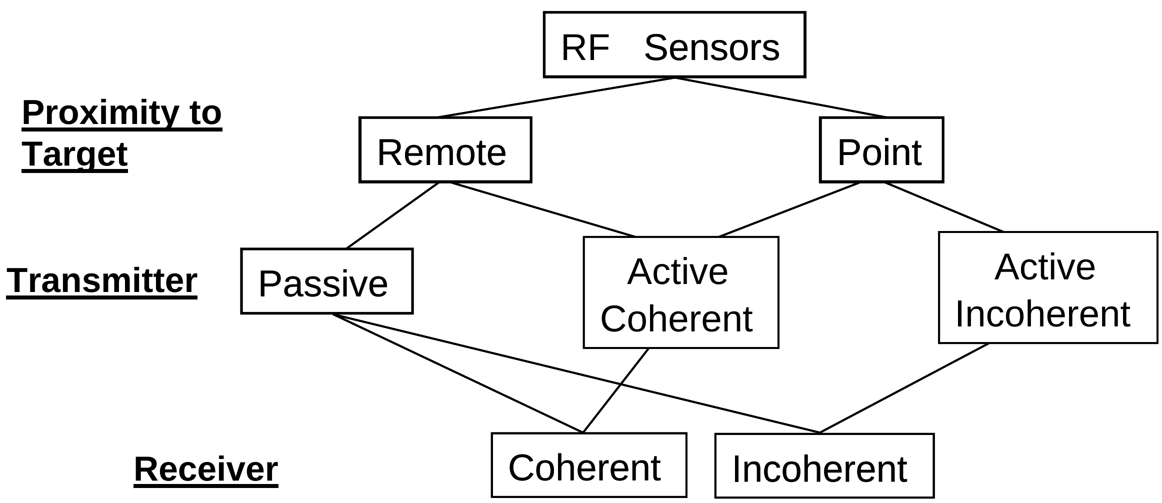
Overview of RF Sensors

Definition: an RF system designed to detect the presence of objects or materials through their electromagnetic reflection or emission.

Hierarchy



An *active* sensor detects radiation reflected from an object that the sensor system itself transmits. The paradigm active sensor is the radar – an acronym for *ra*dio *d*etection and *ra*nging.

A *passive* sensor detects thermal radiation emitted by an object, or environmental radiation reflected by the object. The paradigm passive sensor is the radiometer.

Coherent receivers are often based on heterodyne down-conversion (more later). Incoherent receivers are often based on square-law detection (more later).

Review of Classical Electromagnetics

Maxwell's Equations

$\vec{\nabla}\cdot\vec{E} = \dfrac{\rho}{\varepsilon} \approx 0$ Coulomb's

In free space

$\vec{\nabla}\cdot\vec{B} = 0$

$\vec{\nabla}\times\vec{E} = -\partial\vec{B}/\partial t$ Faraday's

$\vec{\nabla}\times\vec{H} = \vec{J}_C + \partial\vec{D}/\partial t \approx \partial\vec{D}/\partial t$ generalized Ampere's

In free space

Constitutive Relations

$$\vec{D} = \varepsilon\cdot\vec{E} = \varepsilon_r\varepsilon_0\cdot\vec{E}$$

$$\vec{B} = \mu\cdot\vec{H} = \mu_r\mu_0\cdot\vec{H}$$

$$Z_o = \sqrt{\varepsilon_0/\mu_0} = 377\ \Omega$$

Take curl of Faraday's:

$$\vec{\nabla}\times\vec{\nabla}\times\vec{E} = -\vec{\nabla}^2\vec{E} + \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) = \partial(\vec{\nabla}\times\vec{B})/\partial t = \partial(\mu\vec{\nabla}\times\vec{H})/\partial t$$

In free space:

$$-\vec{\nabla}^2\vec{E} + \vec{\nabla}(\vec{\nabla}\cdot\vec{E}) \approx -\vec{\nabla}^2\vec{E} \approx \partial(\mu_0\partial\vec{D}/\partial t)/\partial t$$

$$= \mu_0\varepsilon_0\cdot\partial^2\vec{E}/\partial t^2$$

And we get the vector wave equation: $\vec{\nabla}^2\vec{E} - \mu_0\varepsilon_0\cdot\partial^2\vec{E}/\partial t^2 = 0$

Sinusoidal solutions: $\vec{E}(\vec{r},t) = \mathrm{Re}\{\tilde{\vec{E}}(\vec{r})e^{j\omega t}\}$

Vector Helmholtz Eqn for E: $\vec{\nabla}^2\tilde{\vec{E}} + \mu_0\varepsilon_0\omega^2\cdot\tilde{\vec{E}} = 0$

Analogous Eqn for H: $\vec{\nabla}^2\tilde{\vec{H}} + \mu_0\varepsilon_0\omega^2\cdot\tilde{\vec{H}} = 0$

Radiation always entails two degrees of freedom: $\tilde{\vec{E}}$ and $\tilde{\vec{H}}$

Simplest solution: Plane Wave: $\tilde{\vec{E}}(\vec{r}) = \vec{E}_0 e^{-j\vec{k}\cdot\vec{r}} = \vec{E}_0 e^{-jk\cdot x}$

For propagation along x axis

Substitution back into vector Helmholtz eqn yields:

$$-k^2\tilde{\vec{E}} + \mu_0\varepsilon_0\omega^2 \cdot \tilde{\vec{E}} = 0 \Rightarrow k^2 = \mu_0\varepsilon_0\omega^2 = \frac{\omega^2}{c^2}$$

This is called the dispersion relation: a very important kinematic relationship for all wave phenomena

<u>Poynting's and Sensor Power Theorem</u>

From plane wave propagation we know that $\vec{E}\times\vec{H}$ is a vector that always points in the direction propagation. To understand what the magnitude means, operate on this quantity with the vector divergence operator:

$$\vec{\nabla}\cdot(\vec{E}\times\vec{H}) = \vec{H}\cdot(\vec{\nabla}\times\vec{E}) - \vec{E}\cdot(\vec{\nabla}\times\vec{H})$$

$$= \vec{H}\cdot(-\partial\vec{B}/\partial t) - \vec{E}\cdot(\vec{J} + \partial\vec{D}/dt)$$

$$= -\vec{E}\cdot\vec{J} - (1/2)\varepsilon\partial|E|^2/dt - (1/2)\mu\partial|H|^2/\partial t$$

$$= -\vec{E}\cdot\vec{J} - \partial U_E/dt - \partial U_M/\partial t$$

Joule heat   Electric energy density   Magnetic energy density

Application of Gauss' divergence theory now yields

$$\int_V \vec{\nabla}\cdot(\vec{E}\times\vec{H})dV = \oint(\vec{E}\times\vec{H})\cdot d\vec{s} \equiv \oint\vec{S}\cdot d\vec{s}$$

surface vector that encloses volume V

So $\vec{E}\times\vec{H}$ represents a flux of power and the the integral

$\oint(\vec{E}\times\vec{H})\cdot d\vec{s}$ represents total power leaving enclosed volume

Poynting's theorem for plane waves:

In phasor form: $\tilde{\vec{E}} = \tilde{E}_X \hat{x} = E_X^+ e^{-jkz} \hat{x}$ $\quad \tilde{\vec{H}} = H_y^+ e^{-jkz} \hat{y} = (E_X^+ / Z_0) e^{-jkz} \hat{y}$

$$\vec{E} \times \vec{H} = \mathrm{Re}\{\tilde{\vec{E}}\} \times \mathrm{Re}\{\tilde{\vec{E}} / Z_0\} = \frac{(E_X^+)^2}{Z_0} \cos^2(-kz + \omega t) \hat{z}$$

Instantaneous value of S ranges between 0 and $\dfrac{(E_X^+)^2}{Z_0}$

Average value is found by noting that long-time average of $\cos^2$ (or $\sin^2$) is 1/2

$$< \vec{S} > = \frac{(E_X^+)^2}{2Z_0} \hat{z}$$

For example : $E_x^+ =$ 1 V/m , $\quad Z_0 = 377 \quad < \vec{S} > = 1.33x10^{-3} \hat{z}$ W/m^2

Nice mathematical trick: Given phasor forms of E and H,

$$< \vec{S} > = \frac{1}{2} \mathrm{Re}\{\tilde{\vec{E}} \times \tilde{\vec{H}}\}$$

Sensor Power Theorem

An important quantity for sensors is the time-averaged power flowing into the sensor aperture, $P_{inc} = \vec{S} \cdot \vec{A} = (1/2)\,\mathrm{Re}\{\vec{E} \times \vec{H}\} \cdot \vec{A}$, where $\vec{A}$ is the sensor areal vector (pointed perpendicular to the sensor surface). An even more important quantity is the power *usefully* absorbed,

$$P_{abs} \equiv \eta \cdot P_{inc} = \frac{1}{2}\eta \cdot \mathrm{Re}\{\vec{E} \times \vec{H}\} \cdot \vec{A}$$

and η is the power coupling efficiency. Since η is the fraction of incident power *absorbed*, it must account for the effects of reflection at the environment-sensor interface, unabsorbed radiation that passes through the sensor, etc. The majority of sensors couple radiation in from free space propagating perpendicular to the surface. In this case $\vec{H}$ is perpendicular to $\vec{E}$, $|\vec{H}| = |\vec{E}|/z_0$,and

$$P_{inc} = ½(E^2 A)/z_0 = ½(\varepsilon_0 cE^2 A) = cU_E A\ ,$$

so that

$$P_{abs} = \eta\ c\ U_E\ A$$

This is very useful in sensor calculations since the energy density is a better quantity to deal with than the Poynting vector when one considers both radiation *signal* and radiation *noise*.

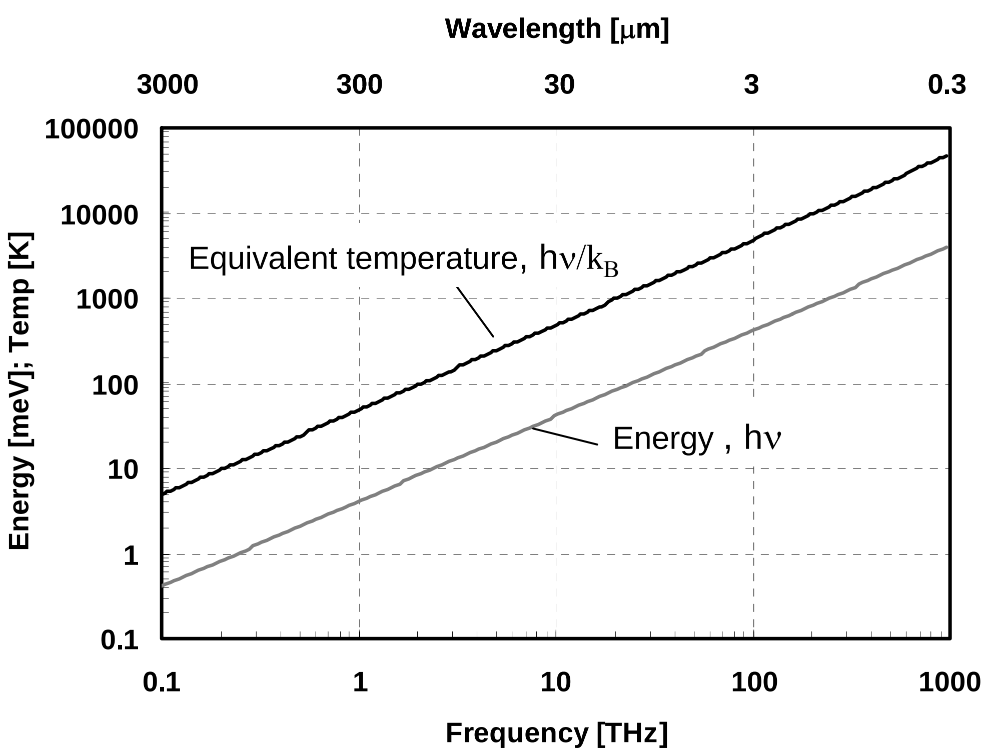
Quantum Picture of Radiation

Like any other physical observable, neither E or H can be measured with arbitrary precision. To understand the measurement of E and H at the finest scale, we need quantum mechanics. The balance between the electric and magnetic field energy densities in Poynting's theorem resembles the balance between potential and kinetic in a harmonic oscillator (e.g., mass on spring). Using methods of quantum field theory, the E and H amplitudes in the classical wave equation can be quantized. The resulting *collective excitation* of E and H can be represented by an equivalent quantized harmonic oscillator :

$$U_K = (n + ½)\, h\nu_K = (n + 1/2)\hbar\omega_K$$

$$\omega_K = k \cdot c \quad \text{in free space}$$

The term $(1/2)\hbar\omega_K$ is called the "zero-point" energy . Effectively, it is energy stored in the electromagnetic field that can not be extracted to do useful work (i.e., can not be used as the basis for remote sensing or communications)



The two most important constants in sensor theory:
Planck's: $h = 6.626 \times 10^{-34}$ J-s
Boltzmann's: $k_B = 1.38 \times 10^{-23}$ J/K

Quantum Statistics of Radiation

How do we find photon number in a statistical sense?
Assume atoms all exchange heat with a bath, at temperature T
Still can apply Maxwell-Boltzman:

In general, $f_k = \dfrac{e^{-U_k/k_B T}}{\sum e^{-U_k/k_B T}}$ $\rightarrow$ but now think as one k state as "subsystem"

$$f_{n_k} = \frac{e^{-\left(n_k+\frac{1}{2}\right)\hbar\omega_k \big/ k_B T}}{\sum e^{-\left(n_k+\frac{1}{2}\right)\hbar\omega_k \big/ k_B T}}$$ $n_k = 0,1,2\ldots.$ arbitrary!

$$\langle n_k \rangle = \frac{\sum_{n_k=0}^{\infty} n_k e^{-\left(n_k+\frac{1}{2}\right)\hbar\omega \big/ k_B T}}{\sum_{n_k=0}^{\infty} e^{-\left(n_k+\frac{1}{2}\right)\hbar\omega \big/ k_B T}} = \frac{e^{-\frac{\hbar\omega_k}{2k_B T}} \sum_{n_k=0}^{\infty} n_k e^{-n_k \hbar\omega/k_B T}}{e^{-\frac{\hbar\omega_k}{2k_B T}} \sum_{n_k=0}^{\infty} e^{-n_k \hbar\omega/k_B T}} = \frac{\sum_{n_k=0}^{\infty} n_k e^{-n_k \hbar\omega/k_B T}}{\sum_{n_k=0}^{\infty} e^{-n_k \hbar\omega/k_B T}}$$

denominator $\sum = \left(e^{-0\hbar\omega/k_B T} + e^{-\hbar\omega/k_B T} + e^{-2\hbar\omega/k_B T} + \cdots\right)$,

a geometric series, each term has magnitude < 1

So $\displaystyle\sum_{n_k=0}^{\infty} e^{-n_k\hbar\omega/k_B T} = \sum_{n_k}^{\infty}\left(e^{-\hbar\omega/k_B T}\right)^{n_k} = \frac{1}{1-e^{-\hbar\omega/k_B T}}$

Numerator; trick-notice:

$$n_k = \left(\frac{-k_B T}{\hbar} \frac{d}{d\omega_k} e^{-(n_k)\hbar\omega_K / k_B T} \right) \left(e^{n_k \hbar\omega / k_B T} \right)$$

So $\displaystyle\sum_{n_K=0}^{\infty} n e^{-(n_k)\hbar\omega/k_B T} = \sum -\frac{k_B T}{\hbar} \frac{d}{d\omega_k} e^{-(n_k)\hbar\omega_k/k_B T} = \frac{-k_B T}{\hbar} \frac{d}{d\omega_k} \sum_{n_k=0}^{\infty} e^{-(n_k)(\hbar\omega_k/k_B T)}$

$$= \frac{-k_B T}{\hbar} \frac{d}{d\omega_K} \left(\frac{1}{1-e^{-\hbar\omega_K/k_B T}} \right) = +\frac{k_B T}{\hbar} \frac{\hbar}{k_B T} \frac{e^{-\hbar\omega/k_B T}}{\left(1-e^{-\hbar\omega/k_B T}\right)^2}$$

So we get $\displaystyle\langle n_k \rangle \equiv \frac{\sum_{n_k=0}^{\infty} n_k e^{-n_k \hbar\omega_k/k_B T}}{\sum_{n_k=0}^{\infty} e^{-n_k \hbar\omega_k/k_B T}} = \frac{\dfrac{e^{-\hbar\omega_k/k_B T}}{\left(1-e^{-\hbar\omega_k/k_B T}\right)^2}}{\left(\dfrac{1}{1-e^{-\hbar\omega_k/k_B T}} \right)} = \frac{1}{\left(e^{\hbar\omega_k/k_B T} - 1 \right)}$

This is the famous Planck distribution

Mean energy and other thermodynamic quantities are average over k states

$$\langle U_{tot} \rangle = \sum_{k,p} \left(\langle n_k \rangle + \frac{1}{2} \right) \hbar\omega_k = \sum_{k,p} \left[\frac{1}{e^{\hbar\omega_k/k_B T} - 1} + \frac{1}{2} \right] \hbar\omega_k \, , \; \omega_k = \omega(k)$$

Note that when $\hbar\omega \ll k_B T$ $\quad \exp(\hbar\omega / k_B T) \approx 1 + \hbar\omega / k_B T$ **and**

$$\langle n_k \rangle \approx \frac{k_B T}{\hbar\omega}$$

a very important condition called the "classical" limit in statistical mechanics, and the Rayleigh-Jeans limit in radiation theory.

It is almost always true in RF sensors

Thermal (Blackbody) Radiation

Cavity Derivation

Imagine doing a mean energy calculation inside a large cubic cavity of side L

Convert to sum over ω_k since <U> is explicit in ω_k ,

$\langle U \rangle = \sum \left(\langle n_k \rangle + \frac{1}{2} \right) \hbar\omega_k \cdot g_m$ where g_m is the degeneracy factor = 2 for two possible polarizations. Thus

$$\langle U \rangle = 2 \cdot \int_0^\infty \left[\frac{1}{e^{\hbar\omega_k / k_B T} - 1} + \frac{1}{2} \right] \hbar\omega_k D(\omega) d\omega$$

$D(\omega) \rightarrow$ density of states in frequency space. (i.e., # states per unit volume in frequency space). To get $D(\omega)$, we must do state counting noting that any closed cavity has a minimum frequency separation between modes $\Delta\nu = c/L \Rightarrow \Delta k = \Delta\omega/c = 2\pi/L$

"Volume" per state in k space $(2\pi/L)^3 \Rightarrow$ #states/vol [k space] = $(L/2\pi)^3 = V/(2\pi)^3$

So # states between 0 and k = $N(k) = V/(2\pi)^3 \, (4/3)\pi k^3$

And
$$D(\omega) = \frac{dN}{dk}\frac{dk}{d\omega} = \frac{Vk^2}{2\pi^2}\frac{dk}{d\omega} = \frac{V\omega^2}{2\pi^2 c^3}$$

Excluding the zero-point term, we thus get an energy per unit volume of

$$\langle U' \rangle = 2 \cdot \int_0^\infty \frac{\hbar\omega}{e^{\hbar\omega/k_B T} - 1} \frac{\omega^2}{2\pi^2 c^3} d\omega$$

And from the power theorem, we get a total power passing through any unit area of

$$P = cA \cdot \langle U' \rangle = \int_0^\infty \frac{A\hbar}{e^{\hbar\omega/k_B T} - 1} \frac{\omega^3}{\pi^2 c^2} d\omega$$

In thermodyamics and heat transfer, we are interested in this integral and its derivatives.

In remote sensing, the spectrum is usually "band limited" so we are interested in the fraction of power δP power over a limited range $\delta\omega$: $\delta P = (dP/d\omega)\,\delta\omega$

And because the limits of the integral are constants, we have

$$\frac{dP}{d\omega} = \frac{A\hbar}{e^{\hbar\omega/k_B T}-1}\cdot\frac{\omega^3}{\pi^2 c^2} \text{ or } \frac{dP}{d\nu} = \frac{Ah}{e^{h\nu/k_B T}-1}\cdot\frac{8\pi\nu^3}{c^2}$$

In this derivation the blackbody radiation is isotropic so radiation within any small solid angle $\delta\Omega$ is simply $\delta P(\delta\Omega/4\pi)$. If we also divide out the area, we get a very special function in remote sensing called the *brightness*

$$B(\delta\nu,\delta\Omega) \equiv \frac{1}{A}\frac{d^2P}{d\nu\cdot d\Omega} = \frac{1}{e^{h\nu/k_B T}-1}\cdot\frac{2h\nu^3}{c^2}$$

The brightness is so useful because it describes the power per unit area emanating from a blackbody into a small solid angle and limited frequency band. This is the quantity that remote sensors usually measure, although from objects that do not behave like blackbodies because of finite surface reflection. To show this deviation, we define an emissivity, ε, which goes to 1 for a perfect blackbody and to zero for a perfect reflector.

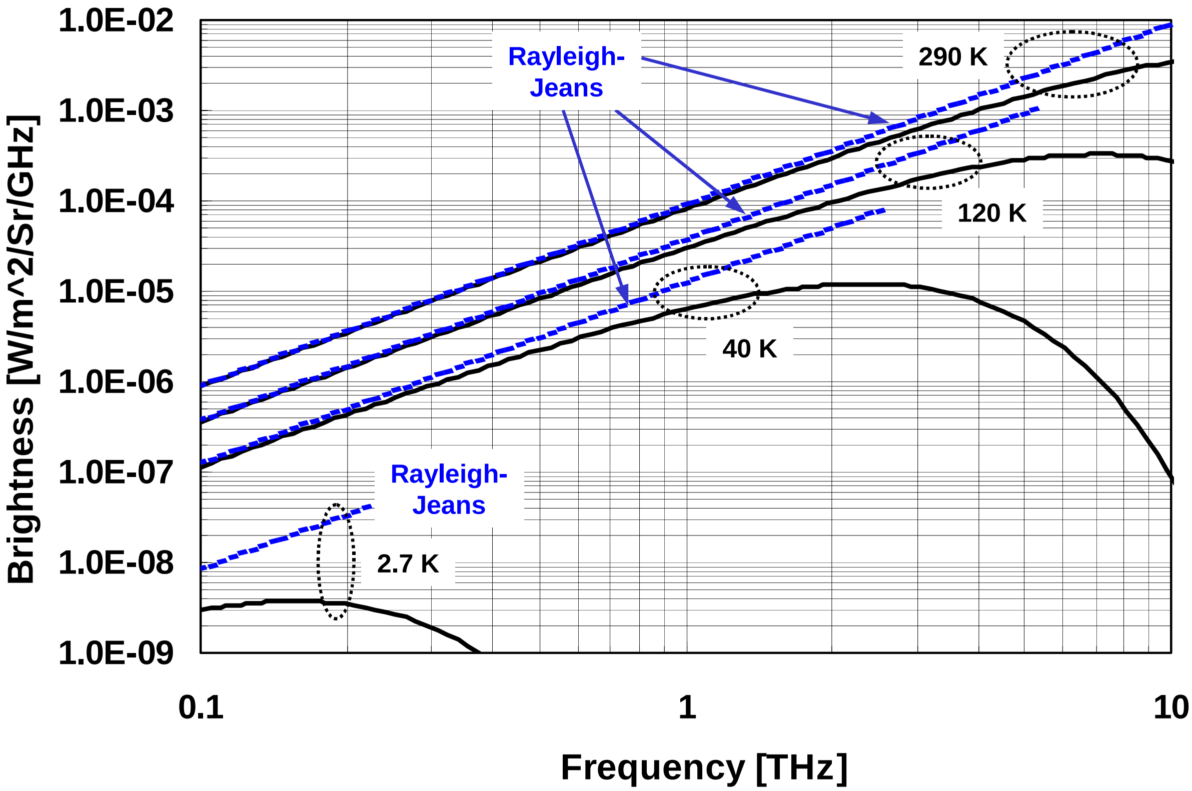
$$B(\delta\nu,\delta\Omega) = \frac{\varepsilon}{e^{h\nu/k_B T}-1}\cdot\frac{2h\nu^3}{c^2}$$

For remote sensing in the RF region with terrestrial targets, one generally has T > 200 K so that $h\nu \ll k_B T$. This leads to the Rayleigh-Jeans limit of thermal radiation,

$$B(\delta\nu,\delta\Omega) = \frac{\varepsilon}{e^{h\nu/k_B T}-1}\cdot\frac{2h\nu^3}{c^2} \to 2\varepsilon\cdot k_B T\,\frac{\nu^2}{c^2} = 2\varepsilon\cdot k_B T\,\frac{1}{\lambda^2}$$

This simple result was originally derived using the principle of equipartition: that there is a total of (1/2) $k_B T$ of energy per degree-of-freedom. The thermal radiation is made up of electromagnetic waves, each having four degrees of freedom: two for the electric and magnetic fields, respectively, and two for the different polarizations.

Blackbody brightness spectra for different source temperatures



Modal Derivation

- Real sensors are not located in radiation boxes !

- To get the power received and usefully absorbed by real sensors, it is necessary to decompose the radiation into orthogonal spatial modes

- The most useful spatial modes are the "spatial" modes defined by the antenna of the sensor system

- From statistical mechanics, we know the Planck function is valid for any orthogonal set of modes, no matter what their origin. So the mean thermal energy *incident* from free space at frequency ν is just the energy quantum, hν, times the mean number of photons in that mode, summed over all spatial modes

$$<U>=\sum_{m=1}^{M} f_P(\nu)\cdot h\nu$$

Where m is the spatial mode index and M is the maximum number of spatial modes (RF sensors sometimes accept more than one spatial mode, but almost never couple to so many that the summation can be approximated by an integral).

- In addition to the spatial modes, we need to address the issue of "longitudinal" modes. In the Planck distribution each ν corresponds to a unique harmonic oscillator, and therefore a unique mode. The total energy in a frequency range Δν is to be thought of as a sum over all the possible "longitudinal" modes for each lateral mode

$$<U>=\sum_{m=1}^{M}\sum_{n=1}^{N} f_P(\nu)\cdot h\nu$$

where n is the longitudinal-mode index.

We estimate the number of longitudinal modes by three practical assumptions:

(1) Thermal radiation is separated from the sensor antenna by a distance L and boundary conditions require that the electromagnetic intensity be a maximum at both ends.

(1) The lowest frequency longitudinal mode corresponds to a half-wavelength between the two ends, ν_{min} = c/2L (note: in open-cavities and Fabry-Perot resonators, this quantity is called the free spectral range).

(3) The sensor is filtered so that it responds only to radiation lying within a “passband” ν_0 to $\nu_0 + \Delta\nu$, and that the sensor responds only to the half of the longitudinal modes propagating in the direction from the source.

The number of longitudinal modes N(ν) and mean energy incident on the sensor at each increment of ν are then given by:

$$N(\nu) = \frac{1}{2} \cdot \frac{\nu - \nu_0}{c/2L}$$

$$<U> = \sum_{\nu_0}^{\nu_0+\Delta\nu} \sum_{m=1}^{M} N(\nu) \cdot h\nu \cdot f_P(\nu)$$

For most RF sensors the “range” L is great enough that c/2L << ν_0, where ν_0 is the bottom of signal passband. So we can approximate the sum by

$$<U> = \sum_{m=1}^{M} \int_{\nu_0}^{\nu_0+\Delta\nu} f_P(\nu) \cdot \frac{dN}{d\nu} \cdot h\nu \cdot d\nu = \sum_{m=1}^{M} \frac{L}{c} \int_{\nu_0}^{\nu_0+\Delta\nu} f_P(\nu) \cdot h\nu \cdot d\nu$$

The energy density becomes:

$$<U'> = \frac{1}{AL} <U> = \sum_{m=1}^{M} \frac{1}{cA} \int_{\nu_0}^{\nu_0+\Delta\nu} f_P(\nu) \cdot h\nu \cdot d\nu$$

And the incident and absorbed powers can be written:

$$<P_{inc}> = \sum_{m=1}^{M} \int_{\nu_0}^{\nu_0+\Delta\nu} f_P(\nu) \cdot h\nu \cdot d\nu$$

$$<P_{abs}> = \sum_{m=1}^{M} \int_{\nu_0}^{\nu_0+\Delta\nu} \eta_m(\nu) f_P(\nu) \cdot h\nu \cdot d\nu$$

No dependence on c or A !

Both expressions can be generalized to account for other forms of radiation:

$$< P_{inc} >= \sum_{m}^{M} \int_{\nu_0}^{\nu_0+\Delta\nu} h\nu < n_m(\nu) > \cdot d\nu \qquad < P_{abs} >= \sum_{m}^{M} \int_{\nu_0}^{\nu_0+\Delta\nu} \eta_m(\nu) \cdot h\nu < n_m(\nu) > \cdot d\nu$$

In the RF bands and in the terrestrial environment, one generally has T > $h\nu/k_B$ over the entire band $\Delta\nu$, so that again, $e^{h\nu/k_B T} - 1 \approx h\nu/k_B T$.

$$< P_{abs} >= M \int_{\nu_0}^{\nu_0+\Delta\nu} f_P h\nu \cdot d\nu = M \int_{\nu_0}^{\nu_0+\Delta\nu} k_B T \cdot d\nu = M \cdot k_B T \cdot \Delta\nu$$

This is the Rayleigh-Jeans limit again, typical at all RF frequencies and below. For example, if ν = 1 THz (top end of RF regime) and T = 290 K (room temperature), we have $h\nu$ = 4.1 meV, and $k_B T$ = 25.0 meV, so that $h\nu/kT = 0.164$ and $e^{h\nu/k_B T} = 1.178$ (i.e., the Rayleigh-Jeans approximation is accurate to about 9%)

Quantum Picture of Coherent Radiation

- The sinusoidal waveforms from the classical wave equations are both replaced by a wave function called the *coherent state* of frequency $\nu = \omega/2\pi$.
- The amplitude, or occupancy, of this state corresponds to the instantaneous power associated with the classical field amplitudes E or H, and the occupancy number represents the number of *photons* in this state.
- Each photon still has energy $h\nu$.
- Because of quantum uncertainty, the photon number in the coherent state itself is random and obeys Poisson statistics.
- Probability of measuring n photons in the mode in an arbitrary time interval is given by

$$\boldsymbol{p(n) = \frac{<n>^n}{n!} e^{-<n>}}$$

where <n> is the mean number of photons measured in this same interval over many different measurements.

(Note: like Gaussian distribution, Poisson distribution is a bonafide probability distribution function with the required properties

$\sum_0^\infty p(n) = 1$ and that $\sum_0^\infty np(n) = <n>$

- Connection to classical behavior is realized by noting that for purely sinusoidal E and H field

$$p(n) = \frac{[<n>(t)]^n}{n!} e^{-<n>(t)}$$

where $<n>(t) \propto \sin^2(\omega t)$

- Given the photon picture, an equivalent way to represent a coherent wave and a sensor is through the average measured photon rate J_P ($\equiv$average number of photons usefully absorbed by the sensor per unit time)

$$J_P = \frac{P_{abs}}{h\nu} = \frac{c \cdot \eta \cdot U_E \cdot A}{h\nu} \text{ (a photon flux)}$$

(same as Einstein's photoelectric expression)

- Even for the relatively weak RF coherent sources, this flux is astronomically high. For example, a source putting out 1 μW at 600 GHz ($h\nu$ = 2.48 meV) is emitting a photon rate of $2.5\text{x}10^{15}$ photon/s !