Quick Overview of THz Transmission Lines

Three Types

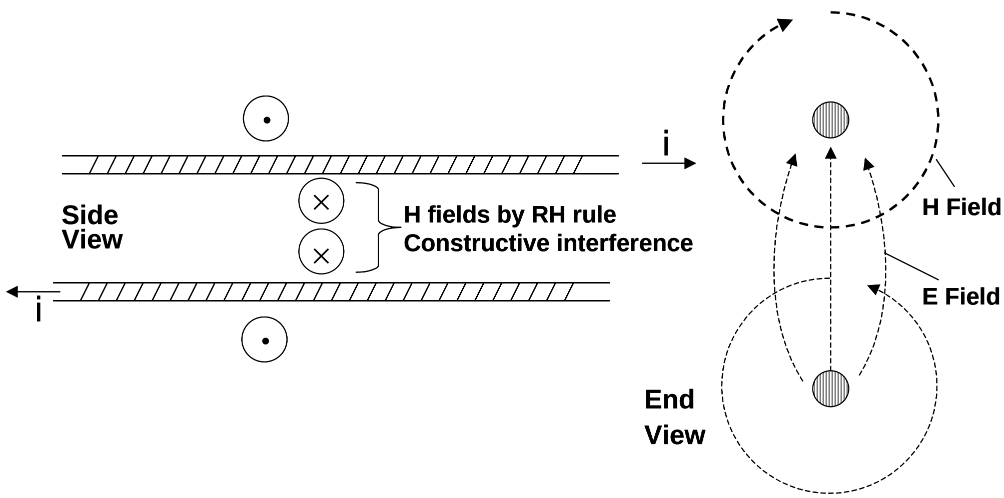
1). TEM
- e.g., coax structures by micromachining

2). TE or TM
- Waveguide: rectangular and circular

3). Quasi-TEM (best approach for integrated circuits on semiconductor substrates
- Coplanar strips, coplanar slots (i.e., coplanar waveguide)
- Microstrip

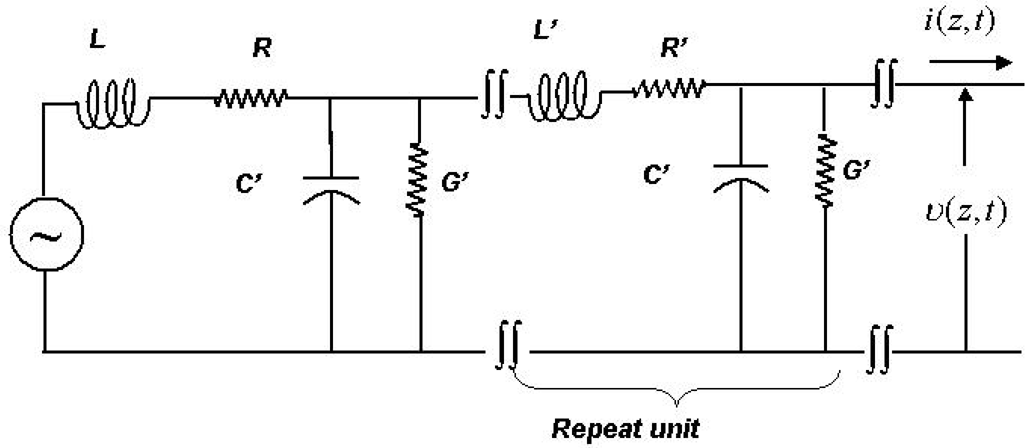
Uniform transmission line & TEM mode

- Two separate conductors with translational symmetry AND homogeneous cross-section
- If conductors are assumed to extend to ± infinity along translational axis, then electric and magnetic fields must be perpendicular to coordinates



- The electric field between conductors can be associated with a *specific* capacitance C'
- The magnetic field can be associated with a *specific* inductance L'

Lumped-Element Model (O..Heaviside)



Can solve by phasor technique of circuit theory

$$v(z,t) = \mathrm{Re}\{\tilde{V}(z)e^{j\omega t}\}$$

$$i(z,t) = \mathrm{Re}\{\tilde{I}(z)e^{j\omega t}\}$$

Application of circuit rules (KCL & KVL) along with Faraday and Maxwell laws for lumped elements

(v = L di/dt , i = C dv/dt)

yields:

$$\tilde{V}(z) = V_+ e^{-jz} + V_- e^{jz}$$

wave moving toward positive z — wave moving toward negative z

$\gamma \rightarrow$ propagation constant (analogous to k in free space)

$$\gamma^2 = (R' + j\omega L') (G' + j\omega C')$$

Also $\tilde{I}(z) = I_+ e^{-\gamma z} + I_- e^{\gamma z}$

Key result: $\dfrac{V_+}{I_+} = -\dfrac{V_-}{I_-} = \sqrt{\dfrac{R' + j\omega L'}{G' + j\omega C'}} \equiv Z_o$ { Characteristic Impedance

Special (and very useful) case: Lossless Line

⇨ R' = G' = 0

$$\gamma^2 = (j\omega L')(j\omega C') = -\omega^2 LC$$

$$\gamma = j\omega\sqrt{L'C'}$$

$$\tilde{V}(z) \rightarrow V_+ e^{-j\omega\sqrt{L'C'}\,z} + V_- e^{j\omega\sqrt{L'C'}\,z}$$

has the form e^{-jkz} iff $k = \omega\sqrt{\boldsymbol{L'C'}} \equiv \omega / v$

$$\Rightarrow \quad v = \frac{\mathbf{1}}{\sqrt{\boldsymbol{L'C'}}} \quad \text{phase velocity}$$

$$Z_o \rightarrow \sqrt{\frac{j\omega L'}{j\omega C'}} = \sqrt{\frac{L'}{C'}}$$ <u>purely real</u>

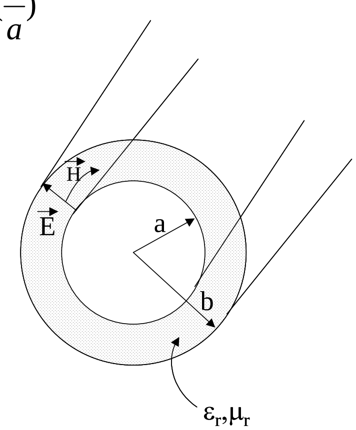
But this is not dissipative ! It's just a ratio of fields

<u>Example: Coaxial line</u>
From electrostatics:

$$C' = \frac{2\pi\varepsilon}{\ln(\frac{b}{a})} \qquad L' = \frac{\mu}{2\pi}\ln(\frac{b}{a})$$

.

$$\varepsilon = \varepsilon_r \varepsilon_o \qquad \mu = \mu_r \mu_o$$



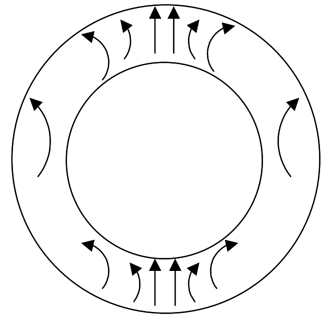
$$\Rightarrow Z_o = \sqrt{\frac{\frac{\mu}{2\pi}\ln(\frac{b}{a})}{\frac{2\pi\varepsilon}{\ln(\frac{b}{a})}}} = \frac{120\pi}{2\pi}\sqrt{\frac{\mu_r}{\varepsilon_r}}\ln(\frac{b}{a}) = 60\sqrt{\frac{\mu_r}{\varepsilon_r}}\ln(\frac{b}{a}) \ [\Omega]$$

$$.\ \upsilon_\rho = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\varepsilon_r\mu_r}}.\frac{1}{\sqrt{\varepsilon_o\mu_o}} = \frac{c}{\sqrt{\varepsilon_r\mu_r}} \quad [\text{m/s}]$$

Bandwidth is determined by presence of higher-order mode
1st higher mode is TE_{11} mode

In air filled coax, turn-on of this mode is determined by $k_c \approx \frac{2}{a+b} = \frac{2\pi}{\lambda_c}$

Lines of electric force for TE_{11} mode



$$\Rightarrow \; f_c = \frac{c}{\lambda_c} \approx \frac{c}{\pi(a+b)}$$

Absorption in TEM Transmission Line (example of coaxial line)

- In THz region absorption in the metal walls of transmission lines establishes the lower limit on attenuation per unit length. And the fundamental absorption mechanism is caused by the *skin effect*.
- The skin effect is a classical consequence of the tendency in metals and other good conductors for the ac electric field to penetrate less in the material as the frequency increases (recall that at dc or, say, 60 Hz, the current flows uniformly through a good conductor, such as the copper power lines). The penetration depth, called the "skin depth" is given by $\delta = (\pi f \mu \sigma)^{-1/2}$, where μ is the magnetic permeability and σ is the (ac) electrical conductivity. Its value in various common metals is shown in the plot below.
- From analysis of, total current in center conductor

$I_0 = \int_S J \cdot dS = \int_0^{2\pi} \cdot \int_0^{a} J(r) r dr d\theta \approx \sigma E_0 \cdot 2\pi a \cdot \delta$ if $\delta << a$

But the small but non-zero longitudinal component of E field must also generate a voltage drop $V_0 = \int_0^L E \cdot dz = E_0 L$, so the series resistance is

$R \equiv V_0/I_0 \approx L / (2\pi a \sigma \delta)$ and the specific series resistance is

$R' \equiv R/L = 1/(2\pi a \sigma \delta) \equiv R_s/(2\pi a)$

Where Rs is the "surface" resistance = $(\sigma\delta)^{-1}$