Coupling of THz Radiation to Free Space: Antennas*

- A critical aspect of any remote sensor is the coupling from the circuit (or transmission line) medium of the sensor to the external medium in which the target is embedded (usually free space), and/or the coupling of the external medium to the sensor
- The component that carries out this coupling is traditionally called the "antenna".
- RF sensor antennas generally fall into one of two categories: (1) wire antennas, and (2) aperture antennas.
- At the low end of the RF spectrum, roughly up to 10 GHz, the wire antennas take on the form of dipoles, spirals, helices and other simple shapes. The aperture antennas usually take on the form of parabolic or elliptical dishes.
- At the high end of the RF spectrum, the wire antenas usually occur on substrates in the form of patches, slots, or other "printed-circuit" antennas. The aperture antennas usually have the form of feedhorns or small dishes.

- *All antennas are also classified by their electromagnetic properties (i.e. radiation or beam "pattern" in the external medium) and their circuit properties (i.e., the impedance) in the internal medium. Because the antenna is a reciprocal passive element, the properties in reception are related to those in transmission, but the transmit case is easier to do first*

Electromagnetic Properties in Transmit

(1) The radiated electric field at a far distance from the antenna will tend to display a *modified spherical-wave* of the form

$$E(r,\theta,\phi) \propto \sqrt{F(\theta,\phi)}\,\frac{e^{-jkr}}{r}$$

where k is the free-space propagation constant (= $\omega/c = 2\pi/\lambda$) and F is the (normalized) intensity pattern function, $F \equiv |S(r,\theta,\phi)|/S_{max}$ with S being the Poynting vector and S_{max} is its maximum magnitude, wherever in space that occurs.

- All antennas display a limited direction in space where $F(\theta,\phi)$ is large and other regions where it is negligible, in contrast to isotropic (point) sources. Therefore, a useful metric is the directivity, D.

$$D = 4\pi\left(\iint_{4\pi} F(\theta,\phi)\,d\Omega\right)^{-1} \equiv 4\pi/\Omega_B$$

where Ω_B is the beam solid angle. Conceptually D defines how much greater the intensity is at the peak of F compared to the isotropic radiator emitting the same total power, for which $\Omega_B = 4\pi$ and D = 1.

*Good reference on antennas:
R.S. Elliott, "Antenna Theory and Design," (Prentice Hall, Englewood Cliffs, 1981).

- All RF antennas are generally designed to have a pattern function that displays a predominant, symmetric or quasi-symmetric peak (i.e, "major lobe") in a single direction of space θ_P, ϕ_P

In this case it is useful to approximate $F(\theta,\phi)$ by an equivalent spherical cone or sector having a symmetry axis along θ_P, ϕ_P , and polar angular width (or widths) equal to the full-widths at the half-maximum points $\beta(\phi)$ of the real major lobe.

- Throughout the cone or sector, $F(\theta,\phi) = 1.0$
- If the pattern has perfect conical symmetry (generally true for parabolic dishes and lenses, and often the design goal for feedhorns), then one finds

$$\Omega_B \equiv \iint_{4\pi} F(\theta,\phi) d\Omega \approx \int_0^{2\pi} d\phi \int_0^{\beta/2} \sin\theta \cdot d\theta = 2\pi[1 - \cos(\beta/2)]$$

and

$$D \approx \frac{2}{1 - \cos(\beta/2)}$$

In the limit of a narrow "pencil" beam where β is small (<< 1 rad), one can Taylor expand the denominator, yielding

$$D \approx \frac{16}{\beta^2}$$

Note that in most books make the simpler approximation $\Omega_B \approx \beta^2$, so that $D \approx 4\pi/\beta^2$ - a less precise expression but one easier to remember.

The characterization of the parabolic dish then reduces to knowing the -3-dB full-width the main lobe, β

Non-Ideal Behavior of the Radiation Pattern: Diffraction

- An important aspect of all antennas is their degree of non-ideal behavior related to radiation in "sidelobes". These are peaks of radiation in addition to the main lobe that arise from the phenomenon of diffraction.

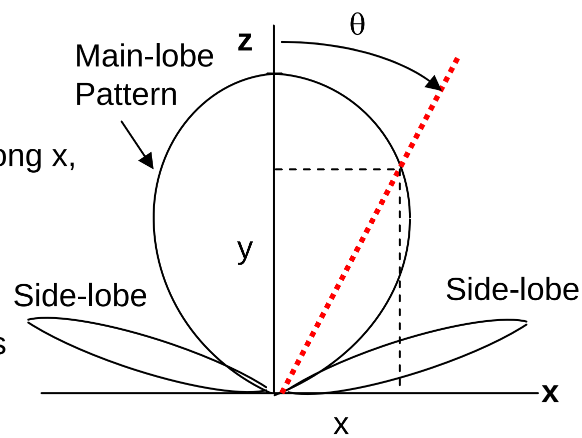
- Diffraction is most easily explained for aperture antennas because they are wider than a wavelength, so that scalar diffraction theory applies:

• Scalar diffraction theory provides an approximate solution to the vector EM wave (Helmholtz) equation for radiation passing through the aperture.

• The scalar formalism results in the famous Kirchoff-Fresnel integral which, in essence, approximates the radiation pattern as the superposition of point sources filling the aperture, each point source radiating a spherical wave .

• A key issue in using this integral is the geometric shape of the aperture and the amplitude distribution of the point sources inside the aperture. In the special case of *uniform* illumination, scalar diffraction predicts a far-field pattern for a rectangular aperture that goes as:

$$|F(\theta,\varphi)|^2 \propto \left[\frac{\sin(kax/r)}{kax/r}\right]^2 \left[\frac{\sin(kby/r)}{kby/r}\right]^2$$



Where k=2π/λ, a is the half-width of the rectangle along x, b is the half-width along y, and r is the range to the measurement point

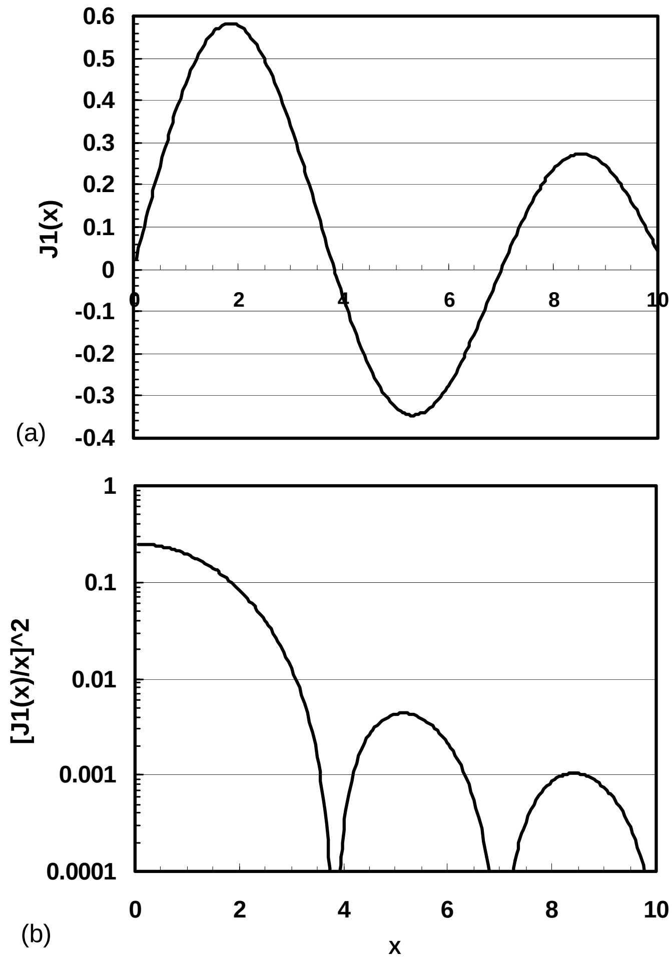
For a uniformly illuminated circular aperture, one gets

$$|F(\theta,\varphi)|^2 \propto \left[\frac{J_1(ka\theta)}{ka\theta}\right]^2$$

where J_1 is the ordinary Bessel function of 1st order, a is the radius, and θ is the angle of the measurement point relative to the optical axis

• Both of these functions oscillate with respect to polar angle, θ , reaching relative minima and relative maxima in between. Each portion of the radiation pattern between the relative minima is called the “sidelobe”. The totality of oscillating behavior of the radiation pattern is called “diffraction.”

Example: the uniformly illuminated circular aperture



• $J_1(x)/x$ peaks at x = 0. But its peak value is 0.25, not 1.0 as for the sinc(x).
• The first null occurs at the first zero of the J_1 function, x = 3.835 or $\theta = 3.835\ \lambda/(2\pi a) = 0.610\ \lambda/a$.
• A secondary peak of magnitude 0.00437 occurs at approximately x = 5.14, corresponding to $\theta = 0.818\ \lambda/a$. Note that this secondary peak (first sidelobe) has a value of 0.0175 or –17.6 dB relative to the main lobe (this is to be contrasted to the more familiar value of –13.2 dB for the first sidelobe for a square aperture of uniform illumination

• From this case, the –3-dB point is at x = 1.616, so the beam full-width is given by
□$\beta = 2\theta = 3.232\ \lambda/(2\pi a)$.
□ Substitution into directivity expressions yields (for $\beta << 1$)
$D \approx 16/\beta^2 = [16(2\pi a)^2]/(3.232\cdot\lambda)^2 = 1.53 \cdot[4\pi A/\lambda^2]$ where $A = \pi a^2$ is the circular area .

•The last quantity arises frequently in the analysis of aperture and wire-like antennas and so has a special place in the electromagnetic field as the maximum or "diffraction-limited" directivity

$$\boxed{\mathbf{D_{max} = 4\pi\, A/\lambda^2}}$$

• This is imprecise but often used as a rule of thumb and for system calculations

• Note: all of the above analysis rests on measuring the radiation pattern in the "far-field", defined approximately by $r > 2d^2/\lambda$. But this criteriion applies only to some antenna types, and is often replaced by $r > 10d^2/\lambda$ to be safe

Antenna Circuit Properties

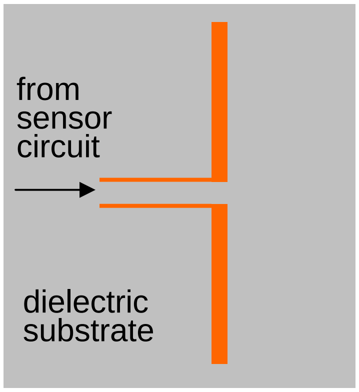
• Every antenna can be characterized by its "driving-point" impedance $Z_A = R_A + jX_A$ where R_A is the driving point resistance and X_A is the driving point reactance.

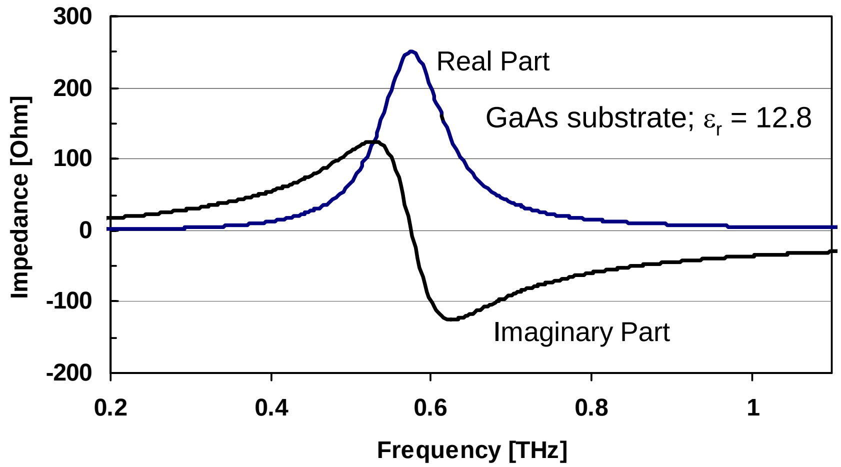
• Since antennas are usually driven through transmission lines, their impedance is related to the measured reflection coefficient, $\Gamma \equiv S_{11} = (Z_A - Z_0)/Z_A + Z_0)$.

• The electrical characterization is based on the behavior of Z_A.
(1) For $R_A >> X_A$ over a wide frequency range, the antenna is called "wideband" assuming that R_A usefully large
(2) For $X_A = 0$ at some frequency and R_A usefully large at this frequency, the antenna is called "resonant".

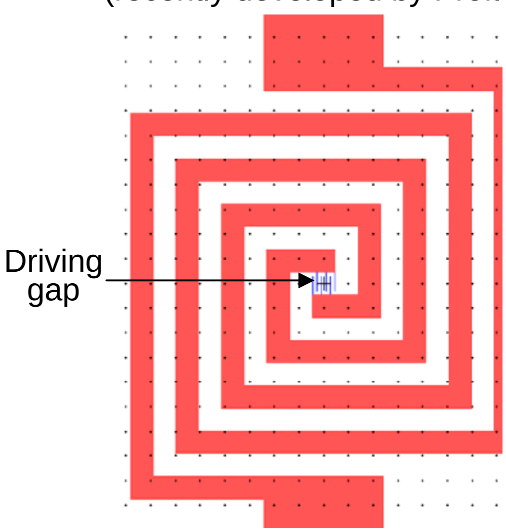
• *Both types of antennas are common, depending on the sensor design and application.*

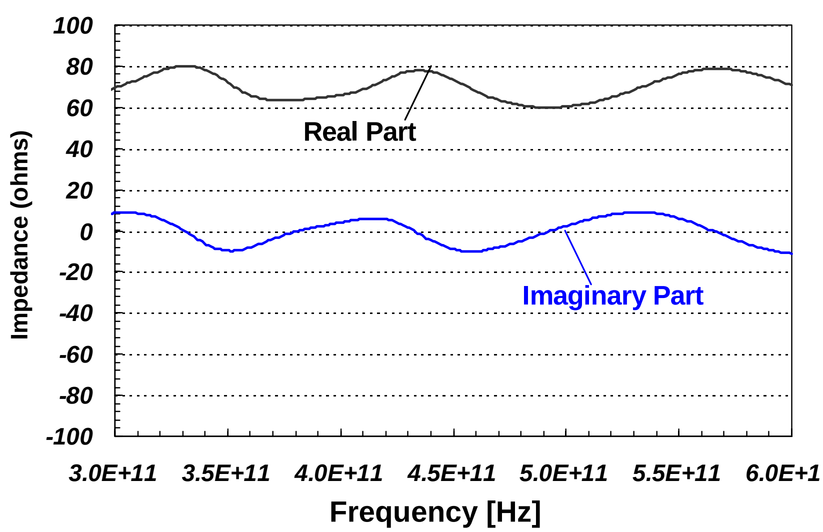
(1) The canonical resonant wire-like antenna. The printed dipole:





(2) A useful "wideband" wire-like antenna. The self-complementary rectangular spiral (recently developed by Prof. Brown's research group).



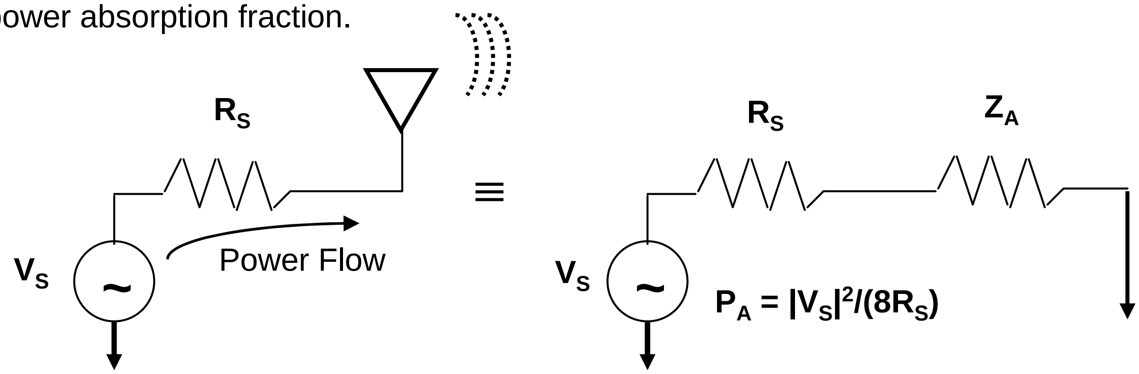


Antenna Gain

• Because it is difficult to get a perfect beam pattern and a perfect impedance match at the same time, a new metric is used that includes both effects. It is called the antenna gain G_A, and is defined simply by

$$G_A = |\tau|^2 D = (1 - |\Gamma|^2)(1 - \varepsilon) \cdot D$$

Where $|\tau|^2$ is the power transmission coefficient from the drive circuit to the environment and D is the directivity. Note that $|\Gamma|^2 = |S_{11}|^2$ is the power reflection coefficient and ε is the power absorption fraction.



Hence, G defines the maximum intensity produced at a particular point in space from an antenna driven by a generator having available power P_A, and taken relative to the intensity from a point source $P/(4\pi r^2)$.

Notes: (1) for a perfectly matched antenna having no absorptive losses, $|\Gamma| = 0$ and $\varepsilon = 0$, so that G = D. At low frequencies, roughly below 10 GHz, many antennas will provide negligible absorptive losses, so that $\varepsilon \approx 0$ and $G_A \approx (1 - |\Gamma|^2)D$. This approximation is very common in antenna textbooks.

At the maximum of the antenna pattern, the intensity (or magnitude of Poynting vector) on a target at a distance r from the antenna will be

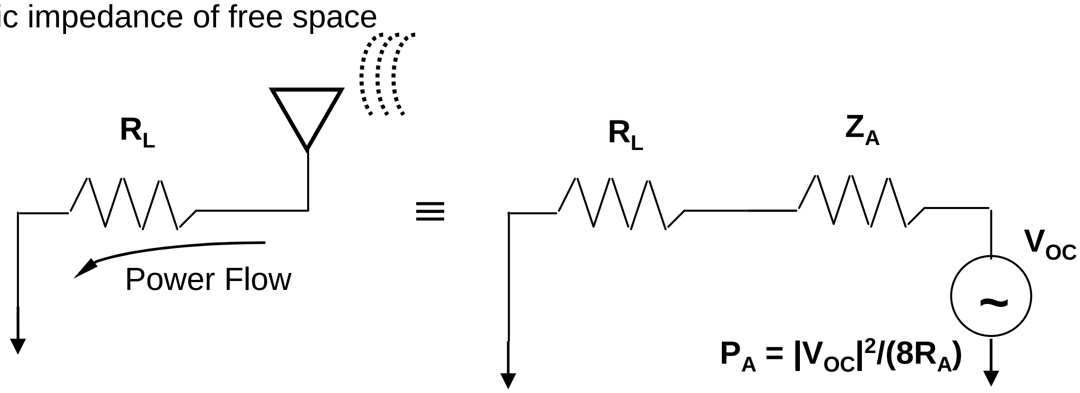
$$\boxed{I_{max} = G_A P_A/(4\pi r^2)}$$

Antennas in Reception

• It was proven early by Lorentz that antennas obey electromagnetic *reciprocity*. In words, if a given drive current I in a transmit antenna produces a voltage V in a receive antenna, than the same I applied to the receive antenna will produce V in the transmit antenna. It leads to the fact that the antenna pattern in transmission is the same as the pattern in reception. And this leads to the representation of all antennas in reception by an "effective aperture"

$$A_{eff} = \frac{P_{\text{int}}}{S_{\max}} = \frac{P_{avail}}{S_{\max}} = \frac{|V_{OC}|^2 / 8R_A}{|E_{inc}|^2 / 2\eta_0}$$

Where P_{int} is the "intercepted" power, S_{inc} is the incident Poynting vector magnitude at the antenna, P_{avail} is the available power, V_{OC} is the open circuit voltage and η_0 Is the intrinsic impedance of free space



The derivation is easiest for wire-like antennas for which $V_{OC} \approx E_{inc} \cdot L_{eff}$ where L_{eff} is the effective length of the antenna (close to the physical length for dipoles).

So we get $$A_{eff} = \frac{\eta_0 L^2}{4R_A} \equiv \frac{120 \cdot \pi \cdot L^2}{4R_A} = \frac{30 \cdot \pi \cdot L^2}{R_A}$$

From elementary antenna theory, we know that for a short (Hertzian) dipole, $R_A = 80(\pi L/\lambda)^2$ and D = 1.5. Hence

$\boxed{A_{eff} = \lambda^2 D/(4\pi)}$ or $\boxed{D = 4\pi A_{eff}/ \lambda^2}$

Note: This is a further reason why this expression is so ubiquotous:
While being the diffraction-limited expression for aperture antennas, it is also the exact definition of effective aperture for short dipole antennas !

The Number of Spatial Modes and the Antenna Theorem

• Independent of the type of radiation, the number of spatial modes collected by a sensor is an important issue from a system standpoint, is difficult to estimate, and is even more difficult to measure.
• A useful approximation technique is based on a result from electromagnetic theory called the antenna theorem, which derives from two separate definitions of directivity given above,

$$D_{\max} = \frac{4\pi A}{\lambda^2} = \frac{4\pi}{\Omega_B}$$

• Since D_{max} is the maximum possible value of the directivity as predicted by diffraction theory, then the corresponding Ω_B is the minimum possible beamwidth and, therefore, corresponds to the *fundamental spatial mode* of the antenna.
• So if we rotate the Ω_B beam in spherical coordinates to *just fill up* surface of a sphere, it would take approximately $4\pi/\Omega_B$ rotations to do this. And since Ω_B is the fundamental spatial mode, D_{max} represents the number of spatial modes required to fill the entire sphere. Usually, sensors are designed to respond to a much smaller solid angle, called the field-of-view, Ω_{FOV}. And the number of spatial modes then becomes

$$M = \frac{\Omega_{FOV}}{\Omega_B}$$

The above relation is often re-stated as the following "antenna theorem"

$$A \cdot \Omega_B \geq \lambda^2$$

Conceptually, it means that if the antenna is diffraction-limited, the product will reach its minimum value of λ^2. But it reminds us that practical antennas have electromagnetic or mechanical limitations which usually cause the fundamental beamwidth to grow beyond this minimum value.

In the diffraction limited case, we get the interesting result

$$M = \frac{\Omega_{FOV} A}{\lambda^2} = \frac{\Omega_{FOV} A \cdot \nu^2}{c^2}$$

This states a ν^2 dependence on the number of modes,which we already found was the case for free-space radiation in a "box" as defined through the photonic density-of-states function for thermal radiation. Here, we see it in more general terms that can be applied to other coupling structures. A good example is a feedhorn coupled to (WR-10) rectangular waveguide. Here, the sharp cutoff frequency of the waveguide modes allows us to calculate M separately. Indeed, the number of spatial modes approaches ν^2 as the mode index gets large !

Examples of modal function for (a) WR-10 waveguide, and (b) a lens-coupled Golay cell