ECE594I, Prof. Brown, Fall Quarter 2009

HW#5: Noise and Detection

1. (Device noise) So many devices or parts of devices in RF electronics display the canonical junction rectification I-V curve, $I = I_0\,[\exp(eV/k_BT) - 1]$. Suppose the device is a one-port that displays generalized Johnson-Nyquist noise and full shot noise, and operates at 300 K into a load resistance R_L .

   (a) For $R_L = 0$ (short-circuit case) and zero bias, what is the RMS current fluctuation through the device in terms of I_0 ? Interpret the answer in terms of the necessary device physics behind I_0 .
   (b) If $R_L = 0$ (short-circuit case), at what bias voltage does the RMS shot-noise current equal the RMS thermal noise current ?
   (c) Suppose this device is connected to a standard $R_L = 50\text{-}\Omega$ load. At what bias voltage does the noise power generated in the load from the rectifier equal the noise power from the load itself ?
   (d) Express the noise power in (c) as a power spectral density of current fluctuations.

2. (System noise figure) Every linear component in a receiver chain can be represented by a noise figure, $F_i = (SNR_{in}/SNR_{out})i = 1 + (N_{add})_i/(G_i\, k_BT_{300}\, B_i)$, where $(N_{add})_i$ is the noise added by the ith component, G_i is its gain, and B_i is its RF bandwidth.
   (a) Use these definitions to derive Friis' formula for total noise figure, F_T.
   (b) Suppose a receiver chain consists of a bandpass filter (BPF) having G = -3.0 dB and F = 0.5 dB, an LNA having G = Y dB and F = 2.0 dB, a mixer having G = -5 dB and NF = 6 dB, and a power amp having G = 30 dB and F = 10 dB. What is the total noise figure if Y = 30 dB ? [clue: easy and fun with an Excel spreadsheet]
   (c) In the limit of very large Y, the noise figure approaches asymptotically a "front-end" limited value. What is this value ? And to what value can Y be reduced so that the actual F is within 1 dB of the "front-end" limit ?
   (d) Suppose the BPF and LNA are interchanged. What is the new F_T if Y = 30 dB ?

3. (Power-Law Detection) A common detector type used in RF sensors, particularly at low signal levels, is the square-law device represented by $X_{out}(t) = A[X_{in}(t)]^2 \equiv \Re P_{in}$.

(a) Of all the possible power-law detectors $X_{out} = A(X_{in})^n$, show that the square-law is the most effective at maximizing the ratio of baseband (dc) power to RF (ac) power.
(b) Assume a nonlinear diode is biased into a region where the I-V curve can be approximated to have the form $I = BV^2$. Use Taylor's theorem to write a general expression for the rectified current $\delta I = I - I_0$ in terms of the voltage difference δV induced at some bias point V_0. Re-express this result as $\delta I = \Re\, P$, where $P = (\delta V)^2/R$ and $\Re$ is the short-circuit responsivity. Write an expression for $\Re$ in terms of known derivatives.
(c) One of the most useful devices through the history of THz sensors has been the metal-semiconductor, or Schottky, diode. A convenient model I-V curve for Schottky diodes is $I = I_S\{\exp(eV/nk_BT) - 1\}$, where n ("ideality" factor) > 1. Formulate the square-law short-circuit $\Re$ for a Schottky diode. Evaluate this responsivity at 300 K assuming n = 1.0 – the best possible case.