ECE 594C: Problem Set/Lab 1

Assigned: October 6
Due: October 20 (in class)
Preparation: Browse fastICA algorithm description in the relevant papers on course home page, download fastICA code or alternative ICA code (e.g., extended Infomax, JADE) or write your own.

Generate the following discrete-time signals of length N:
$s_1[n] = 1 + \cos(\pi n/2)$, $n = 0, 1, ..., N-1$. (This is meant to model line noise in EEG).
$s_2[n] = \sum_{i=1}^{K} p[n - N_i]$ is a sum of pulses occurring at random instances $\{N_i\}$, where $N_m = T_1 + ... + T_m$ and $\{T_i\}$ are i.i.d. geometric random variables with mean 10. The number of terms K in the summation is random, and is the largest integer such that $N_K \leq N$.
We can choose the pulse p to be triangular: set $p(n) = n$ for $0 \leq n \leq 3$ and $p[n] = 0$ otherwise. (This is meant to model randomly occurring impulsive events.)

Each group should turn in a report with answers to the following questions. Attach any Matlab code that you have written, and mention which downloaded software, if any, you have used.

(a) Plot the empirical densities of the two sources, either using histograms from simulations or via analysis, or both.

(b) Determine for each source if it is superGaussian or subGaussian.

(c) Express the sources as an $2 \times N$ matrix $\mathbf{S}$, with each row corresponding to the time course of a source, with ith column $\mathbf{S}_i = (s_1[i], s_2[i])^T$, $i = 0, 1., , , .N-1$. Generate a 10×2 mixing matrix $\mathbf{A}$ with entries given by i.i.d. Gaussian random variables with mean 0 and variance one. Generate the mixed observations $\mathbf{Y} = \mathbf{AS}$, which is a $10 \times N$ matrix, with the ith column $\mathbf{Y}_i = \mathbf{AS}_i$ representing a snapshot of the observations at time i, $i = 0, 1, ..., N-1$.

(d) Compute the empirical mean vector for the observations $\mathbf{m}_Y$ (this is a 10×1 vector) and subtract it out from the observations to get $\tilde{\mathbf{Y}} = \mathbf{Y} - \mathbf{m}_Y$ (in Matlab, this subtracts out the mean from every column of $\mathbf{Y}$)

(e) Find the empirical covariance matrix $\mathbf{C}_Y$ for the observations and do an eigendecomposition. Document the significant drop you see after the first two eigenvalues.

(f) Letting λ_1 and λ_2 denote the two largest eigenvalues and $\mathbf{v}_1$ and $\mathbf{v}_2$ the corresponding orthonormal eigenvectors, project the observations $\tilde{\mathbf{Y}}$ down to the space spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$ and whiten them to get a $2 \times N$ observation matrix $\mathbf{Z}$. This is called a Principal Component Analysis. While you can and should do this compactly using matrices in Matlab, let us write out what it does to the observations at time i: we get

$$\mathbf{Z}_i = (\frac{1}{\sqrt{\lambda_1}}\mathbf{v}_1^T(\mathbf{Y}_i - \mathbf{m}_Y), \frac{1}{\sqrt{\lambda_2}}\mathbf{v}_2^T(\mathbf{Y}_i - \mathbf{m}_Y))^T$$

The matrix $\mathbf{Z}$ now represents zero mean, “sphered” observations.

(g) Use your favorite ICA algorithms to separate the two sources using the observations $\mathbf{Z}$. Specifically, find a 2×2 matrix $\mathbf{W}$ such that

$$\hat{\mathbf{S}} = \mathbf{WZ}$$

optimizes some contrast function. If you use fastICA, I suggest trying a couple of different functions, such as $G(u) = u^4$ and $G(u) = \log\cosh u$.

(h) Plot the time courses of the extracted sources and compare with those of the original sources.

(i) How would you estimate the original mixing matrix $\mathbf{A}$? (up to scale factor and permutation)

(j) Discuss any big picture observations, as well as any difficulties or questions that you encountered in this lab. Try with other sources if you have time.

Please feel free to discuss questions or difficulties across groups and with the instructor.