## Final Exam, ECE 137A

## Wednesday March 19, 2014 7:30-10:30 PM

Name: $\qquad$
Closed Book Exam: Class Crib-Sheet and 3 pages ( 6 surfaces) of student notes permitted Do not open this exam until instructed to do so. Use any and all reasonable approximations (5\% accuracy), after stating \& justifying them.
Show your work:
Full credit will not be given for correct answers if supporting work is missing.
Good luck

| Time function | LaPlace Transform |
| :--- | :--- |
| $\delta(t)$ impulse | 1 |
| $U(t)$ unit step-function | $1 / s$ |
| $e^{-\alpha t} U(t)$ | $\frac{1}{s+\alpha}$ |
| $e^{-\alpha t} \cos \left(\omega_{d} t\right) U(t)$ | $\frac{s+\alpha}{(s+\alpha)^{2}+\omega_{d}^{2}}$ |
| $e^{-\alpha t} \sin \left(\omega_{d} t\right) U(t)$ | $\frac{\omega_{d}}{(s+\alpha)^{2}+\omega_{d}^{2}}$ |


| Part | Points <br> Received | Points <br> Possible | Part | Points <br> Received | Points <br> Possible |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 1a |  | 5 | 2c |  | 15 |  |
| 1b |  | 6 | 2d |  | 10 |  |
| 1c |  | 4 | 3a |  | 7 |  |
| 1d |  | 10 | $3 b$ | 8 |  |  |
| 1e |  | 10 | 3c |  | 7 |  |
| 2a |  | 10 | 3d |  | 8 |  |
| 2b |  | 10 |  |  |  |  |
| total |  | 100 |  |  |  |  |

## Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts


All the transistors have the same (matched) $I_{S}$, have $\beta=\infty$, and $V_{A}=\infty$ Volts. $V_{C E(s a t)}=0.5 \mathrm{~V} . V_{b e}$ is roughly 0.7 V , but use $V_{b e}=(k T / q) \ln \left(I_{E} / I_{S}\right)$ when necessary and appropriate. The supplies are +4.5 Volts and -4.5 Volts. The DC voltage drops across Ree 3 and Ree 11 are both 500 mV .

The DC collector currents of Q3,4,5,6,7,11,12 are all $2.0 \mathrm{~mA} . R_{L}=250 \Omega$

Part a, 5 points
DC bias:


On the circuit diagram above, label the DC voltages at ALL nodes, the DC currents through ALL resistors, and the DC drain currents of all transistors.

Part b, 6 points
DC bias:
Find the value of all resistors.
Rbias $=\quad$ Ree $1=$ $\qquad$ Ree $2=$ $\qquad$ Ree3=
Ree8= $\qquad$ Ree9= $\qquad$ Ree10= $\qquad$ Ree11=

## Part c, 4 points

Find the transconductance of the transistors below:
gm4 $=$ $\qquad$ gm5= $\qquad$ gm6= $\qquad$ gm7 $=$

## Part d, 10 points.

The circuit is fully differential. Assuming a differential input signal, $V_{i n, \text { diff }}=V_{i+}-V_{i-}$, and defining a differential output signal $V_{\text {out, ifff }}=V_{0+}-V_{0-}$, compute the differential gain
$A_{d}=V_{\text {out,diff }} / V_{\text {in,diff }}$
$A_{d}=$

## Part e, 10 points

Maximum peak-peak output voltage at the positive output Vo+ (show all your work)

|  | magnitude and sign of <br> maximum output signal <br> swing due to cutoff | magnitude and sign of <br> maximum output signal <br> swing due to saturation |
| :--- | :--- | :--- |
| Transistor Q1 |  |  |
| Transistor Q4 |  |  |
| Transistor Q6 |  |  |
| Transistor Q9 |  |  |

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1 -sentence statement below as to why it is not relevant.

## Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input $\mathrm{Vi}+$ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.


All NMOS: $I_{D}=1(\mathrm{~mA} / \mathrm{V})\left(W_{g} / 1 \mu \mathrm{~m}\right)\left(V_{g s}-V_{t h}-\Delta V\right)\left(1+\lambda V_{D S}\right)$

$$
\Delta V=0.1 \mathrm{~V}, V_{t h}=0.2 \mathrm{~V}, 1 / \lambda=5 \mathrm{~V}
$$

All PMOS: Also velocity-limited, with $g_{m}=0.5(\mathrm{~mA} / \mathrm{V})\left(W_{g} / 1 \mu \mathrm{~m}\right)$

$$
\Delta V=-0.1 \mathrm{~V}, V_{t h}=-0.2 \mathrm{~V}, 1 / \lambda=5 \mathrm{~V}
$$

$V_{D D}=+1 \mathrm{~V},-V_{S S}=-1 \mathrm{~V}$, The load resistor is $R_{L}=10 \mathrm{k} \Omega$

Part a, 10 points
DC bias.
Approximation: ignore the term $\left(1+\lambda V_{D S}\right)$ in DC bias analysis.

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi- ) necessary to obtain this.

Q1 is to be biased at 0.2 mA drain current.
The transistor gate widths are as follows

| Q1 | Q 2 | Q 3 | Q 4 | Q 5 | Q 6 | Q 7 | Q 8 | Q 9 | Q 10 | Q 11 | Q 12 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $2 \mu \mathrm{~m}$ | $4 \mu \mathrm{~m}$ | $4 \mu \mathrm{~m}$ | $2 \mu \mathrm{~m}$ | $2 \mu \mathrm{~m}$ | $1 \mu \mathrm{~m}$ | $1 \mu \mathrm{~m}$ | $2 \mu \mathrm{~m}$ | $4 \mu \mathrm{~m}$ | $2 \mu \mathrm{~m}$ | $10 \mu \mathrm{~m}$ | $20 \mu \mathrm{~m}$ |

Find:


R1= $\qquad$

Part b, 10 points
DC bias


On the circuit diagram above, label the DC voltages at ALL nodes and the drain currents of ALL transistors

Part c, 15 points.
To compute the op-amp differential gain, we will ground the positive input and apply a signal to the negative input. Assume that the DC bias conditions do not change when we do this.


Find the following

|  | Voltage Gain | Input impedance |
| :--- | :--- | :--- |
| Transistor combination <br> Q4,5,6,7 |  |  |
| Transistor Q8 |  |  |
| Transistor combination <br> Q11,12 |  |  |
| Overall differential <br> Vout/Vin |  |  |

## Part d, 10 points

Maximum peak-peak output voltage at the positive output Vo+ (show all your work) Recall that the FETs are velocity-limited, hence $V_{D S, \text { knee }}=\Delta V=0.1 \mathrm{~V}$.

|  | magnitude and sign of <br> maximum output signal <br> swing due to cutoff | magnitude and sign of <br> maximum output signal <br> swing due to: <br> knee voltage (saturation) |
| :--- | :--- | :--- |
| Transistor Q3 |  |  |
| Transistor Q8 |  |  |
| Transistor Q11 |  |  |
| Transistor Q12 |  |  |

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant". But, give a 1 -sentence statement why below.

Problem 3, 30 points


Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.

## Part b, 8 points

gm=9 mS. $\mathrm{C}=1 \mathrm{nF} . \mathrm{R}=1000$ Ohms
Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with ${ }^{* *}$ correct units** and in ratio-of-polynomials form, i.e.
$\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=K \cdot \frac{1+b_{1} s+b_{2} s^{2}+\ldots}{1+a_{1} s+a_{2} s^{2}+\ldots}$ or (as appropriate) $\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=K \cdot(s \tau)^{n} \cdot \frac{1+b_{1} s+b_{2} s^{2}+\ldots}{1+a_{1} s+a_{2} s^{2}+\ldots}$
Note that an expression like
$\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=\frac{1}{1+\left(3 \cdot 10^{-6}\right) s}$ is dimensionally wrong; $\frac{1}{1+\left(3 \cdot 10^{-6} \text { seconds }\right) s}$ is dimensionally correct
$\qquad$

## Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz :
$\qquad$ , $\qquad$ , $\qquad$ ,

Draw a clean Bode Plot of Vout/Vin,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes

|  |  |  | I\| |  |  | , | TI\| |  | - | ] | I\| |  | \| | T\| | \|| |  | - |  | \| |  |  |  | T | T |
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## Part d, 8 points

$\operatorname{Vin}(\mathrm{t})$ is a 0.2 V amplitude step-function.
Find $\operatorname{Vout}(\mathrm{t})=$ $\qquad$
Plot it below. Label axes, show initial and final values, show time constants.


