# ECE 594C: Addendum to Lab 2 (Hypothesis Testing for fMRI?)

**Assigned:** November 4, 2010
**Due:** Date to be negotiated based on other course to-dos.

We have seen in class how noisy fMRI data is. Yet we are able to recognize the active voxels quite well, at least for Ben's data, using a variety of methods. It would be nice to get some insight into this. To this end, I am proposing a hypothesis testing model as follows.

Suppose that we wish to decide whether a group of voxels is active or not. As in class on November 3, consider the time instants at and immediately following a stimulus as the observed waveform (e.g., consider 5 samples, including the TR at which the stimulus is applied). Collecting these readings over both stimuli and voxels, we have a collection of waveforms $\{y_i(t), i = 1, ..., N\}$, where $N$ is the product of the number of stimuli and the number of voxels (about $80 \times 27 = 2160$ in our case). As in class, assume that the time course for each voxel has already been de-meaned.

Let us now consider the following simple-minded hypothesis testing model.

$$\begin{array}{l} H_1 : \mathbf{y} = \mathbf{s} + \mathbf{n}_i \ , \quad i = 1, ..., N \\ H_0 : \mathbf{y} = \mathbf{n}_i \ , \quad i = 1, ..., N \end{array} \tag{1}$$

where $\mathbf{s}$ is a BOLD response (assumed fixed neither of which assumptions actually hold), $\mathbf{n}_i$ are i.i.d. Gaussian noise vectors, with each element having mean zero and variance $\sigma^2$. The signal $s(t)$ being fixed across $i$ does not hold because the BOLD response right after a stimulus also depends on the sequence of stimuli before it. The noise independence across $i$ does not hold, since many of the observations are overlapping for closely spaced stimuli. The noise variance may not be the same for each hypothesis. And of course, Gaussianity need not hold either. Despite all the potential inaccuracies in the model, we hope to gain some insight into achievable performance.

1) Using Ben's data, estimate $\mathbf{s}$ by averaging across the active voxels (let us agree to use length 5, so we get consistent results across groups). For many of the estimates that follow, we assume that this is the true signal vector $\mathbf{s}$, and that the model (1) holds.

2) Estimate the noise variance $\sigma^2$.

3) For the model (1, show that the maximum likelihood (ML) decision rule is given by

$$Z = \frac{1}{N} \sum_{i=1}^{N} \langle \mathbf{y}_i, \mathbf{s} \rangle \underset{H_0}{\overset{H_1}{\gtrless}} ||\mathbf{s}||^2/2 \tag{2}$$

4) Using the estimated value of $\mathbf{s}$ as in 1), compute the decision statistic $Z$ in (2) for the active and inactive voxels and check their value relative to the threshold. Would you be making the correct decision?

5) Find an expression for the error probability of the ML rule under the model (1) as a function of $N$, $\mathbf{s}$ and $\sigma^2$: show that it depends only on $N$ and the $SNR = \frac{||\mathbf{s}||^2}{\sigma^2}$. What is the SNR for our dataset using the estimates in 1) and 2)? What is the estimated error probability for $N = 2160$? Comment on whether the numbers give you confidence in fMRI imaging.

6) Now, go back to the model (1), but this time assume that the waveform $\mathbf{s}$ is unknown, but that we approximate its shape with a waveform $\hat{\mathbf{s}}$ such that the normalized correlation between $\mathbf{s}$ and $\hat{\mathbf{s}}$ is $\rho$. For simplicity, normalize $\hat{\mathbf{s}}$ to unit norm. Thus, let us now use the decision statistic

$$\hat{Z} = \frac{1}{N} \sum_{i=1}^{N} \langle \mathbf{y}_i, \hat{\mathbf{s}} \rangle \tag{3}$$

and compare it with a threshold $\gamma$. Find an expression for the probabilities of false alarm and miss as a function of $\gamma$, $\rho$, $\mathbf{s}$, $\sigma^2$ and $N$.

7) Consider several fixed choices of $\hat{\mathbf{s}}$, including a standard BOLD response as in the correlation method, as well as simple triangular or rectangular pulses. For each case, estimate $\rho$ based on taking the

normalized correlation with the estimate of $\mathbf{s}$ in 1). Based on parameter estimates from the data, plot the miss probability versus the false alarm probability as the threshold varies (note: this is just an estimated plot based on an unrealistic model). Comment on the efficacy of the correlation method.

8) For each of the waveform choices in 7), compute the decision statistic $\hat{Z}$ and compare it with the threshold you would choose if you used the $\rho$ values computed in 7). Would you have made the right decision for the active versus inactive voxels?

9) Is the assumption that the noise variance is the same for $H_0$ and $H_1$ accurate? If not, how would your decision rules get modified? It is up to you as to how far you pursue this line of thought.

9) If you were using the Generalized Likelihood Ratio Test (GLRT) detector, assuming that $\mathbf{s}$ and $\sigma^2$ are unknown, what is the form of decision statistic that you would get? Again, it is up to you as to how far to pursue this.

The goal of Lab 2 and this addendum is to explore a set of ideas in the controlled setting of Ben's data. In the project, different groups will apply these and other ideas to different fMRI data.