Final Exam, ECE 137A

Wednesday March 19, 2014 7:30-10:30 PM

Name: Solution B.

Closed Book Exam: Class Crib-Sheet and 3 pages (6 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), ***after stating & justifying them.***
Show your work:
Full credit will not be given for correct answers if supporting work is missing.

Good luck

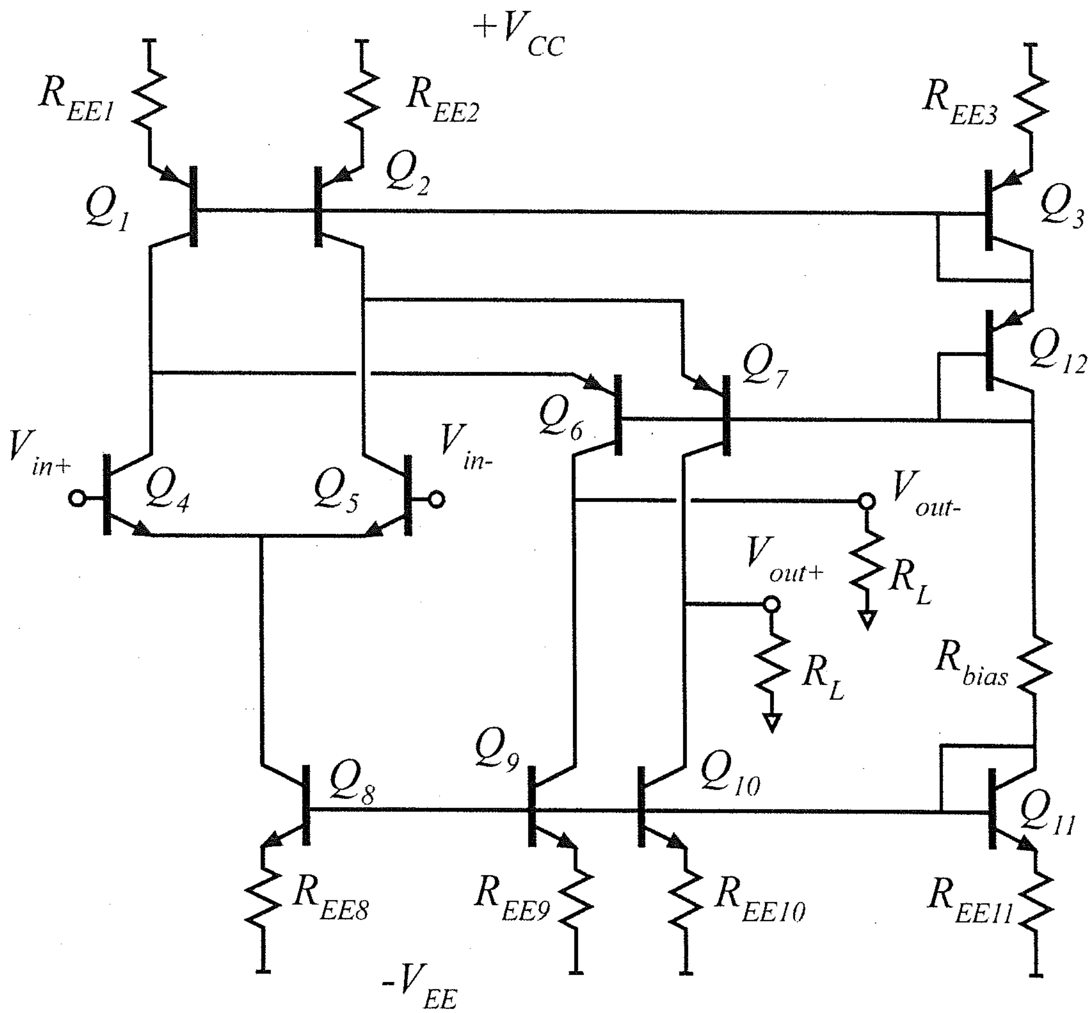
Time function	LaPlace Transform
$\delta(t)$ impulse	1
$U(t)$ unit step-function	$1/s$
$e^{-\alpha t}U(t)$	$\frac{1}{s+\alpha}$
$e^{-\alpha t}\cos(\omega_d t)U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		5	2c		15
1b		6	2d		10
1c		4	3a		7
1d		10	3b		8
1e		10	3c		7
2a		10	3d		8
2b		10			
total		100			

b

Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts



All the transistors have the same (matched) I_S, have $\beta = \infty$, and $V_A = \infty$ Volts. $V_{CE(sat)} = 0.5\text{V}$. V_{be} is roughly 0.7 V, but use $V_{be} = (kT/q)\ln(I_E / I_S)$ when necessary and appropriate. The supplies are +4.5 Volts and -4.5 Volts. The DC voltage drops across Ree3 and Ree11 are both 500mV.

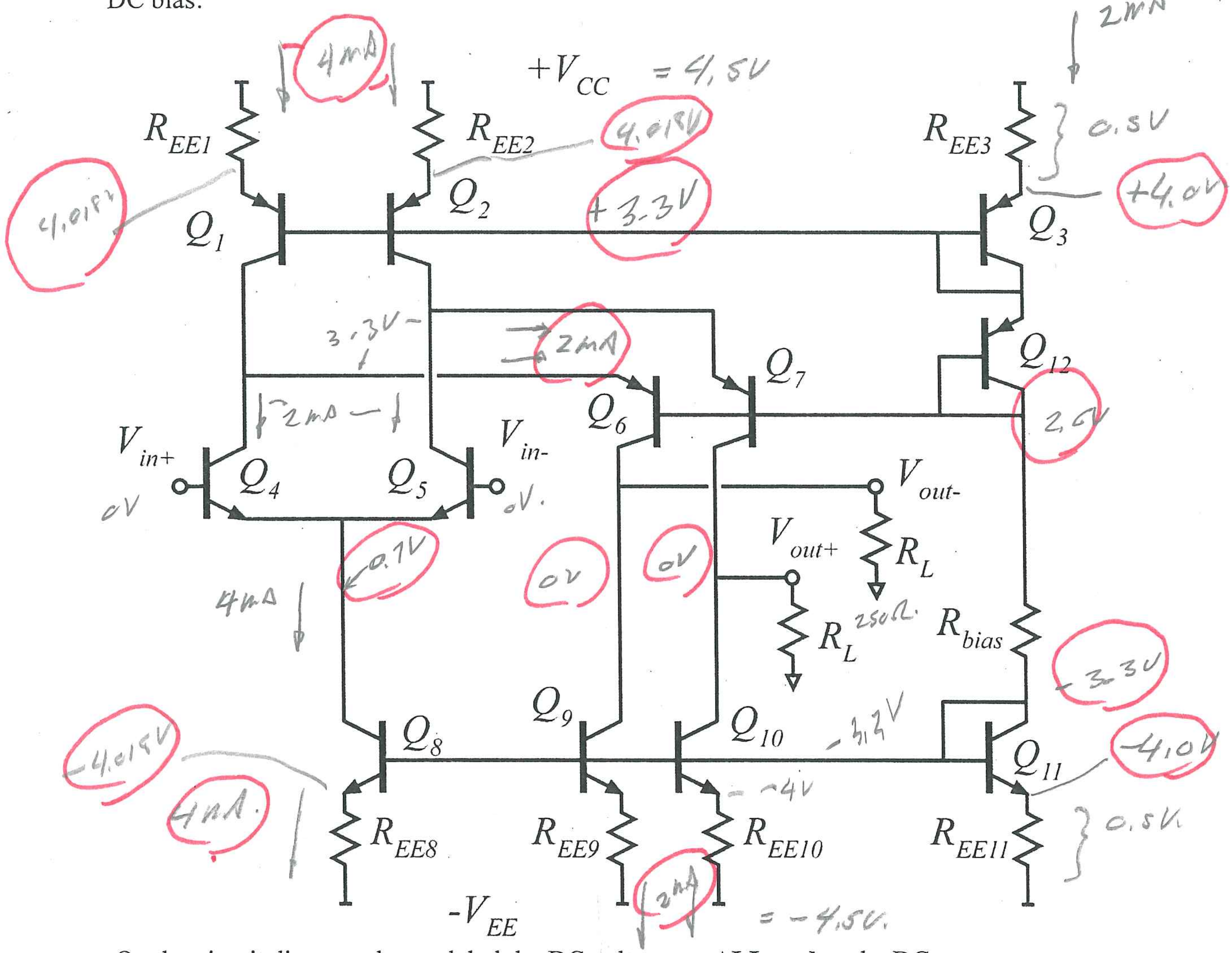
The DC collector currents of Q3,4,5,6,7,11,12 are all 2.0 mA. $R_L = 250\Omega$

b

@ 1/3 pt each

Part a, 5 points

DC bias:



On the circuit diagram above, label the DC voltages at **ALL nodes**, the DC currents through **ALL resistors**, and the DC drain currents of **all transistors**.

Part of part b

note that $V_{RE9} = V_{RE10} = V_{RE11} = 0.5V$] 0.5

note that $V_{BE8} = V_{BE11} + V_T \ln(2) = 0.7V + 18mV$] 1

→ Drop across R_{EE8} is $500mV - 18mV = 482mV$] 0.5

Note that $V_{EB1} = V_{EB2} = V_{EB3} + V_T \ln(2) = 0.7V + 18mV$] 1

drops across $R_{EE1,2}$ are 482mV] 0.5

Part b, 6 points
DC bias:

Find the value of all resistors.
Rbias=2.95kΩ Ree1=170Ω Ree2=120Ω Ree3=250Ω
Ree8=170Ω Ree9=250Ω Ree10=250Ω Ree11=250Ω

$R_{bias} = 5.9V / 2mA = 2.95 k\Omega$] 0.5

$R_{EE1} = R_{EE2} = 482mV/4mA = 120.5\Omega$] 0.5

$R_{EE3} = 0.5V/2mA = 250\Omega$] 0.5

$R_{EE8} = R_{EE9} = 482mV/4mA = 120.5\Omega$] 0.5

$R_{EE11} = R_{EE9} = R_{EE10} = 0.5V/2mA = 250\Omega$] 0.5.

b

Part c, 4 points

Find the transconductance of the transistors below:

gm4=_____ gm5=_____ gm6=_____ gm7=_____] all 76.9mS.

$$\frac{1}{g_{m4}} = \frac{1}{g_{m5}} = \frac{1}{g_{m6}} = \frac{1}{g_{m7}} = \frac{26mV}{2mA} = 13\Omega$$]. 1pt each

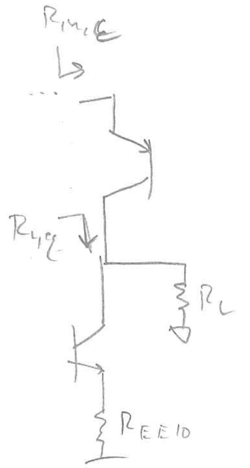
$1/13\Omega = 76.9mS$

b

Part d, 10 points.

The circuit is fully differential. Assuming a differential input signal, $V_{in,diff} = V_{i+} - V_{i-}$, and defining a differential output signal $V_{out,diff} = V_{0+} - V_{0-}$, compute the differential gain $A_d = V_{out,diff} / V_{in,diff}$

$A_d =$ +19.2



$$R_{L,eq} = R_L \,||\, R_{CE}\left(1 + g_m R_{EE10}\left(\frac{r_{be}}{r_{be} + R_{BE} + R_B}\right)\right)$$

$$= R_L = 250\,\Omega$$

$$A_{v,Q6} = \frac{R_L}{r_e} = g_m R_L = \frac{250\,\Omega}{13\,\Omega} = 19.2$$] 4

$$R_{in,E} = \left(r_e + \frac{R_B}{\beta+1}\right)\left(\frac{r_{ce} + R_{CC}}{r_{ce}}\right) = r_e = \frac{1}{g_m}$$

$$\Rightarrow A_{v,Q4} = -g_m \cdot R_{eq} = -\frac{g_{m,Q5}}{g_{m,Q7}} = -1$$] 4

$$A_{v,Tot} = -A_{v,Q4} \cdot A_{v,Q6} = +19.2$$] 2

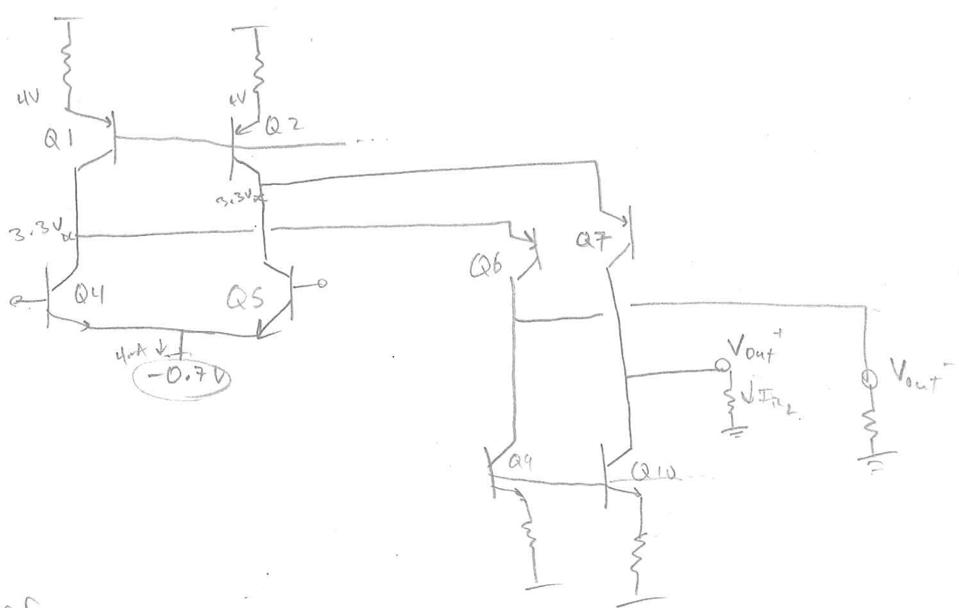
b

Part e, 10 points

Maximum peak-peak output voltage at the positive output Vo+ (***show all your work***)

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to ***saturation***
Transistor Q1	N/A (current mirror)] 1	−3.84V] 2
Transistor Q4	−500mV] 2	+67.2V] 1
Transistor Q6	+500mV] 1	−2.8V] 1
Transistor Q9	N/A (current Mirror)] 1	+3.5V] 1

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant.



Cutoff

Q4: $I_{c,4} = 0mA \Rightarrow I_{c,5} = 4mA \Rightarrow I_{c,7} \rightarrow 0mA$ from 2mA

$\Rightarrow V_{out}^{+} = -R_L \cdot 2mA = -500mV$

Q6: $I_{c,6} = 0mA \Rightarrow I_{c,4} = 4mA \Rightarrow I_{c,5} = 0mA \Rightarrow I_{c,7} = 4mA \Rightarrow I_{R_L} = +2mA$

$\Rightarrow V_{out} = (2mA)(250\Omega) = +500mV$

Q1 & Q9: no cutoff since these are part of current mirrors.

Saturation

Q1: $V_{CE1,Q} = 0.7V$, $V_{CE1,sat} = 0.5V \Rightarrow \Delta V_{C1} = 0.2$

$\Rightarrow \Delta V^-_{out,Q1\,sat} = 0.2 \times 19.2 = 3.84V$

$\Rightarrow \Delta V^+_{out,Q1\,sat} = -3.84V$

Q4

$V_{CE4,Q} = 4V$, $V_{CE4,sat} = 0.5V \Rightarrow \Delta V_{C,4} = -3.5V$

$\Rightarrow \Delta V^-_{out,Q4\,sat} = (19.2)(-3.5V) = -67.2V$

$\Delta V^+_{out,Q4,sat} = +67.2V$

Q6

$V_{CE,Q6} = 3.3V$, $V_{CE,sat} = 0.5V$, $V_B \sim$ fixed $\Rightarrow |\Delta V_{CE}| = |\Delta V_C|$ ≅

$\Delta V_{C6} = +2.8V$

$\Rightarrow \Delta V^-_{out,Q6\,sat} = +2.8V \Rightarrow \Delta V^+_{out,Q6\,sat} = -2.8V$

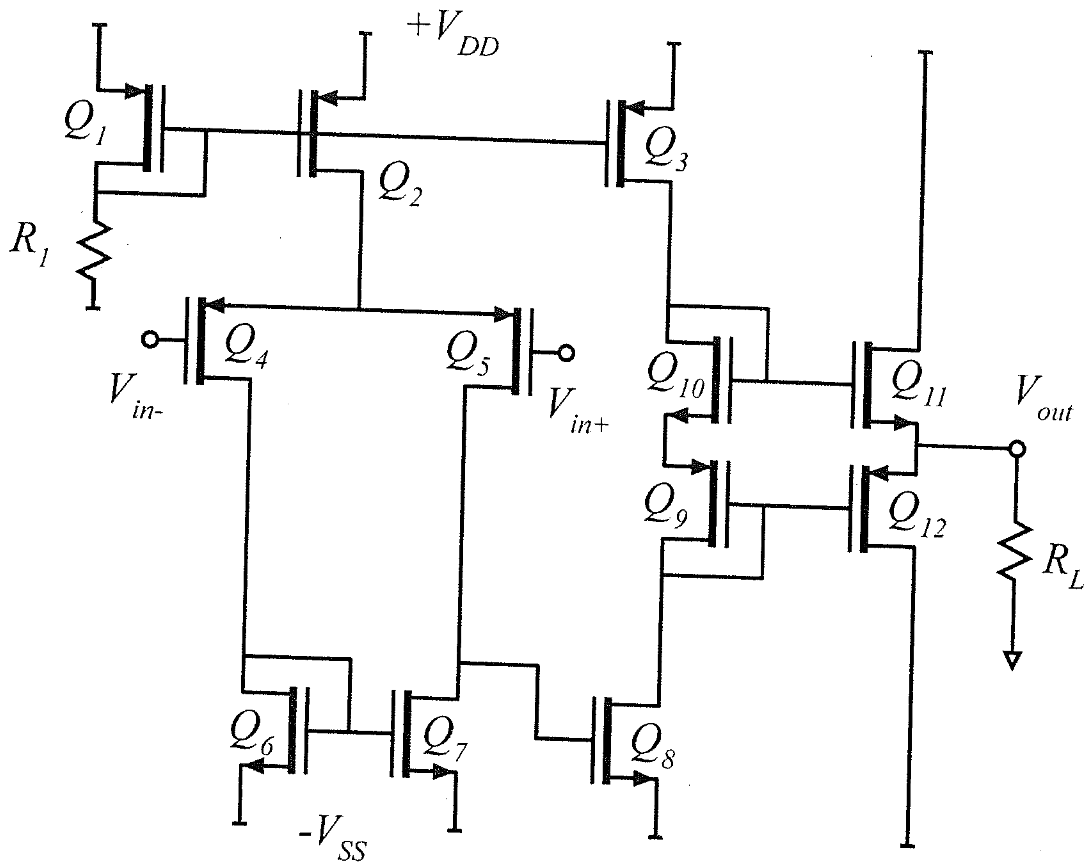
Q9

$V^-_{out} \to -3.5V \Rightarrow \Delta V^+_{out,Q9} = +3.5V$

b

Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.



All NMOS: $I_D = 1(\text{mA/V})(W_g/1\mu\text{m})(V_{gs} - V_{th} - \Delta V)(1 + \lambda V_{DS})$

$\Delta V = 0.1\text{V}$, $V_{th} = 0.2\text{V}$, $1/\lambda = 5\text{V}$

All PMOS: Also velocity-limited, with $g_m = 0.5(\text{mA/V})(W_g/1\mu\text{m})$

$\Delta V = -0.1\text{V}$, $V_{th} = -0.2\text{V}$, $1/\lambda = 5\text{V}$

V_{DD}=+1 V, $-V_{SS}$= -1 V, The load resistor is R_L=10 kΩ

b

Part a, 10 points
DC bias.
Approximation: ignore the term $(1+\lambda V_{DS})$ in DC bias analysis.

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.

Q1 is to be biased at 0.2 mA drain current.
The transistor gate widths are as follows

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Q10	Q11	Q12
2μm	4μm	4μm	2μm	2μm	1μm	1μm	2μm	4μm	2μm	10μm	20μm

Find:
ID1= 0.2mA ID2= 0.4mA ID3= 0.4mA ID4= 0.2mA ID5= 0.2mA ID6= 0.2mA
ID7= 0.2mA ID8= 0.4mA ID9= 0.4mA ID10= 0.4mA ID11= 2mA ID12= 2mA
R1= 7.5kΩ

$I_{D_1} = 0.2mA = (0.5mA/V)\left(\frac{2\mu m}{1\mu m}\right)(|V_{GS_1}| - |V_{th}| - |\Delta V|)$ (+1)

$|V_{GS_1}| = 0.2V + 0.2V + 0.1V = 0.5V$

$R_1 = \frac{1.5V}{0.2mA} = 7.5K\Omega$ (+1)

$|V_{GS_2}| = |V_{GS_3}| = 0.5V$

(+2) $I_{D_2} = \frac{W_{g_2}}{W_{g_1}} \cdot I_{D_1} = 2 \times 0.2mA = 0.4mA = I_{D_3}$ (+1)

(+1.5) $I_{D_4} = I_{D_5} = I_{D_2}/2 = 0.2mA = (0.5mA/V)\left(\frac{2\mu m}{1\mu m}\right)(|V_{GS_4}| - |V_{th}| - |\Delta V|)$

$|V_{GS_4}| = |V_{GS_5}| = 0.2V + 0.3V = 0.5V$

(+1.5) $I_{D_6} = I_{D_4} = I_{D_7} = I_{D_5}$; $W_{g_6} = \frac{1}{2} W_{g_4}$; $g_{m_6} = 2 g_{m_4}$

$\therefore |V_{GS_6}| = |V_{GS_7}| = |V_{GS_4}| = |V_{GS_5}| = 0.5V$

b

$$I_{D_{10}} = \left(1\,mA/V\right)\left(\frac{2\mu m}{1\mu m}\right)\left(|V_{GS_{10}}| - |V_{TH}| - |\Delta V|\right) = 0.4\,mA$$

$$|V_{GS_{10}}| = 0.2V + 0.3V = 0.5V$$

$$W_{g9} = 2W_{g10} \;;\; g_{m_9} = \frac{1}{2} g_{m_{10}} \;;\; I_{D_{10}} = I_{D_9}$$

$$\Rightarrow |V_{GS_{10}}| = |V_{GS_9}|$$

$$I_{D_8} = I_{D_{10}} \;;\; W_{g8} = W_{g10} \;;\; g_{m_8} = g_{m_{10}}$$

$$\Rightarrow |V_{GS_8}| = |V_{GS_{10}}|$$

$$I_{D_{11}} = \left(1\,mA/V\right)\left(\frac{10\mu m}{1\mu m}\right)\left(0.5V - 0.2V - 0.1V\right) = 2\,mA$$

$$I_{D_{11}} = I_{D_{12}} = 2mA$$

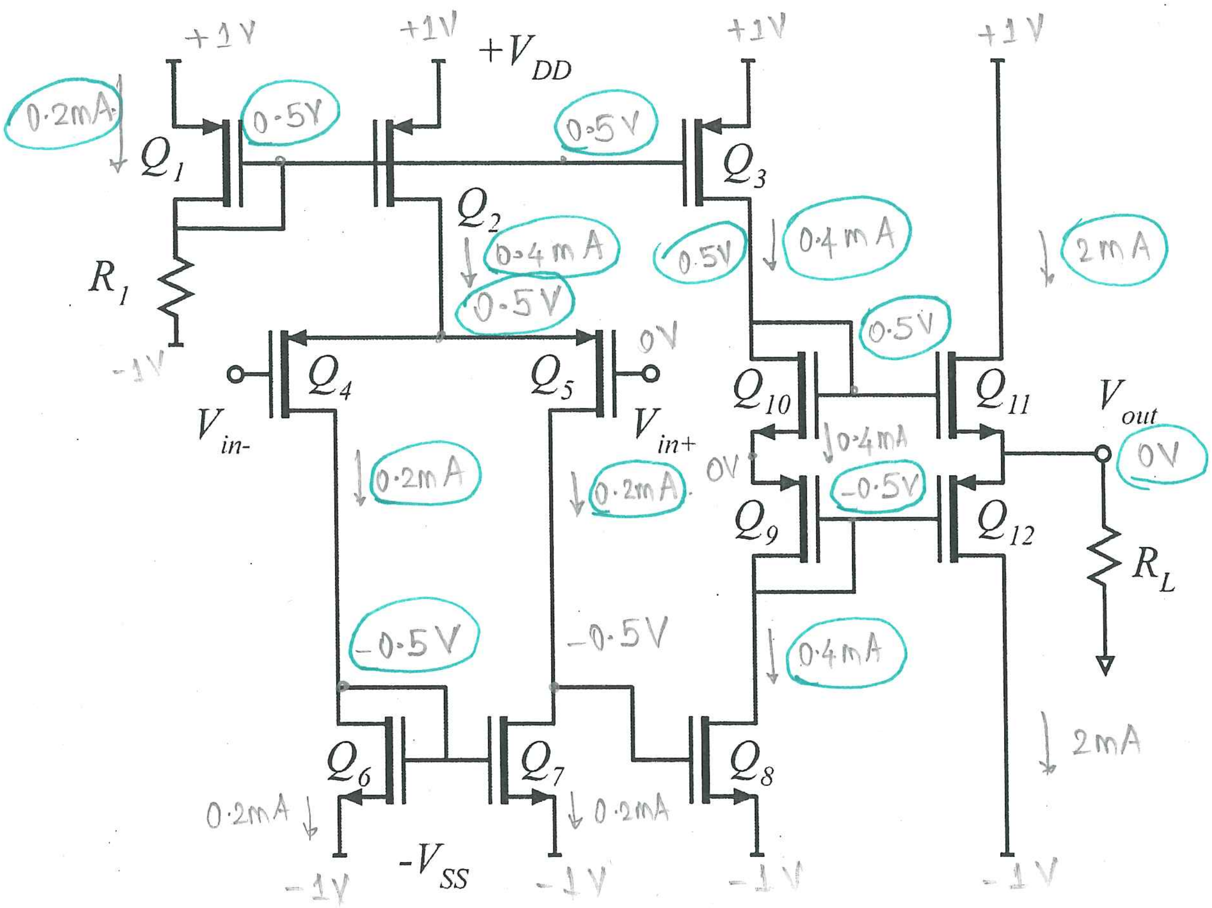
+2

b

2/3 pt each

Part b, 10 points

DC bias



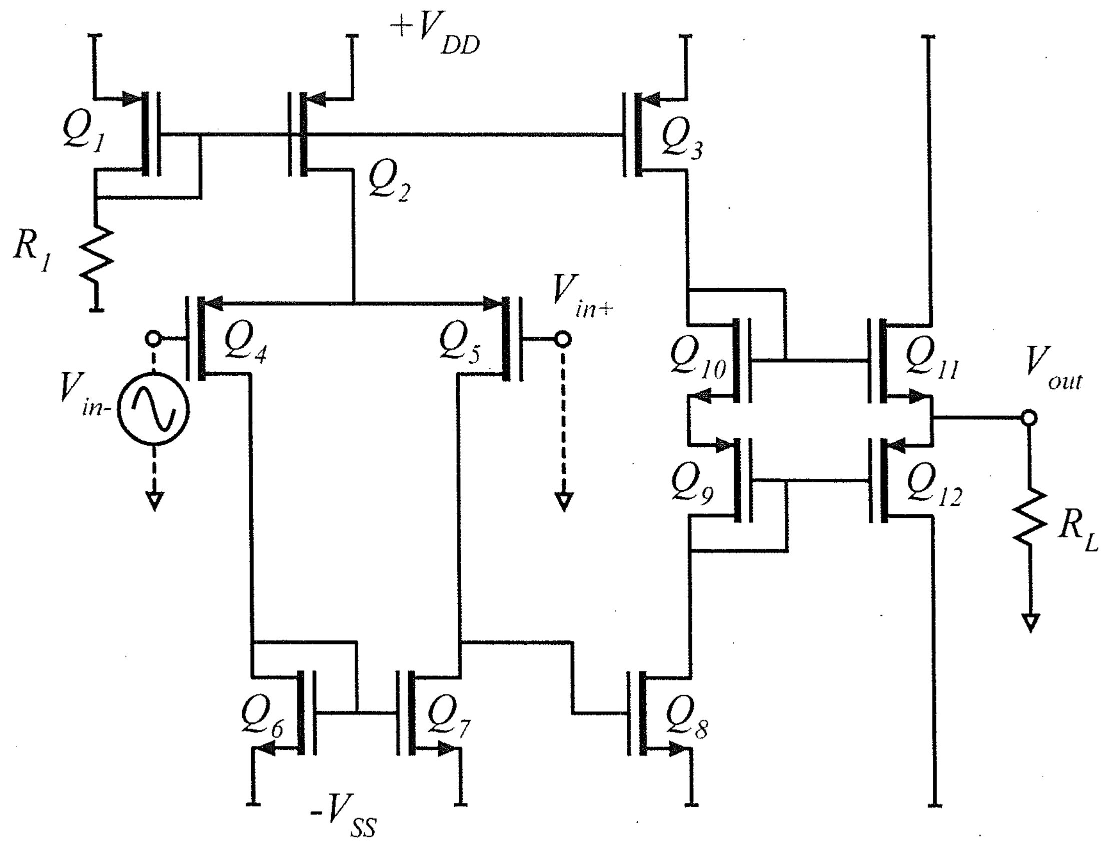
On the circuit diagram above, label the DC voltages at **ALL nodes** and the drain currents of **ALL transistors**

b

Define gain = $\frac{V_{out}}{V_{in}^{+} - V_{in}^{-}}$

Part c, 15 points.

To compute the op-amp differential gain, we will ground the positive input and apply a signal to the negative input. Assume that the DC bias conditions do not change when we do this.



Find the following

	Voltage Gain	Input impedance
Transistor combination Q4,5,6,7	± 15	∞ Ω
Transistor Q8	− 13.75	∞ Ω
Transistor combination Q11,12	1 (or) 0.5	∞ Ω
Overall differential Vout/Vin	∓ 103.125	∞ Ω

b

$A_{V,4,5,6,7}$

$$A_{V,4567} = \pm g_{m_5} R_{Leq}$$

$$R_{Leq} = R_{DS_7} \parallel R_{DS_5}$$ (+1.5)

$$R_{DS_7} = \frac{1}{\lambda I_{D_7}} = \frac{5V}{0.2mA} = 25k\Omega = R_{DS_5}$$ (+1.5)

$$R_{Leq} = 12.5k\Omega$$

(+1.5) $$g_{m_5} = (0.5mA/V)\left(\frac{2\mu m}{1\mu m}\right)\left(1 + \frac{1V}{5V}\right) = 1mS(1.2) = 1.2mS$$

$$A_{V,4567} = \pm 15$$ (+1.5)

$A_{V,8}$

$$A_{V,8} = -g_{m_8} R_{Leq}$$

$$R_{Leq} = R_{DS_8} \parallel R_{DS_3}$$

$$R_{DS_8} = \frac{1}{\lambda I_{D_8}} = \frac{5V}{0.4mA} = 12.5k\Omega$$

$$R_{DS_3} = R_{DS_8} = 12.5k\Omega$$ (+1.5)

$$R_{Leq} = 12.5k\Omega \parallel [12.5k\Omega]$$

$$R_{Leq} = 6.25k\Omega$$ (+1.5)

$$g_{m_8} = (1mA/V)\left(\frac{2\mu m}{1\mu m}\right)\left(1 + \frac{0.5}{5}\right) = 2.2mS$$

$$A_{V,8} = -2.2mS \times 6.25k\Omega = -13.75$$ (+1.5)

$A_{V_{11,12}}$ (Assuming Q_{11}, Q_{12} as 'ON')

$$R_{Leq} = R_L \parallel \left(\frac{1}{g_{m_{11}}}\right) \parallel \left(R_{DS_{11}} \parallel R_{DS_{12}}\right)$$

$$g_{m_{11}} = \left(1\,mA/V\right)\left(\frac{10\mu m}{1\mu m}\right)\left(1 + \frac{1}{5}\right) = 12\,mS \quad (+1.5)$$

$$R_{DS_{11}} = \frac{1}{\lambda I_{D_{11}}} = \frac{5V}{2mA} = 2.5\,k\Omega = R_{DS_{12}}$$

$$R_{Leq} = 10\,k\Omega \parallel (83.3\,\Omega) \parallel (1.25\,k\Omega)$$

$$R_{Leq} = 83\,\Omega \quad (+1.5)$$

$$A_{V,11,12} = \frac{R_{Leq}}{R_{Leq} + 1/g_{m_{12}}} = \frac{83\,\Omega}{83\,\Omega + 83.3\,\Omega} \approx 0.5 \quad (+1.5)$$

Overall gain: $A_{V_{4567}} \times A_{V,8} \times A_{V_{11,12}} = (\pm 15)(-13.75)(0.5)$

$$\boxed{A_{V_{overall}} = \mp 103.125} \quad (+1.5)$$

{ The difference in the value of $A_{V,11,12}$ comes from assuming both 11 & 12 as being 'ON' (or) either of them being 'ON'. BOTH ARE OK

Ambiguity in the signs of the overall gain comes from the definition of the gain for the first stage (i) wrt V_{in} signal input @ V_{in}^{-}
(ii) wrt $(V_{in}^{+} - V_{in}^{-})$

Part d, 10 points (Assuming Q_{11}, Q_{12} are both 'ON')

Maximum peak-peak output voltage at the positive output Vo+ (***show all your work***)
Recall that the FETs are velocity-limited, hence $V_{DS,knee} = \Delta V = 0.1V$.

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to: ***knee voltage*** (saturation)
Transistor Q3	N/A current mirror	(+0.2V ↑) (or) (+0.4V ↑)
Transistor Q8	(+1.25 V ↑) (or) (+2.5V ↑)	(−0.2V ↓) (or) (−0.4V ↓)
Transistor Q11 (+1)	N/A push-pull	+ 0.9V ↑
Transistor Q12 (+1)	N/A push-pull	− 0.9V ↓

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant". But, give a 1-sentence statement why below.

Q_{11} : (Knee) $V_{DS,Q} = 1V$

$\Delta V_{out} = V_{DS,Q} - \Delta V = +0.9V$ ↑ (+1)

Q_{12} : (Knee) Same as Q_{11} (+1)

(+2) Q_8 : (cutoff) $I_{CQ_8} = 0.4mA$

$\Delta V_{D_8} = I_{CQ_8} \times R_{Leq} = (0.4mA)(6.25 k\Omega)$

$= +2.5$ ↑ ⇒ $\Delta V_{out} = +1.25V$ ↑

(+2) Q_8 : (Knee) $V_{DS,Q} = 0.5V$

$\Delta V_{D_8} = V_{DS,Q} - V_{DS,knee} = 0.5V - 0.1V = 0.4V$ ↓

$\Delta V_{out} = -0.2V$ ↓

b

Q3 : $V_{DS,Q} = 0.5V$

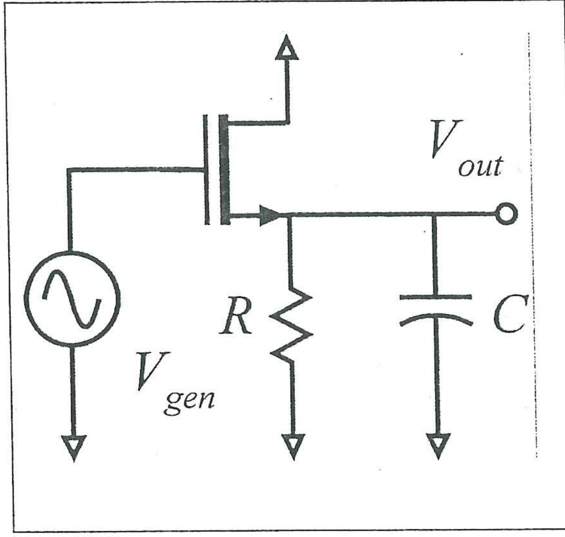
$\Delta V_{DS} = 0.5V - 0.1V = 0.4V$

$\Delta V_{out} = +0.2V$ ↑

+2

b

Problem 3, 30 points



You will be working on the circuit to the left

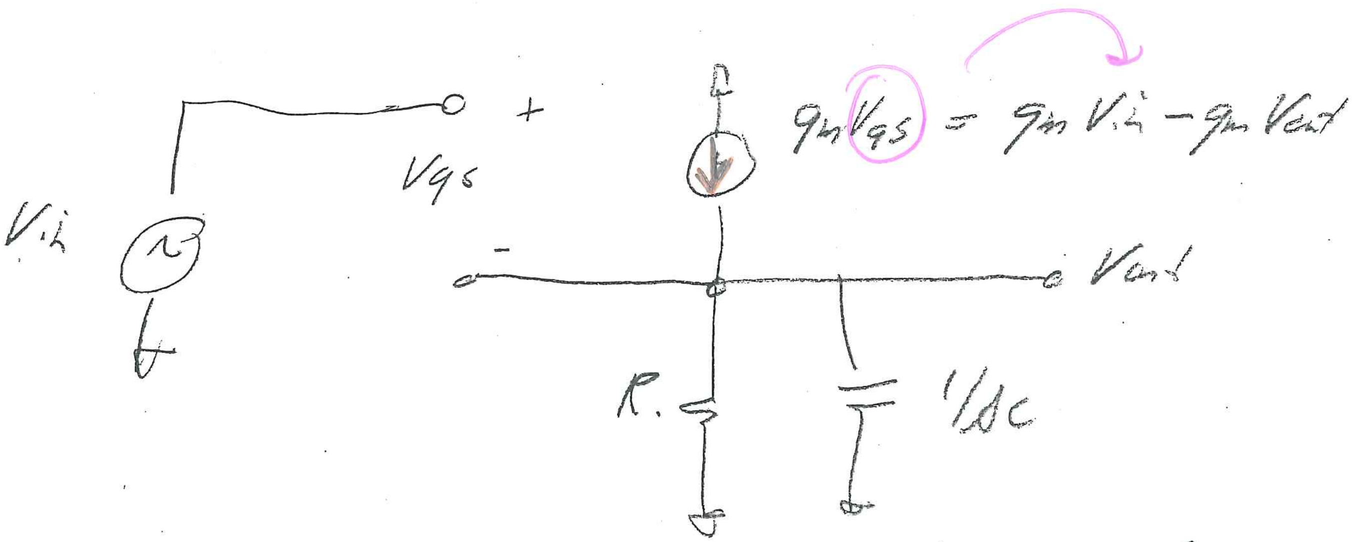
Ignore DC bias analysis. You don't need it.

The transistor has transconductance gm.

Its output resistance Rds is infinity...so you don't need to include this element in the circuit diagram !

Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.



check controlling voltage of $g_m V_{gs}$!

b

Part b, 8 points

gm=9 mS. C=1 nF. R= 1000 Ohms

Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with **correct units** and in ratio-of-polynomials form, i.e.

$$\frac{V_{out}(s)}{V_{gen}(s)} = K \cdot \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots} \text{ or (as appropriate) } \frac{V_{out}(s)}{V_{gen}(s)} = K \cdot (s\tau)^n \cdot \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots}$$

Note that an expression like

$$\frac{V_{out}(s)}{V_{gen}(s)} = \frac{1}{1+(3\cdot 10^{-6})s} \text{ is dimensionally wrong; } \frac{1}{1+(3\cdot 10^{-6}\text{ seconds})s} \text{ is dimensionally correct}$$

Vout(s)/Vin(s)= ___________________

$\sum I = 0$ @ Vout

4 $-g_m V_{in} + g_m V_{out} + V_{out}/R + sC \cdot V_{out} = 0$

$g_m V_{in} = V_{out}[g_m + 1/R + sC]$

$$\frac{V_{out}}{V_{in}} = \frac{g_m}{g_m + 1/R + sC_1}$$

b

$g_m = 9mS$, $R = 1k\Omega$

$C = 10^{-9}F$

$$\frac{V_{out}}{V_{in}} = \frac{g_m}{g_m + 1/R} \cdot \frac{g_m + 1/R}{g_m + 1/R + sC}$$

$$= \frac{g_m}{g_m + 1/R} \cdot \frac{1}{1 + sC(g_m + 1/R)^{-1}}$$

$$= 0.9 \cdot \frac{1}{1 + s\tau}$$

where $\tau = (10mS)^{-1} 10^{-9}F = 100\Omega \cdot 10^{-9}F$

$= 10^{-7}$ seconds

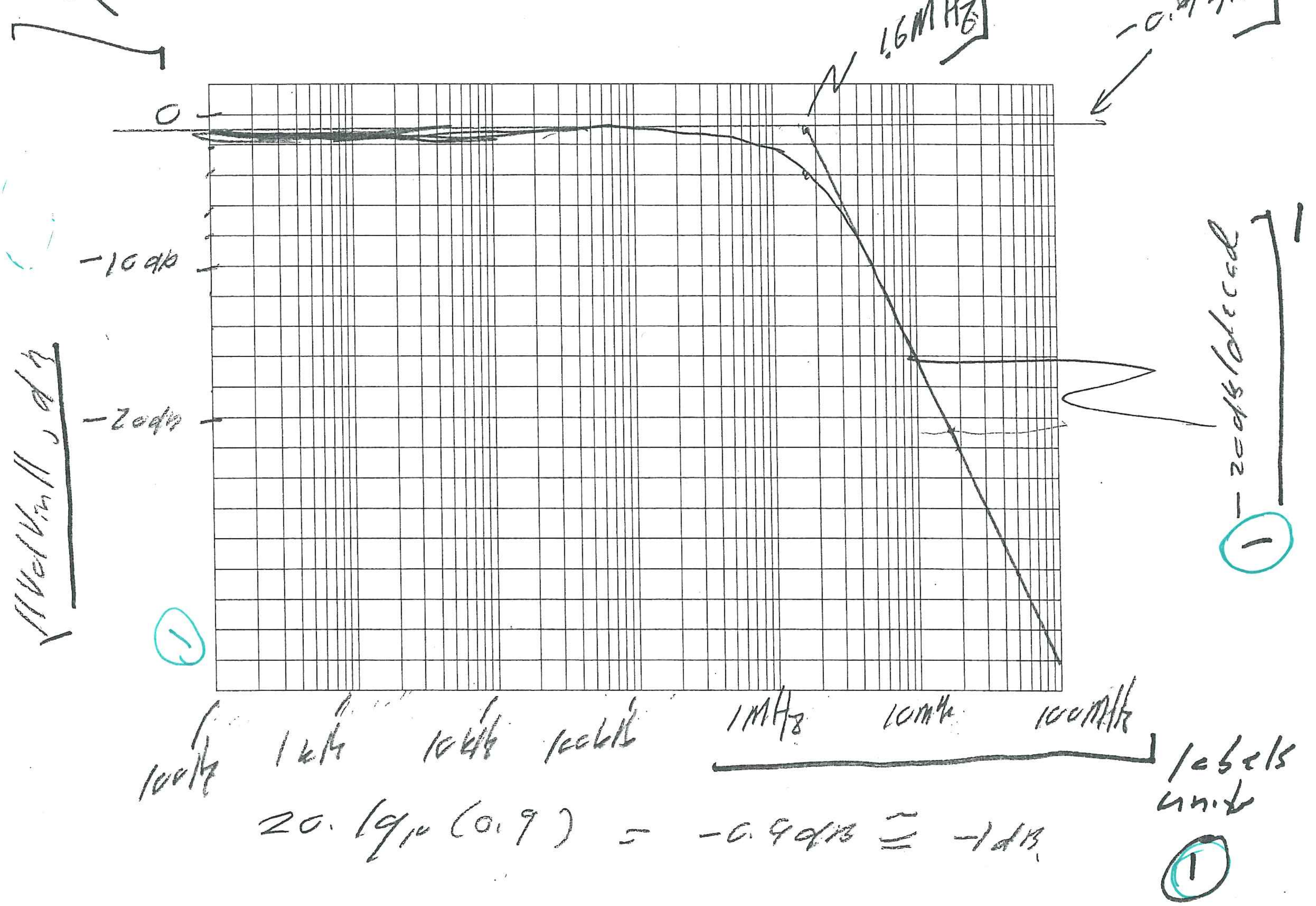
$$f_0 = \frac{1}{2\pi\tau} = 1.59MHz \cong 1.6MHz$$

Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz: pole @ 1.6 MHz.

______________, ______________, ______________, ______________

Draw a clean Bode Plot of Vout/Vin,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes



b

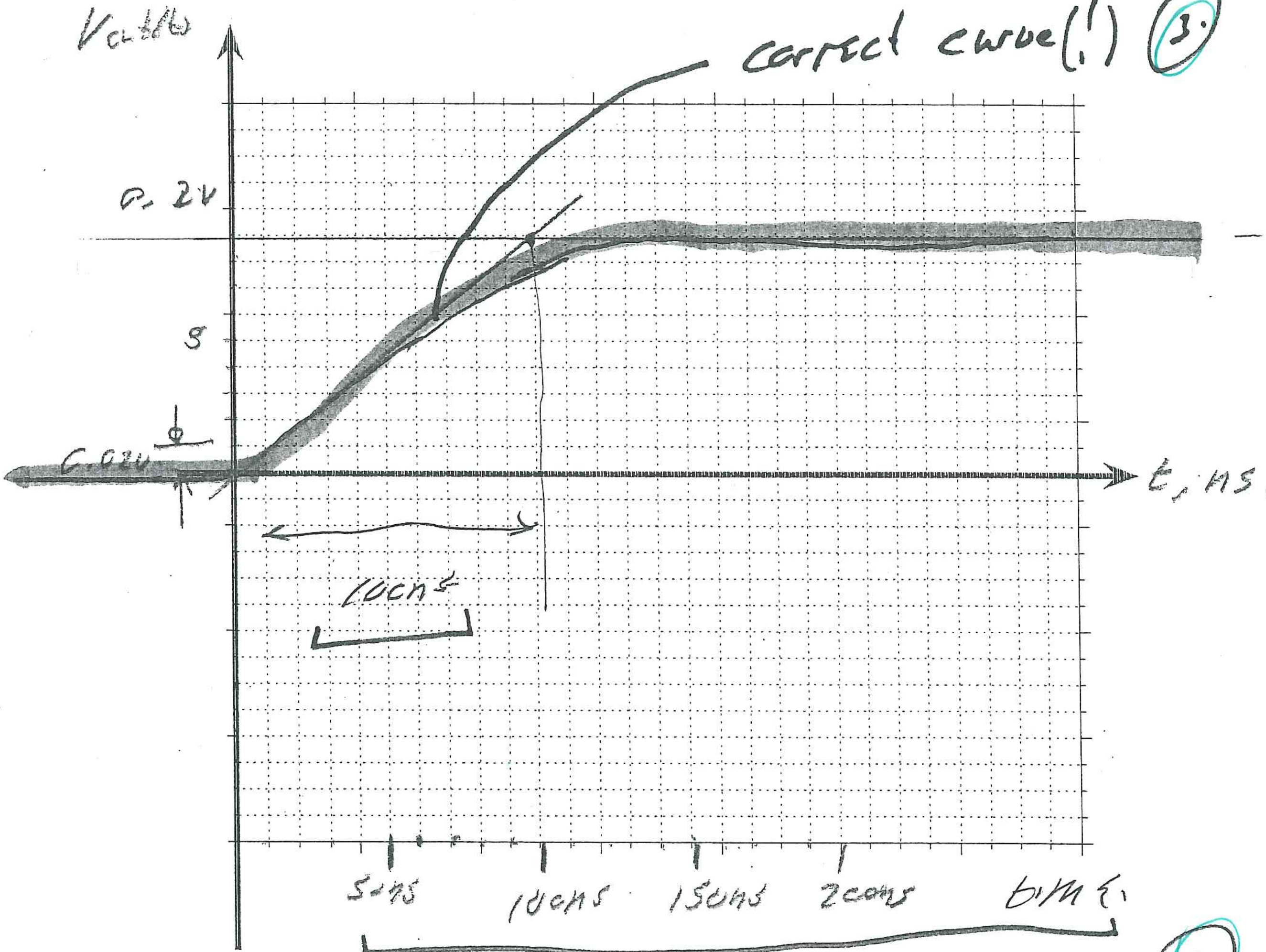
labels, units (1)

Part d, 8 points

Vin(t) is a 0.2 V amplitude step-function.

Find Vout(t)= $0.18V \cdot u(t) [1-e^{-t/(100ns)}]$ (2)

Plot it below. Label axes, show initial and final values, show time constants.



b

$$H(s) = \frac{0.9}{1+s\tau} \quad \text{where } \tau = 10^{-7}\text{ sec}$$

$$V_{in}(s) = \frac{0.2V}{s}$$

$$V_{out}(s) = \frac{0.18V}{s} \frac{1}{1+s\tau}$$

$$V_{out}(t) = 0.18V \cdot u(t) \left[1 - e^{-t/\tau} \right]$$

where $\tau = 10^{-7}$ sec = 100ns