# ECE 137 A Mid-Term Exam

## Thursday February 6, 2014

Do not open exam until instructed to.

Closed book: Crib sheet and 1 page personal notes permitted

There are 2 problems on this exam, and you have 75 minutes.

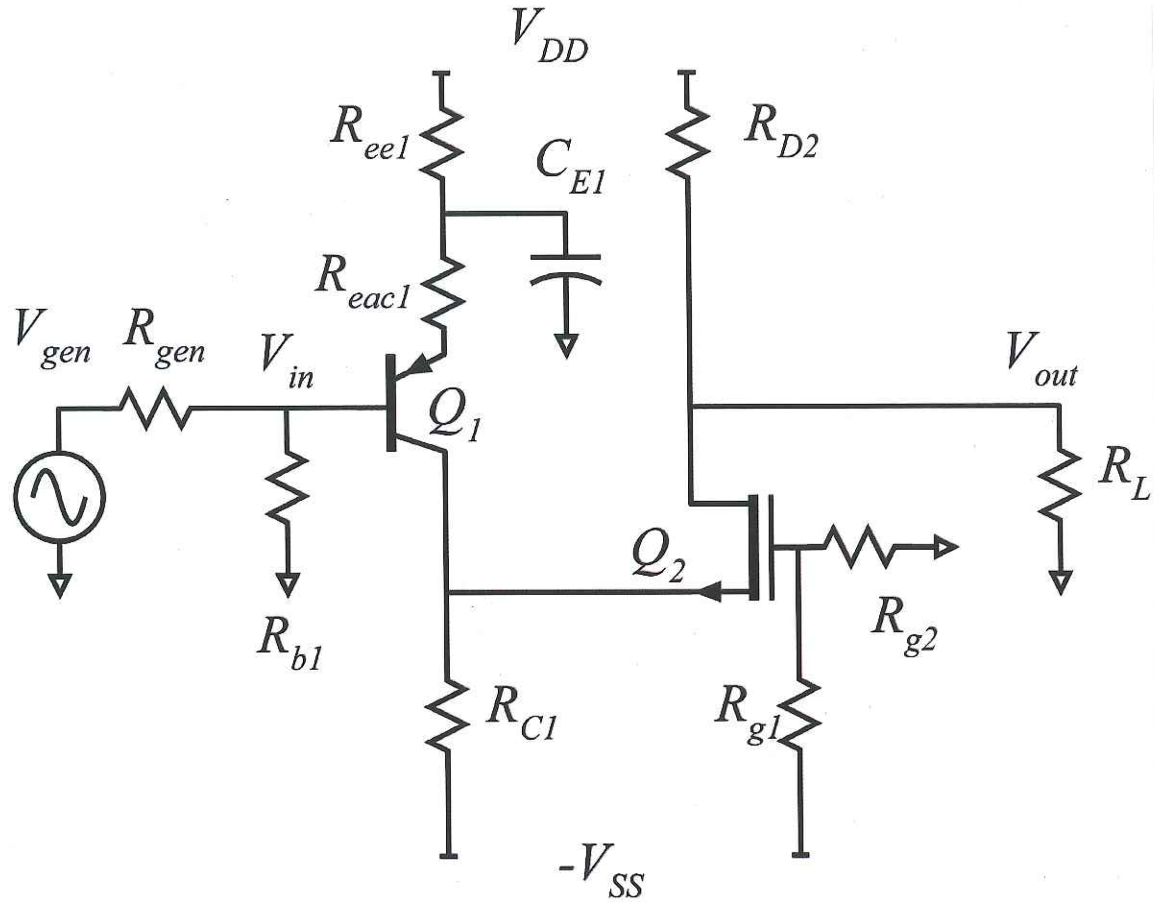
Use any and all reasonable approximations (5% accuracy is fine. ) , ***AFTER STATING and approximately Justifying them.***

**Name:** Solution "A"

| Part | Points Received | Points Possible |
|---|---|---|
| 1a | | 7 |
| 1b | | 7 |
| 1c | | 6 |
| 1d | | 15 |
| 1e | | 15 |
| 1f | | 6 |
| 1g | | 14 |
| 2a | | 12 |
| 2b | | 13 |
| 2c | | 5 |
| TOTAL | | 100 |

**Problem 1, 70 points**

You will be working on the circuit below:




Q1: $\beta = 100$, $V_A$=100 V

Q2: Velocity-limited $V_{th} = 0.2V$, $1/\lambda$ = infinity, $\Delta V = L_g v_{th} / \mu$=0.1V, $c_{ox} v_{th} W_g$ =5mA/V

The supplies are +3V and -3 V

Rgen=1000 Ohms, RL=10,000 Ohms. Rg2=100kOhms, Reac1=74 Ohms, Rb1=10kOhms

Ce1 is very large (AC short-circuit)

Part a, 7 points
DC bias.
$V_{in}$ is at (approxiately) zero volts DC.
The gate of Q2 is to be biased at -1.5 Volts
The drain is to be biased at zero volts.
Q1 is to be biased at 1 mA emitter current
Q2 is to be biased at 2 mA drain currrent.

Find the following:
Rc1= 267Ω  Rg1= 100kΩ  Ree1= 2.26kΩ

Rg1= 100kΩ

(1) $I_{D2} = 2mA = (5mA/V)(V_{gs1} - V_{th} - \Delta V)$

$2mA = (5mA/V)(V_{gs1} - 0.2V - 0.1V)$

$V_{gs1} = 0.2V + 0.1V + 2mA/(5mA/V) = 0.7V$

(1) → $V_{S1} = -0.7V - 1.5V = -2.2V$

$R_{C1}$ carries 3mA $(I_{C1} + I_{D2})$

(1) $R_{C1} = \frac{3V - 2.2V}{3\,mA} = 267\Omega.$

(1) Base of Q1 @ ~0V → $V_{B1} \cong 0.7V$

(1) $R_{EE1} = \frac{3V - 0.7V}{1mA} - R_{EAC1} = 2.3k\Omega - 74\Omega = 2.226k\Omega$

(1) $R_{D2} = \frac{3V - 0V}{2mA} = 1.5k\Omega$

$R_{g2}$ carries $1.5V/100k\Omega = 15\mu A$

(1) $R_{g1} = \frac{3V - 1.5V}{15\mu A} = 100k\Omega.$



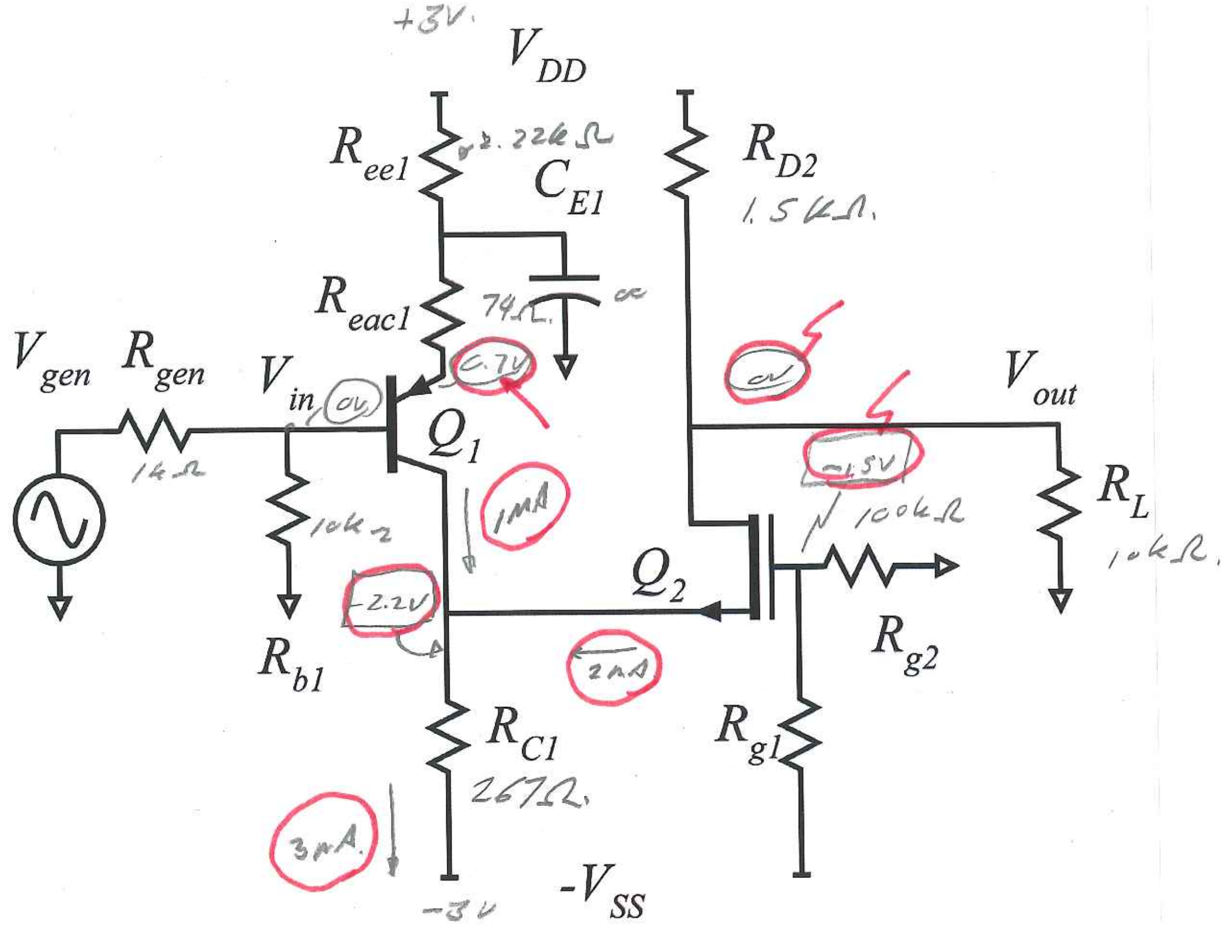
a

Q2 : $V_{tl} = 0.2V$, $\lambda = 0$, $\Delta V = 0.1V$, 5mA/V.

Q1 : $\beta = 100$, $V_A = 100V$

Part b, 7 points

DC bias




On the circuit diagram above, label the DC voltages at **ALL nodes** and the DC currents through **ALL resistors**

Part c, 6 points

Find the small signal parameters of Q1 and Q2.

Transistor Q1: gm= 38.5mS Rce= 100kΩ Rbe= 2.6 kΩ

Transistor Q2: gm= 5mS Rds= ∞ Ω.

Q1: $I_C = 1mA$, $V_A = 100V$, $\beta = 100$

1.5 [ $g_m = \frac{1mA}{26mV} = 38.5\,mS = \frac{1}{26\Omega}$

1.5 [ $R_{BE} = \beta / g_m = 100 \cdot 26\Omega = 2.6\,k\Omega$

1 [ $R_{CE} = \frac{V_{CE} + V_A}{I_C} \cong \frac{V_A}{I_C} = \frac{100V}{1mA} = 100\,k\Omega$.

Q2: $I_D = 2mA$. velocity limited

1 [ $g_m = 5mA/V$

1 [ $R_{DS} = \infty$ because $\lambda = 0V^{-1}$

Part d, 15 points.

Find the small signal voltage gain (Vd2/Vs2) of Q2 and Q2's small-signal input resistance.

Vd2/Vs2= + 6.5.

Rin,q2= 200 Ω.

3.5 [Q2: Common-gate.

3.5 [Because $R_{DS2} = \infty$, $R_{in2} = 1/g_{m2} = \frac{1}{5mS} = 200\Omega$.

3.5 [$R_{Leq2} = R_{D2} \| R_L = 1.5k\Omega \| 10k\Omega = 1.3k\Omega$

3.5 [$\frac{V_{D2}}{V_{S2}} = \frac{R_{Leq2}}{R_{in2}} = \frac{1.3k\Omega}{200\Omega} = 6.5$

Part e, 15 points

Find the small signal voltage gain (Vc1/Vb1) of Q1 and the *** amplifier *** input resistance.

Vc1/Vb1= $-1.14$

Rin,amplifier = $5k\Omega$

2 [ Q1 → common-emitter with degeneration.

$1/g_m = 26\Omega$, $R_{EAC} = 74\Omega$

2 [ $R_{out,Q1} = R_{CE}(1+g_m R_{EAC}) = 100k\Omega(1+74/26\Omega)$
$=$ very large, can neglect.

3 [ $R_{Leq1} = R_{C1} \| R_{in2} \| R_{outQ1}$
$\cong 267\Omega \| 200\Omega = 114\Omega$

3 [ $A_{v1} = \dfrac{-R_{Leq}}{r_{EAC} + 1/g_{m1}} = \dfrac{-114\Omega}{26\Omega + 74\Omega} = -1.14$

3 [ $R_{in\,Q1} = \beta(1/g_m + R_{EAC1}) = 100 \cdot (74\Omega + 26\Omega)$
$= 10{,}000\,\Omega$

2 [ $R_{in\,A1} = R_B \| R_{inQ1} = 10k\Omega \| 10k\Omega = 5k\Omega$

Part f, 6 points

Find (Vout/Vin), (Vin/Vgen) and (Vout/Vgen)

(Vout/Vin) = $-7.41$

(Vin/Vgen) = $0.833$

(Vout/Vgen) = $-6.17$

2 $V_{in}/V_{gen} = \dfrac{R_{inA}}{R_{inA} + R_{gen}} = \dfrac{5k\Omega}{5k\Omega + 1k\Omega}$

$= 0.833$

2 $V_o/V_{in} = A_{v1} A_{v2} = -1.14 \cdot 6.5 = -7.41$

2 $V_o/V_{gen} = V_o/V_{in} \cdot V_{in}/V_{gen}$

$= -7.41 \cdot 0.833$

$= -6.17$

Part g, 14 points

Now you must find the maximum signal swings. Find the output voltage due to saturation and cutoff in Q2. ***Give the sign (+ or -) in your answers below.***

Cutoff of Q1; Maximum ΔVout resulting = - 0.741V.

Saturation of Q1; Maximum ΔVout resulting = + 15.6V

Cutoff of Q2; Maximum ΔVout resulting = +2.6V.

Knee voltage of Q2; Maximum ΔVout resulting = -2.1V.

Note: $A_{v2} = +6.5$

Cutoff Q1:

1 [ $\Delta I_c|_{max}$ = 1mA - 0mA = 1mA (.) decrease

1 [ $R_{Leq1}$ = 114 Ω

3.5 1 [ $\Delta V_{c1}|_{max}$ = 1mA · 114Ω = 114 mV, negative-going.

1/2 [ $\Delta V_{out}$ = 114mV · $A_{v2}$ = 114mV (6.5) = 0.741V

Saturation of Q1

1 [ $V_{CEQ}$ = 0.7V + 2.2V = 2.9V, $V_{CEsat} \cong 0.5V$ ]

3.5 1 [ $\Delta V_{CE}|_{max}$ = 2.4V, neglecting $\Delta V_E$, $\Delta V_c|_{max}$ = 2.4V ] 1

1/2 [ $\Delta V_{out}$ = 2.4V · (6.5) = +15.6 V

Cutoff Q2

1.5 [ $I_{DQ}$ = 2mA, ⇒ $\Delta I_D|_{max}$ = 2mA (decrease)

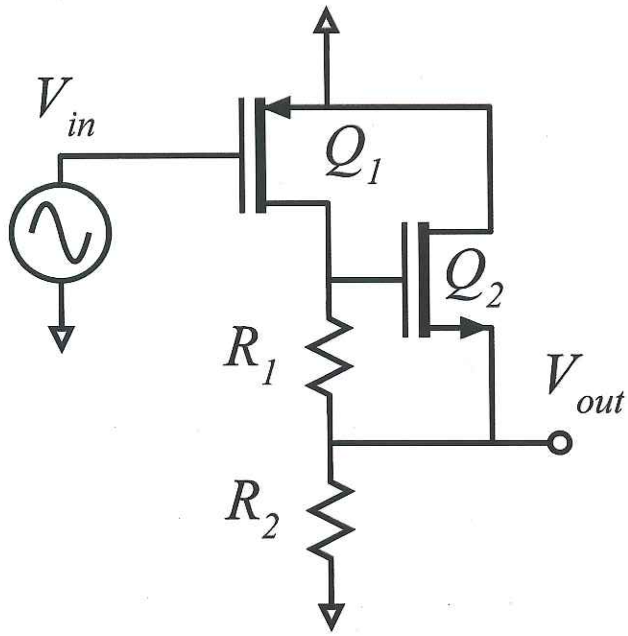
3.5 2 [ $\Delta V_{out}$ = 2mA · $R_{Leq2}$ = 2mA · 1.3 kΩ = 2.6V

Knee Q2

1 [ $V_{DSQ}$ = 2.2V, [ $V_{DS}|_{knee}$ = 0.1V (velocity-saturated) ] 2

3.5 1/2 [ $\Delta V_{DS} \cong \Delta V_{out}$ = 2.1V

**Problem 2, 30 points**
*nodal analysis*




You will be working on the circuit to the left.

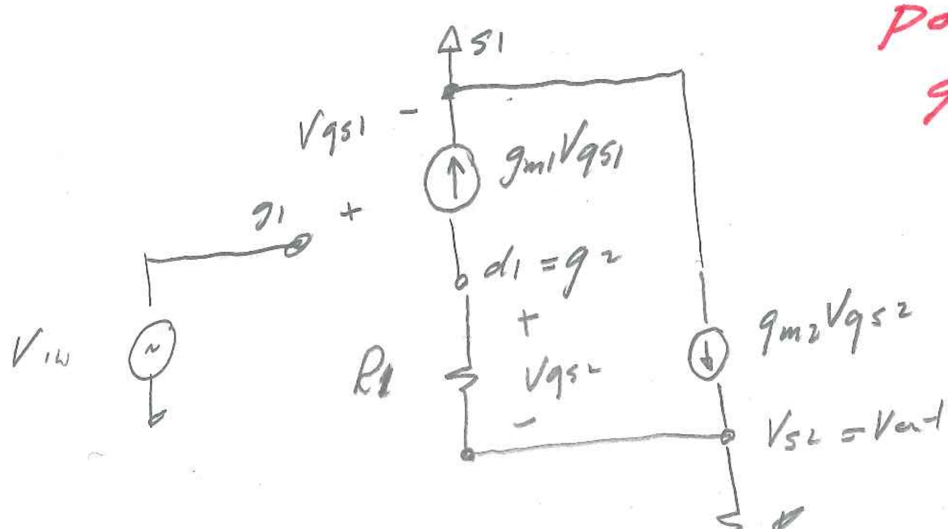
Ignore DC bias analysis. You don't need it.

Transistor 1 has transconductance gm1.
Transistor 2 has transconductance gm2.

The drain-source resistances Rds of both transistors are infinity (so you don't need to draw it!)

Part a, 12 points

Draw the small-signal equivalent circuit




points off if gm generators are not labelled or controlling voltages not identified

Part b,13 points

Find, by nodal analysis, a small-signal expression for Vout/Vin.

Vout/Vin= ___________________

there are two unknowns: $V_{d1} = V_{g2}$ and $V_{out}$.

$\Sigma I = 0$ @ $V_{g2}$ (+ = leaving)

4 $[+g_{m1} V_{gs1} + (V_{g2} - V_{out})/R_1 = 0]$

$+ g_{m1}(V_{in}) + V_{g2}/R_1 - V_{out}/R_1 = 0$

$V_{g2}(1/R_1) + V_{out}(-1/R_1) = -g_{m1} V_{in}$

$\Sigma I = 0$ @ $V_{out}$ (+ = leaving)

4 $[(V_{out} - V_{g2})/R_1 + V_{out}/R_2 - g_{m2} V_{gs2} = 0]$

$(V_{out} - V_{g2})/R_1 + V_{out}/R_2 - g_{m2}(V_{g2} - V_{out}) = 0$

$V_{g2}(-1/R_1 - g_{m2}) + V_{out}(1/R_1 + 1/R_2 + g_{m2}) = 0$

$$\begin{bmatrix} 1/R_1 & -1/R_1 \\ -1/R_1 - g_{m2} & 1/R_1 + 1/R_2 + g_{m2} \end{bmatrix} \begin{bmatrix} V_{g2} \\ V_{out} \end{bmatrix} = \begin{bmatrix} -g_{m1} \\ 0 \end{bmatrix} V_{in}$$

so $\dfrac{V_{out}}{V_{in}} = \dfrac{N}{D}$ where

$$D = \begin{vmatrix} 1/R_1 & -1/R_1 \\ -1/R_1 - g_{m2} & 1/R_1 + 1/R_2 + g_{m2} \end{vmatrix}$$

$$= (1/R_1)(1/R_1 + 1/R_2 + g_{m2}) - (1/R_1)(1/R_1 + g_{m2})$$

$$N = \begin{vmatrix} 1/R_1 & -g_{m1} \\ -1/R_1 - g_{m2} & 0 \end{vmatrix} = -g_{m1}(1/R_1 + g_{m2})$$

$$\frac{V_{out}}{V_{in}} = \frac{-g_{m1}\,(1/R_1 + g_{m2})\,R_1}{(1/R_1 + 1/R_2 + g_{m2}) - (1/R_1 + g_{m2})}$$

$$\boxed{\frac{V_{out}}{V_{in}} = -g_{m1}(1 + g_{m2}R_1)R_2} \leftarrow 5.$$

Part c, 5 points

gm1= 1 mS gm2= 10 mS , R1=1kOhm, R2=2kOhm
Give a numerical value for Vout/Vin.

Vout/Vin= — 22.

5

$$\frac{V_{out}}{V_{in}} = -1mS\,(1 + 10mS \cdot 1k\Omega)\,2k\Omega$$

$$= -1mS\,(11)\,2k\Omega$$

$$= -11mS \cdot 2k\Omega$$

$$= -22$$