Homework 2  (due 1/27)

2.1 Rimless Wheel (RW) return map. In this problem, you are asked to annotate Figure 1, in which the solid (blue) lines show the return map relating the post-collision angular velocity after a given step, $\Omega_n^+$, to the velocity after the $n$th step, $\Omega_{n+1}^+$. (Note that Coleman02 [1] plots return maps on “Z”; Figure 1 maps $\Omega$ itself.) Hint: Grab a ruler! Also, download the “RW cheatsheet” from the homework website. (Assume a unit length leg ($L = 1$ [m]), point mass ($J = 0$), and 8 spokes ($2\alpha = 2\pi/8$).)

![Figure 1: Rimless Wheel return map.](image-url)
2.1 Rimless Wheel (RW) return map. (continued...)

a) From Figure 1, what (approximately) is the value of the rolling fixed point, \( \Omega^+ \)?

b) Given your estimate of \( \Omega^+ \) (from part a), estimate the angle of the ground, \( \gamma \).

c) What is the slope of the return map near \( \Omega^+ \)?

d) Given: "\( \Omega^+_n = \Omega^+_* + 0.01 \)”, estimate \( \Omega^+_n \). *Hint: Use part c."

e) Graphically find the basins of attraction for the standing and rolling fixed points, using a “stair step” approach. Shade in the portions corresponding to the “rolling” fixed point. *Hint: download the m-files called “RW_return.m” and “RW_stairstep.m” from the homework webpage. Run RW_return to create a return map. Then run RW_stairstep to graphically click on the figure to create stair steps to the appropriate fixed point for this initial condition. You can continue to run RW_stairstep again and again, to overlay many stair step traces. Note that the return map shown will (intentionally) be for a different value of ground slope, \( \gamma \)."

2.2 Limping Rimless Wheel. Consider a rimless wheel with 8 evenly-space spokes (i.e., same angular spacing) but where every other leg is 1.05 meters while the others are 0.95 meters. For a given range of ground slope, and for the right initial conditions, this wheel will converge to a constant, “limping” gait: every impact landing on a “long” leg will approach some value \( (\Omega^+_*)_{long} \), and every impact landing on a “short” leg will approach some value \( (\Omega^+_*)_{short} \).

a) Derive these two values, \( (\Omega^+_*)_{short} \) and \( (\Omega^+_*)_{long} \) for rolling on a downhill slope of 10°. *Hint 1: Use the “RW cheatsheet” you downloaded for problem 1. Hint 2: Recall the law of cosines, to calculate any unknown-but-needed variables from Figure 2: \( L_a^2 = L_b^2 + L_c^2 - 2bc \cos \theta_a \)

![Figure 2: Limping rimless wheel geometry.](image)
2.3 **Compass Gait (CG) walker phase portrait.** In this problem, you are asked to analyze a model of passive dynamic walking (PDW).

a) From the phase portrait shown in Figure 3, estimate the state $X^* = [\theta_{ns}, \theta_s, \dot{\theta}_{ns}, \dot{\theta}_s]^T$ of the fixed point for a Poincaré section at the “post-collision” state.

![Figure 3: Compass Gait phase portrait.](image)

b) Download all the “compass gait simulation files” for MATLAB from the web, and use the function “cg_step.m” [which requires the other m-files to run] to create the 4x4 Jacobian. 

*Hint 1*) Start with your estimate of the fixed point from part a, above. Use this 4x1 state as an input to the function “cg_step”, and you will get the next post-impact state as an output. e.g., create a “for loop” where “Xfixed_approx = cg_step(Xfixed_approx);”. This will simulate successive steps, each of which should asymptotically approach the true fixed point.
**Hint 2)** Once you have a better estimate of the fixed point, you can now use “cg_step” to generate each of the 4 columns of the Jacobian, $J$. See the DW2008 Tutorial Assignment and Tutorial Answers for better details:


Also look for additional course notes on the Jacobian on the “notes” website.

**c)** Find the eigenvalues of the Jacobian (which give information about the local stability of the step-to-step transitions in a neighborhood near the fixed point). Is the fixed point stable? Why (or why not)?

**d)** One eigenvalue should be very close to zero (numerically). Why is this so? *(Read the DW2008 Assignment and Answers, mentioned in part a...)*

**e)** Looking again at the phase plot in Figure 3, is $\theta_s(t)$ (the stance leg angle) monotonic over a given step? Is $\theta_{ns}(t)$ (the non-stance leg angle) monotonic over a step?

### 2.4 [EXTRA CREDIT] Compass Gait period-doubling bifurcations.

Something interesting happens to the gait of the passive compass gait walker in problem 2.3 as the slope of the terrain gets steeper: It begins to “limp”! Beyond some particular slope angle, a walker which begins with appropriate initial conditions (within the basin of attraction for passive walking) will converge to a gait where every post-collision state for the “left” foot approaches one particular set of values while every footprint with the “right” foot approaches a different set.

**a)** First, just find a slope angle where this occurs!

To do this, you can start with your fixed point solution from problem 2.3, increase the slope of the ground ($\gamma$) in the simulation code by a small amount, and then take several steps until the walker converges to a new post-collision fixed point. Repeat until you notice the post-collision states alternate with every other step. To change the value of $\gamma$, run the MATLAB function called cg_gam_slope. For instance, entering the command “cg_gam_slope(3.2*pi/180)” would reset the ground slope to $3.2^\circ$.

If you change the slope by only a small amount, you should be able to begin with your previous fixed point solution and simulate several steps. Create a loop where you increase the slope and find a new fixed point, again and again – and you should eventually notice that the post-collision state alternates between two sets of values. This is a bifurcation, and as the slope gets steeper and steeper, you should notice a gait which repeats every 4 steps, then every 8 steps, ... and eventually becomes “chaotic” (where you can no longer find any repeating pattern at all – although the walker still continues to walk on and on and on in an apparently stable way).

**b)** Now that you have a slope that exhibits a “period-2” gait, find the one-step Jacobian, as you did in problem 2.3 b). What are the eigenvalues? Do they all have a magnitude less than one (indicating stability for the step-to-step transitions)?
c) Finally, create a Jacobian on a two-step mapping. Use the same general procedure you used in problem 2.3, but execute two steps instead of one step when filling in each column. (e.g., cg_step(cg_step(Xleft))) What are the eigenvalues now, and what do they tell us about the stability of the two-step return map?

References