UNIVERSITY OF CALIFORNIA, SANTA BARBARA
Department of Electrical and Computer Engineering

ECE 277B | Fall 2007 | Instructor: Ken Rose

Homework Assignment #4

(Due on Monday 11/19/2007)

Problem # 1. Text, Prob. 2.29. (Falls within midterm scope).

Problem # 2.

a) Consider N points in general position in two dimensional feature space. Select one of the points. How many linear dichotomies of the rest are possible if the separating line is constrained to contain the chosen point?

b) The previous part should have been very easy to do from your class notes. It was meant to prepare you for this part. Now we consider N such points in three dimensional space and require that the separating plane contains two of the N vectors. How many linear dichotomies are possible?

Problem # 3. It is intuitively obvious that for a fixed dimension l, as we increase the number of vectors in the sample set $N \to \infty$, the probability that a randomly chosen dichotomy is linear goes to zero. Prove it using the following steps:

a) Let $\phi(n, k)$ be given by

$$\phi(n, k) = \sum_{i=0}^{k} \binom{n}{i}$$

Show that $\phi(n, k) \leq n^k + 1$.

b) Use part (a) to show that as $N \to \infty$

$$\text{Pr[linear dichotomy]} = \frac{O(N, l)}{2^n} \to 0.$$

(You may assume the result of part (a) even if you could not prove it.)

Problem # 4. Let $\mathcal{Y} = \{y_1, y_2, \ldots, y_N\}$ be the set of "extended normalized" vectors corresponding to a linearly separable training set $\mathcal{X}$, i.e., there exists some weight vector a such that $a^T y_i > 0$, $i = 1, 2, \ldots, N$. Consider the so-called "batch" update rule:

$$a_{k+1} = a_k + \sum_{y \in \mathcal{Y}_k} y$$

where $\mathcal{Y}_k = \{y \in \mathcal{Y} : a_k^T y \leq 0\}$ is the subset of $\mathcal{Y}$ that is misclassified by a_k. Show that the sequence $\{a_k\}$ converges to a weight vector in the solution region. Note that this is almost but not exactly the algorithm discussed in class.