) Repeat parts (c) and (d) of Problem 2 using the RLS algorithm.

   (b) Suppose that the AR parameter a_1 is made positive; otherwise, its magnitude is left as specified for each of the three cases. How would you assess the effect of this change on the performance of the RLS algorithm?

4. **Discussion**: Compare and discuss your results. What can you say about the computational complexity and speed of convergence for each algorithm?