HW#5 Solutions

1. Canonical junction rectification I-V curve, $I = I_0 [\exp(eV/k_BT) - 1]$ with generalized Johnson-Nyquist noise and full shot noise, and operates at 300 K into a load resistance R_L.

(a) The generalized Nyquist theorem states that $\langle(\Delta I)^2\rangle = 4k_BT\Delta\nu G = 4k_BT\Delta\nu(dI/dV) \Rightarrow 4k_BT (I_0e/k_BT) \exp(eV/k_BT)\,\Delta\nu = 4eI_0 \exp(eV/k_BT)\,\Delta\nu$ for the canonical rectifier. Full shot noise is given by $\langle(\Delta I)^2\rangle = 2eI\Delta\nu \Rightarrow 2eI_0 [\exp(eV/k_BT) - 1]\Delta\nu$ for the canonical rectifier. Both of these can be associated with random noise current generators in parallel with the device nonlinear conductance $G = dI/dV$. So for $R_L = 0$, the current from both generators flows entirely through the load. The total is found by noting that the thermal noise and shot noise are nominally uncorrelated, so $\langle(\Delta I)^2\rangle_{tot} = 4eI_0 \exp(eV/k_BT)\,\Delta\nu + 2eI_0 [\exp(eV/k_BT) - 1]\Delta\nu = 4eI_0\,\Delta\nu$ at $V = 0$, so that $I_{rms} = (4eI_0\,\Delta\nu)^{1/2}$. This has an interesting interpretation in solid state physics. Every solid-state rectifier contains a junction (e.g., p-n, metal-semiconductor, etc) of dissimilar materials designed to display a highly nonlinear I-V curve. But at zero bias, even such a nonlinear device must return to thermodynamic equilibrium with zero average terminal current and a uniform temperature equal to the bath temperature. However, the heterogeneous material nature of the junction always means that there are two competing current mechanisms which just balance at zero bias. In forward bias, one mechanism generally dominate, and in reverse bias, the other one dominates. In the given rectifier device, I_0 is clearly associated with the mechanism that dominates in reverse bias. This gets balanced by just enough of the forward current mechanism – I_0 again - associated with the thermally-activated term, to create zero average current at zero bias. But both terms can contribute to shot noise because they are statistically independent current mechanisms ! So each should contribute $\langle(\Delta I)^2\rangle = 2eI_0\Delta f$ fluctuations through the junction, or $4eI_0\Delta f$ in total.

(b) If we set the generalized Nyquist noise and full shot noise terms equal, we get the expression $4eI_0 \exp(eV/k_BT)\,\Delta\nu = 2eI_0 [\exp(eV/k_BT) - 1]\Delta\nu$ or $2 \exp(eV/k_BT) = -1$. Clearly this has no solution at any bias.

(c) If the device is connected to a 50-Ohm load, there will be a current divider action between the device and load that depends on bias voltage. But according to the normal Johnson-Nyquist theorem, the noise from the load resistor can also be represented by a current generator of $4k_BT\Delta\nu G_L$ connected to the same series pair of the device and RL, so the current divider action will be the same as for the device. And the load noise power will be equal between the when $6eI_0 \exp(eV/k_BT) - 2eI_0 = 4k_BTG_L$. Solving for V, we get $V = (k_BT/e)\ln[(2k_BTG_L+eI_0)/eI_0]$.

(d) For $R_L = 50\ \Omega$ and $T = 300$, the power spectrum is $SI = 4k_BTG_L = 3.3\times10^{-22}\ A^2/Hz$.

2. Every component in a receiver chain can be represented by a noise figure, $F_i = (SNR_{in}/SNR_{out})_i = 1 + (N_{add})_i/(G_i\, k_BT_{300}\, B_i)$, where $(N_{add})_i$ is the noise added by the ith component, G_i is its gain, and B_i is its RF bandwidth.

(a) Use these definitions to derive Friis' formula for total noise figure, F_T.

The output *signal* from the chain $S_{out} = S_{in}\, G_1G_2\cdots G_n \equiv S_{in}\, G_T$. The output *noise* from the chain is: $N_{out} = (N_{add})_n + (N_{add})_{n-1}\, Gn + \ldots (N_{add})_2\, G_3G_4\cdots G_n \ + \ldots (N_{add})_1\, G_2G_3\cdots G_n + \ldots N_{in}\, G_1G_3\cdots G_n$ where, by definition, $N_{in} = k_BT_{300}B$. Thus, $F_T \equiv (SNR_{in})/(SNR_{out}) = (S_{in}N_{out})/(S_{out}N_{in}) =$
$\{(N_{add})_n + (N_{add})_{n-1}\, G_n + \ldots (N_{add})_2\, G_3G_4\cdots G_n \ + \ldots (N_{add})_1\, G_2G_3\cdots G_n + \ldots N_{in}\, G_1G_3\cdots G_n\}/[G_TN_{in}]$
$= k_BT_{300}B\{(F_n-1)G_n + (F_{n-1}-1)G_{n-1}G_n + \ldots (F_2-1)G_2G_3\cdots G_n \ + \ldots (F_1-1)G_1G_2\cdots G_n\}/[G_TN_{in}] + 1$
$= (F_n-1)/\, G_1G_2\cdots G_{n-1} + \ldots .\ (F_3-1)/G_1G_2 + (F_2-1)/G_1 + (F_1-1) + 1$

$= (F_n\text{-}1)/ G_1G_2\cdots G_{n-1} + \ldots. (F_3 -1)/G_1G_2 + (F_2 -1)/G_1 + F_1$

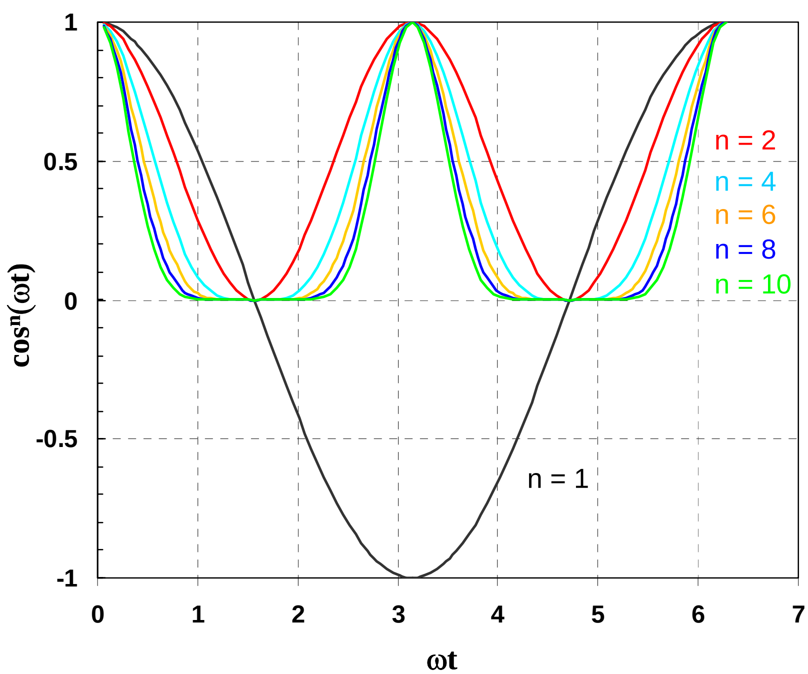
(b) For X = 30 dB, application of the above formula yields $F_T = 2.34$, or 3.69 dB

(c) In the limit of high X, G_2 becomes so large that $F_T \text{->} F_1 + (F_2 -1)/G_1 = 2.29$, or 3.60 dB. Application of Excel Goal Seek (or trial and error) shows that X = 19.1 dB yields a 1-dB degradation to F_T to 4.60 dB (or 2.88 linear).

(d) If the BPF and LNA are interchanged (LNA coming first) with X =30 dB, we find $F_T = 1.63$ (2.13 dB), a 1.56-dB improvement over the case in (b) above ! This is a good example of how important the ordering of components can be in RF receivers, particularly the passive components.

3. Power-law device represented by $X_{out}(t) = A[X_{in}(t)]^n \equiv \Re P_{in,}$ n = 2 being the square law.

(a) There are a number of ways to prove that the square law is the most effective rectifier, the simplest being to let $X_{in}(t)$ be a pure sinusoid, say $\cos(\omega t)$ and averaging $X_{out}(t)$ over one cycle or over a time $t = 2\pi/\omega$. For n odd, the result is clearly zero since any odd power of $\cos(\omega t)$ necessarily spends as much time negative going as positive going. For n even, the function is always positive going, as shown in the plot below up to n = 10. But clearly, n = 2 produces the highest average (i.e., dc term) over the cycle of ½. The other averages are listed below.



Plots of $\cos^n(\omega t)$, n even, up to n = 10. The average values over one cycle $(2\pi/\omega)$ are as follows: ave(n=2) = 0.5, ave(n=4) = 0.375, ave(n=6) = 0.312, ave(n=8) = 0.273, ave(n=10) = 0.246.

(b) If $I_{out} = B\,(V_{in})^2$, then we can Taylor expand about the point V_0,I_0 and write $I = I_0 + (1/2)(d^2I/dV^2)(V\text{-}V_0)^2$ or if I and I_0 are close, $I - I_0 \equiv \delta I = (1/2)(d^2I/dV^2)(\delta V)^2$, where $\delta V = V\text{-}V_0$. .But if we define $P_{in} = (\delta V)^2/R = (\delta V)^2\,(dI/dV)$, then $\delta I = (1/2)[(d^2I/dV^2)/(dI/dV)]P \equiv \Re\,P$ where $\Re = (1/2)[(d^2I/dV^2)/(dI/dV)]$ is the famous Torrey-Whitmer responsivity.

(c) For the Schottky diode, $I = I_S\{\exp(eV/nk_BT) - 1\}$, $\Re = (1/2)(e/nk_BT)$. For n = 1.0 and T = 300 K, this becomes $\Re = 19.3$ A/W a high number indeed.