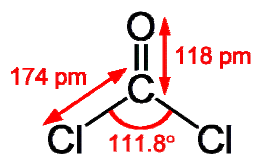
ECE594I, Prof. Brown, Fall Quarter 2009

Homework Problems #2: THz Propagation Phenomenology

1. Vapor State: Molecular Rotational Transitions. Along with water, the Earth's atmosphere can hold many trace molecules of interest for global warming, and homeland security, many of which have measurable resonances in the THz or millimeter-wave regions.
   (a) A molecule of interest for the "greenhouse effect" and global warming is nitrous oxide, N_2O. Using the rigid rotor model to estimate the resonance frequencies between 100 GHz and 1.0 THz
   (b) A molecule of interest for homeland security is phosgene, $COCl_2$ which is a planar molecule having the structure shown in the figure below. It can be shown for any four-atom planar molecule that the moment of inertia about the axis perpendicular to the plane is given by $I = \frac{m_1 m_2 d_{12}^2 + m_1 m_3 d_{13}^2 + m_1 m_4 d_{14}^2 + m_2 m_3 d_{23}^2 + m_2 m_4 d_{24}^2 + m_3 m_4 d_{34}^2}{m_1 + m_2 + m_3 + m_4}$ where d_{ij} is the distance between the i and j atoms. Use this to predict the lowest rotational resonance frequency of phosgene about the perpendicular axis.



2. Liquid State: Debye Model. Water, the "liquid of life", has very interesting dielectric properties in the THz region. Assuming fresh water, a temperature of 25^{o}C, the "single" Debye model, and five frequencies of interest of 10, 100, 300, 1000, and 3000 GHz
   (a) find or calculate the Debye relaxation time and calculate the real and imaginary parts of the dielectric function.
   (b) calculate the complex electric-field transmission and reflection coefficients.
   (c) calculate the power transmission and reflection coefficient
   (d) calculate the power absorption coefficient, the attenuation length, and the emissivity.
3. Solid State: Although a century old, the Drude model is still the first best estimate for the complex dielectric function of solid-state conductors at RF and THz frequencies.
   (a) Boron-doped p-silicon is probably the most common single-crystal semiconductor substrate in the world, being used pervasively in CMOS fabrication. The resistivity of these substrates typically falls in the range 0.1 to 10 Ω-cm, and the permittivity is always $\varepsilon_0 = 12$. Is this resistivity high enough for low-loss THz applications ? To address this, estimate the complex conductivity at 300 GHz assuming a momentum relaxation time of 0.3 ps for resistivity values of 0.1, 1.0, and 10.0 Ω-cm.
   (b) Use the result of (a) to estimate the complex dielectric function at 300 GHz for the same three resistivity values.
   (c) Use the result of c) to estimate the absorption coefficient [cm^{-1}] and penetration depth for the same three resistivity values.
4. Scattering Effects: THz propagation through fog (plus good exercise in probability)
   It is well known that microwaves and millimeter waves have far better transmission through clouds and fog than infrared or visible light. But what about THz transmission ? Provided that the water particles in the fog are all spherical and much smaller than a wavelength so that the scattering is in the Rayleigh limit, the radar cross section of an individual water

droplet can estimated by the expression derived from effective-medium theory of optics:

$$\frac{\sigma}{\pi R^2} = 4\left|\frac{\varepsilon_c - 1}{\varepsilon_c + 2}\right|^2 \left(\frac{2\pi R}{\lambda}\right)^4$$ where R is the radius and ε_c is the complex dielectric function.

Assume the distribution function of droplets can be approximated as a Rayleigh pdf with the most likely radius being 10 micron and the concentration of fog particles being 50 cm^{-3}.

(a) Find the variance of the Rayleigh-distributed fog (or cloud).

(b) Find the mean value of the radar cross section at 10 GHz, 100 GHz, 300 GHz, and 1.0 THz using the “single-Debye” model for the dielectric function of water.

(c) Assuming single-particle scattering only and validity of the Beer law of transmission, estimate the attenuation constant (units of cm^{-1}) and the transmission loss in dB/km at 100 GHz, 300 GHz, and 1.0 THz.