Solutions HW#7

1. Comparison of radar and radiometry.

(a) For the double-sideband receiver at 650 GHz, the antenna gain G = 50 dB = $10^5 \approx 16/\beta^2$, or β = 0.0127 rad = 0.7 deg. The angle subtended by 1-ft-wide body at 30 ft is $\alpha \approx 2\cdot\text{atan}(0.5/30)$ = 0.033 rad = 1.9 deg. So assuming the receiver beam is pointed correctly, its beam is "filled" by the body so the apparent brightness temperature is equal to the body temperature (37 C) times the effective emissivity or $T_B = eT_S$ = 0.8x310 K = 248 K.. This is to be contrasted to the double-sideband receiver noise temperature, which is often close to one half the single-sideband value (assuming that the IF bandwidth is much less than the LO frequency, 1% in the present case). So assuming a single spatial mode is received and the RJ limit, we estimate a pre-detection $SNR_{PD} = T_B /(T_B + T_R)$ = 248/(2500+248) = 0.090. Square law detection and integration improves this $\boldsymbol{SNR_{AD} = SNR_{PD}^2(\Delta\nu / 2\Delta f) \approx SNR_{PD}^2(\Delta\nu \cdot t_i)}$ where $\Delta\nu$ is the IF bandwidth bandwidth and t_i is the integration time. Substitution of $\Delta\nu$ = 0.01*650 GHz = 6.5 GHz and setting SNR_{AD} = 10 yields $t_i \approx 2.3\text{x}10^{-6}$ s. From the definition given in class, the NEΔT = $NEP_{AD}/(k_B\Delta\nu)$ (assuming a single spatial mode received), and $NEP_{AD} \approx k_BT_R\Delta\nu /(\Delta\nu\ t_i)^{1/2}$ = $1.8\text{x}10^{-12}$ W, so NEΔT = 20 K. If the integration time is raised to 1 s, then the SNR_{AD} goes up by $1/(2.3\text{x}10^{-6})$ from 10 to a value of $4.3\text{x}10^6$. And the NEΔT falls by a factor of $(2.3\text{x}10^{-6})^{½}$ to 31 mK.

(b) For the 650-GHz "direct" receiver and 20% THz bandwidth, we again assume the body "fills" the antenna beam. The absolute optical NEP_{AD} is just the specific NEP_{AD} times the square root of post detection bandwidth $\Delta f = 1/(2t_i)$. So for a 1 s integration time, $NEP_{AD} = (NEP_{AD})' *(1/2t_i)^{1/2} = 7.07\text{x}10^{-13}$ W. For the unimodal antenna, the $SNR_{AD} = k_BT_B\Delta\nu / (NEP_{AD})$ = 629 for T_B = 248 K and $\Delta\nu$ = 130 GHz (20% BW). The NEΔT under this condition (assuming R-J limit) is $NEP_{AD}/(k_B\ \Delta\nu)$ = 0.39 K. To achieve SNR_{AD} = 10, the required integration time would only be 1 s $*(10/629)^2$ = 0.25 ms.

(c) For the monostatic radar at 650 GHz, we apply the modified radar range equation (for a specular target) to get a pre-detection SNR_{PD} (= SNR_{BD}) = $P_{rec}/[k_B(T_S+T_R)\Delta\nu] = (P_tG^2 \Gamma^2\lambda^2\varepsilon_p)/[(4\pi)^2(2R)^2]/ [k_B(T_B+T_R)\Delta\nu]$, where Γ^2 is the power reflection (0.5), R is the range (30 ft or 9.14 m), T_R is the single-sideband receiver noise temperature (5000 K), and ε_p is the polarization matching factor which we will assume to be unity. Under the stated system conditions, $P_{rec} = 20\text{x}10^{-6}$ W for, P_t = 1 mW and all other parameters as stated. And for $\Delta\nu$ = 6.5 GHz, the $SNR_{PD} = 4.2\text{x}10^4$. The noise-equivalent transmit power is just $NEP_T = [(4\pi)^2(2R)^2]*[k_B(T_S+T_R)\Delta\nu] /[G^2\ \Gamma^2\lambda^2\ \varepsilon_p]$ = 0.47 μW ! From (a) above, the SNR_{PD} = 0.09 and $SNR_{AD} = 4.3\text{x}10^6$ for a 1-s integration time in the double-sideband radiometer. To achieve these with the radar using a square-law detector and 1 s integration time, we again apply $\boldsymbol{SNR_{AD} = SNR_{PD}^2(\Delta\nu / 2\Delta f) \approx SNR_{PD}^2(\Delta\nu \cdot t_i)}$. Setting $\Delta\nu$ = 6.5 GHz again, we get SNR_{PD} = 0.026. The required transmit power in this case is just $(0.026/4.2\text{x}10^4)$* 1 mW = 0.6 nW. Although these solutions are overly optimistic, they make the point that active systems are often superior to passive ones when the target is reasonably close. And things get even better when the target is quasi-specular, provided that the reflected power is aligned with the receiver main beam. This is the well-known "pointing" challenge of all THz systems (and RF systems too) with high-gain antennas.

(d) Not required.

2. Matched filter problem

a) Physically unrealizable: $h'_m(t) = \begin{cases} \sin\left(-\frac{8\pi t}{T}\right) & -T \le t \le 0 \\ 0 & elsewhere \end{cases}$

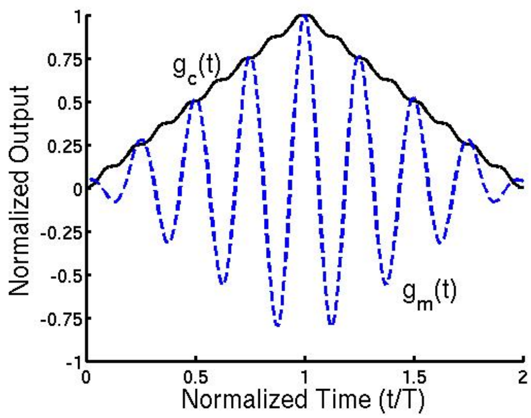
Physically realizable: $h_m(t) = \begin{cases} \sin\left(\frac{8\pi(T-t)}{T}\right) & 0 \le t \le T \\ 0 & elsewhere \end{cases}$

b) $g_m(t) = \int_t^{t+T} \sin\left(\frac{8\pi(u-t+T)}{T}\right) r(u)\,du$

c) t=T; there are other times in this particular case that the outputs are equal but for the illustrative purposes of this problem, t=T is of interest. Moreover t=T is the time which the observation interval time ends.

d)
$$g_c(t) = \frac{(T-|t-T|)}{2} + \frac{T}{36\pi}\sin\left(\frac{16\pi(|t-T|)}{T}\right)$$
$$g_m(t) = \frac{(T-|t-T|)}{2}\cos\left(\frac{8\pi(|t-T|)}{T}\right) + \frac{T}{16\pi}\sin\left(\frac{8\pi(|t-T|)}{T}\right)$$

e) see plot below



3. Since MATLAB offers the inverse error function, the two desired expressions for Pd vs Pfa and SNR for coherent detection are as follows:

(a) $P_d = \frac{1}{2} erfc\left[erf^{-1}(1-2P_{fa}) - \sqrt{SNR}\right]$ if $SNR < \left[erf^{-1}(1-2P_{fa})\right]^2$

$P_d = \frac{1}{2}\{1 + erf\left[\sqrt{SNR} - erf^{-1}(1-2P_{fa})\right]\}$ if $SNR > \left[erf^{-1}(1-2P_{fa})\right]^2$

(b) and(c) The MATLAB code used is as shown here (there are certainly more efficient ways to do the computation, but the code below is very straightforward). The plots are shown below. The results for the coherent receiver are shown in red and contrasted against the known results for an incoherent receiver (shown in blue).

```
%MATLAB code to compute Pd vs Pfa parameterized by SNR for coherent
%and incoherent receiver cases.  Written by  Prof. E. R. Brown
numberSNR=input('what is the number of the signal to noise ratios?');
startingPfa=input('what is the starting PFA?');
numberPfa=input(' what is the number of PFAs , assuming order of magnitude
increments?');
for i = 1:numberSNR
   SNR=input('what is the SNR?');
for j = 1:numberPfa
   Pfa(j) = startingPfa*10.^(j-1);
   if SNR < (erfinv(1 - 2.*Pfa)).^2
      PdC(j)=.5.*erfc(erfinv(1 - 2.*Pfa(j)) - sqrt(SNR));
   else
      PdC(j) =.5.*(1 +erf( (sqrt(SNR) - erfinv(1 - 2.*Pfa(j)))));
   end
end
loglog(Pfa,PdI)
hold on
loglog(Pfa,PdC,'color','red')
hold on
grid on
end
```