Final Exam, ECE 137A

Wednesday March 18, 2015 7:30-10:30 PM

Name: Solution b

Closed Book Exam: Class Crib-Sheet and 3 pages (6 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), ***after stating & justifying them.***
Show your work:
Full credit will not be given for correct answers if supporting work is missing.

Good luck

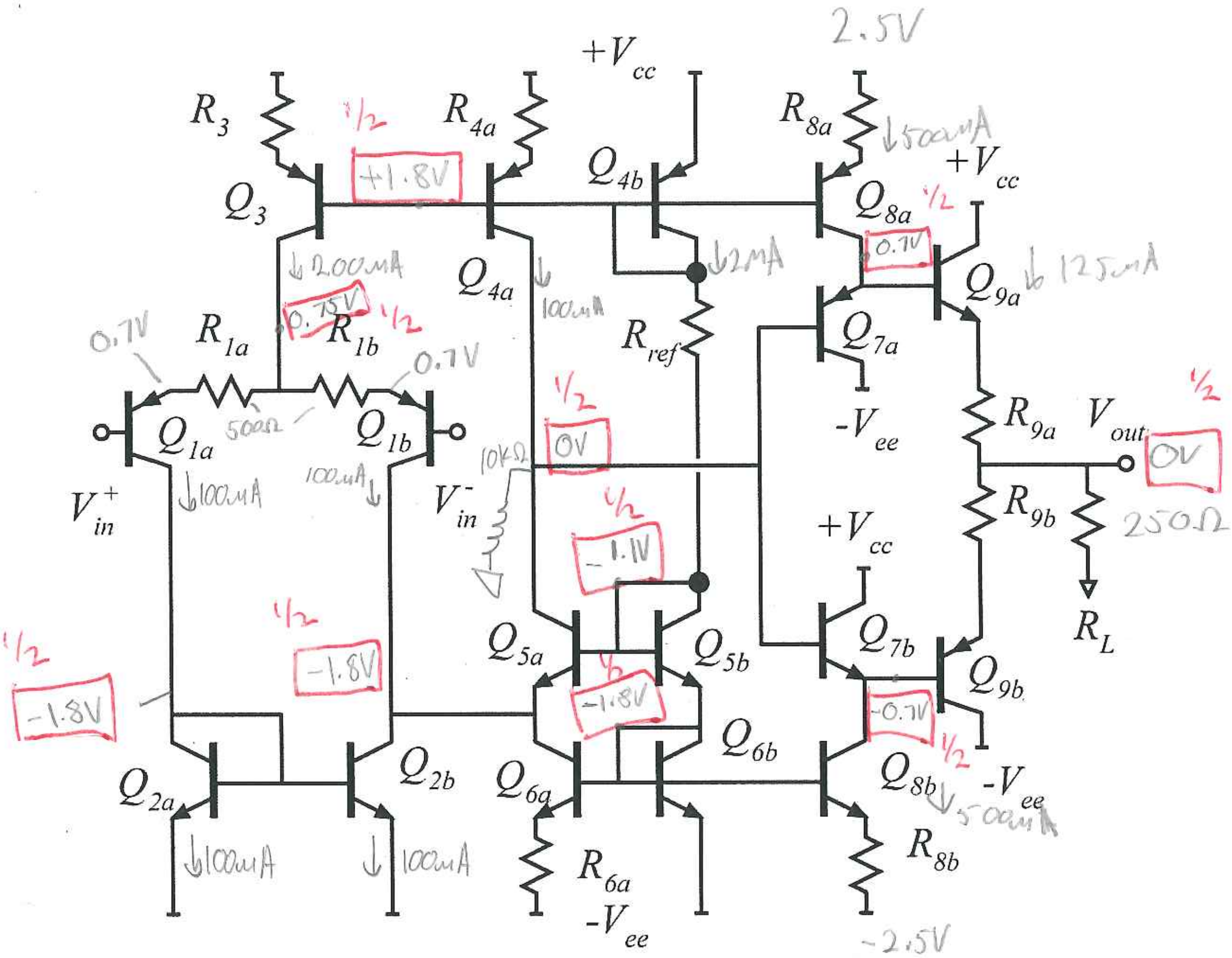
Time function	LaPlace Transform
$\delta(t)$ impulse	1
$U(t)$ unit step-function	$1/s$
$e^{-\alpha t}U(t)$	$\frac{1}{s+\alpha}$
$e^{-\alpha t}\cos(\omega_d t)U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		5	2c		15
1b		6	2d		10
1c		4	3a		7
1d		10	3b		8
1e		10	3c		7
2a		10	3d		8
2b		10			
total		100			

b

Part a, 5 points

DC bias---to simplify ,assume $\beta = \infty$ ***for the DC analysis only.***



On the circuit diagram above, label the DC voltages at **ALL nodes**, the DC currents through **ALL resistors**, and the DC collector currents of **all transistors**.

Part b

① $\left[V_{be6b} - V_{be6a} = \frac{kT}{q}\ln\left[\frac{2mA}{100\mu A}\right] = 78mV \rightarrow R_{6a} = \frac{78mV}{100\mu A} = \boxed{779\,\Omega}\right.$

① $\left[V_{be6b} - V_{be8b} = \frac{kT}{q}\ln\left[\frac{2mA}{500\mu A}\right] = 36.04mV \rightarrow R_{8b} = \frac{36.04mV}{500\mu A} = \boxed{72.1\,\Omega}\right.$

① $\left[V_{be7b} - V_{be9b} = \frac{kT}{q}\ln\left[\frac{500\mu A}{125\mu A}\right] = 36.04mV \rightarrow R_{9a} = \frac{36.04mV}{125\mu A} = \boxed{288\,\Omega}\right.$

b

by same arguments, $R_{8a} = \boxed{72\Omega}$
$R_{9a} = \boxed{288\Omega}$

① $\left[V_{be4b} - V_{be4a} = \frac{kT}{q} \ln\left[\frac{2\,mA}{100\,\mu A}\right] = 78\,mV \rightarrow R_{4a} = \frac{78\,mV}{100\,\mu A} = \boxed{779\,\Omega} \right.$

① $\left[V_{be4b} - V_{be3} = \frac{kT}{q} \ln\left[\frac{2\,mA}{200\,\mu A}\right] = 60\,mV \rightarrow R_3 = \frac{60\,mV}{200\,\mu A} = \boxed{299\,\Omega} \right.$

① $\left[R_{ref} = \frac{1.8V - (-1.1V)}{2\,mA} = \boxed{1450\,\Omega} \right.$

b

Part b, 6 points
DC bias:

Find the value of all resistors.
R3=299Ω R4ba=779Ω R6a=779Ω Rref=____ R8a=72Ω R8b=72Ω
R9a=288Ω R9b=288Ω

↑ 1.45kΩ

b

Part c, 4 points

Find the transconductance of the transistors below:
gm1a=3.85mS gm1b=3.85mS gm5a=3.85mS gm7a=19.2mS
gm7b=19.2mS gm9a=4.8mS gm9b=4.8mS

(1) [$Q_{1a,b}$: $I_c = 100\mu A$, $\frac{1}{g_m} = 260\Omega \rightarrow$ 3.85mS

(1) [Q_{5a}: $I_c = 100\mu A \rightarrow \frac{1}{g_m} = 260\Omega \rightarrow$ 3.85mS

(1) [$Q_{7a,b}$: $I_c = 500\mu A \rightarrow \frac{1}{g_m} = 52\Omega \rightarrow$ 19.23mS

(1) [$Q_{9a,b}$: $I_c = 125\mu A \rightarrow \frac{1}{g_m} = 208\Omega \rightarrow$ 4.8mS

b

Part d, 10 points.

Find the following, ***using the actual value of β, i.e. β=250***

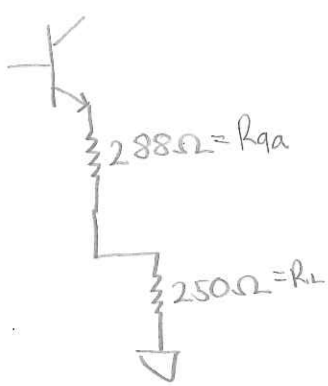
1 pt. each

	Voltage Gain	Input impedance
Transistor combination Q2a,2b,1a,1b	0.342	190kΩ
Q5a	38.46	260Ω
Q7a or 7b	1.0	46.6MΩ
Q9a or 9b	0.335	186.5kΩ
Overall differential Vout/Vin	4.4	190kΩ

☆ We have a push-pull stage. Gain will vary over signal swing if one transistor or the other or both are on.

☆ We will assume either Q9A or Q9b is on, but not both.

Q9a:



$1/g_{m9a} = 208\Omega$

☆ Emitter-follower + Voltage Divider.

Voltage Divider gain:

$$\frac{250}{250+288} = 0.46$$

Emitter Follower gain:

$$R_{L,eq} = 250 + 288 = 538\Omega$$

$$A_v = \frac{538}{538+208} = 0.72$$

overall: $(0.46)(0.72) = \boxed{0.335}$

$R_{in9a} = \beta[208\Omega + 288\Omega + 250\Omega] = \boxed{186.5k\Omega}$

b

$\underline{Q_{7b}}$: $R_{Leq7b} = R_{in9b} = 186.5\,k\Omega$ (neglect $R_{in,c,8b}$)

$1/g_{m7b} = 52\,\Omega$

$A_V = \frac{R_{Leq}}{R_{Leq} + 1/g_m} \cong \boxed{1.0}$

$R_{in7b} = \beta[186\,k\Omega] = \boxed{46.6\,M\Omega}$

$\underline{Q_{5a}}$: $R_{Leq5a} = 10\,k\Omega \| R_{in7a}$ (neglect $R_{in,c,4a}$)

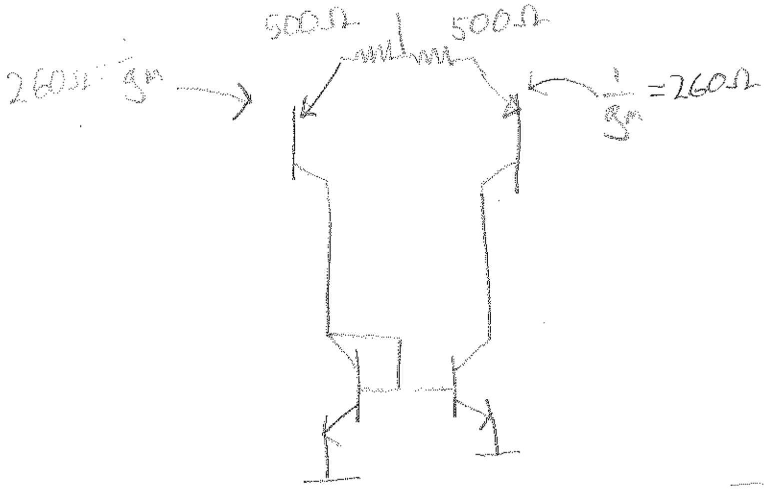
$\cong 10\,k\Omega$

* Impedance presented to base of Q_{5a} is that of diodes Q_{5b} & Q_{6b}, each $13\,\Omega$, for a total of $26\,\Omega = R_{base5a}$

$R_{in5a} = \frac{1}{g_{m5a}} + \frac{R_{base5a}}{\beta} \cong \boxed{260\,\Omega}$

$A_{V5a} = \frac{10\,k\Omega}{260\,\Omega} = \boxed{38.46}$

b



overall transconductance = $\frac{1}{1/g_{m1a} + R_{1a}} = \frac{1}{500\Omega + 260\Omega} = (760\Omega)^{-1}$

gain = $\frac{260\Omega}{760\Omega} = \boxed{0.342}$

Input impedance per side is $\beta(760\Omega + 260\Omega) = \boxed{190k\Omega}$

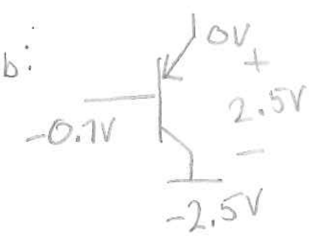
Part e, 10 points

Maximum peak-peak output voltage (***show all your work***)

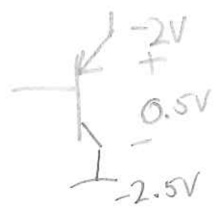
		magnitude and sign of maximum output signal swing due to *cutoff*	magnitude and sign of maximum output signal swing due to ***saturation***
1	Transistor Q4a	+ 0.66V	Not Relevent
2	Transistor Q5a	−0.43V	+0.33V
2	Transistor Q7a	+30V	−0.89V
1	Transistor Q7b	−30V	+0.89V
2	Transistor Q9a	Not Relevant	+0.92V
2	Transistor Q9b	Not Relevant	−0.92V

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant. Q7ab and Qab form a push pull stage, so be careful about your answers here.

(1) Qab:



saturated:



ΔVemitter = −2V ΔVout = −2V(0.46) = −0.92V

↑ voltage divider gain

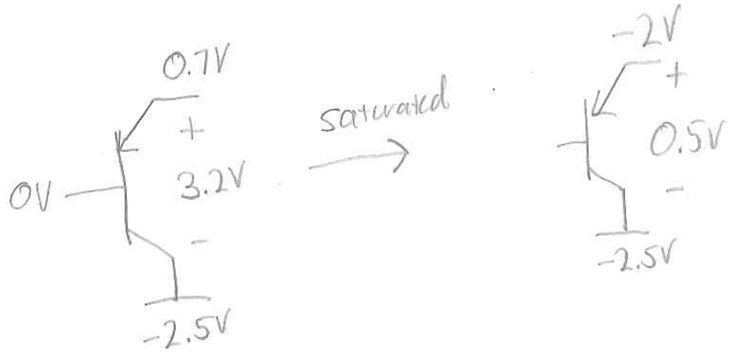
(1) Cutoff: Not relevant because Qaa takes over push-pull

(2) Qaa: Analysis exactly the same but signs are reversed.

b

1

Q7a:



$\Delta V_{emitter} = -2.7V$

$\Delta V_{out} = (-2.7V)(Av_{9a}) = (-2.7V)(0.335) = \boxed{-0.905V}$

1

Cutoff: $R_{Leq} = R_{in9a} = 186k\Omega$

$I_{cq} = 0.5mA$

$\Delta V_{emitter} = 186k\Omega \cdot 0.5mA = +93V$

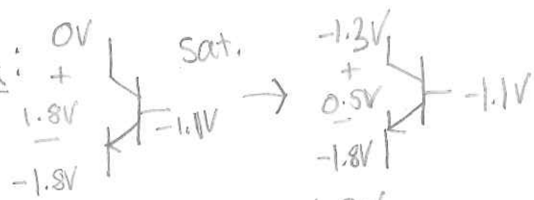
$\Delta V_{out} = 93V \cdot Av_{9a} = \boxed{+30V}$ (Irrelevent)

1

Q7b: Mirror-symmetric with Q7a, so answers the same with signs reversed.

1

Q5a:



$\Delta V_{collector} = -1.3V$

$\Delta V_{out} = -1.3V \cdot Av_7 \cdot Av_9 = \boxed{-0.43V}$

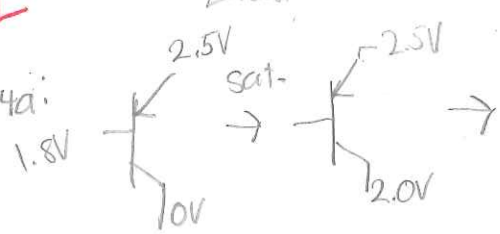
1

Cutoff: $\Delta V_{collector} = 100\mu A \cdot 10k\Omega = +1.0V$

$\Delta V_{out} = +1.0V \cdot Av_7 \cdot Av_9 = \boxed{0.33V}$

1

Q4a:



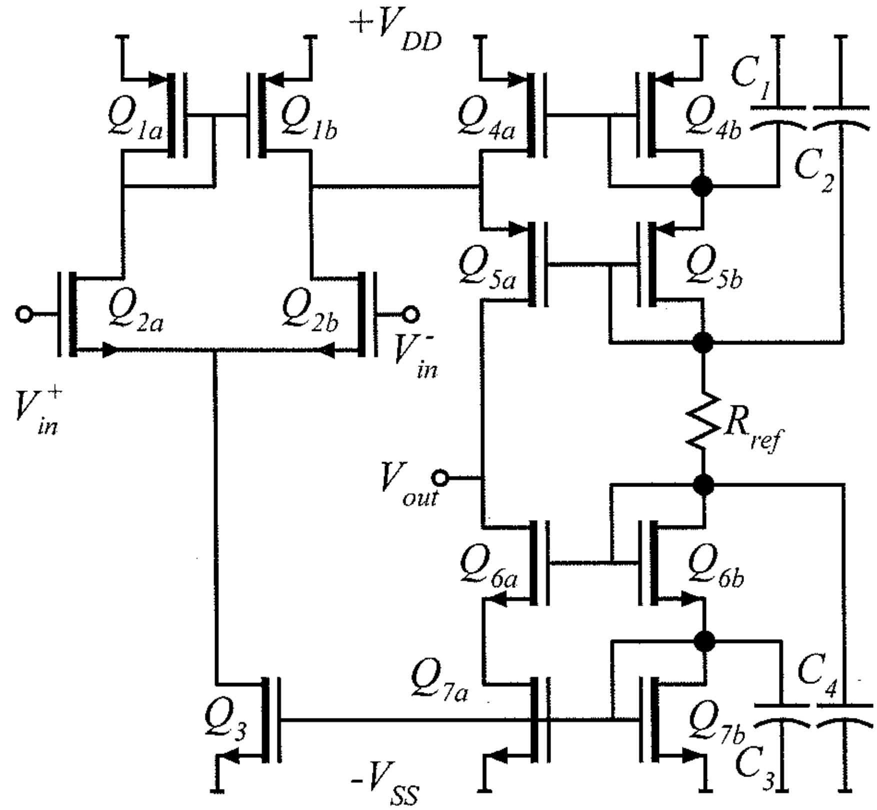
$\Delta V_{collector} = 2.0V$

$\Delta V_{out} = 2V \cdot Av_7 \cdot Av_9 = \boxed{+0.66V}$

b

Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.



The NMOSFETs and the PMOSFETS have a 0.20 V threshold, a 22nm gate length, 300 cm^2/Vs mobility, a 10^7cm/s saturation drift velocity, and $1/\lambda$=3 Volts. The gate oxide thickness is 1.0nm and the dielectric constant is 3.8. This gives

$\mu c_{ox} W_g / 2L_g = 15\ \text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$ and

$v_{sat} c_{ox} W_g = 3.36\ \text{mA/V} \cdot (W_g / 1\mu\text{m})$ (both are a bit unrealistic for a real technology).

and $v_{sat} L_g / \mu = 0.113$ V

V_{DD} =+0.75 V, $-V_{SS}$ = -0.75 V,

Part a, 10 points

DC bias.

Approximation: ignore the term $(1+\lambda V_{DS})$ ***in DC bias analysis.***

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.

Q1ab,2ab are to be biased at 25 μA drain current.

Q4ab,5ab,6ab,7ab are to be biased at 100 μA drain current

All transistors are to operate with $|V_{gs}|$=0.25V.

Find the gate widths of all transistors.

Find:

Wg1a=______ Wg1b =______ Wg2a=______ Wg2b =______ Wg3=______

Wg4a=______ Wg4b =______ Wg5a=______ Wg5b =______

Wg6a=______ Wg6b =______ Wg7a=______ Wg7b =______

Rref=______

(2) [$V_{gs} = 0.25V$ so $V_{gs}-V_{th} = 50mV < \frac{V_{sat} L_g}{\mu}$ so mobility-limited

(2) [$I_D = [15mA/V^2]\left[\frac{W_g}{1\mu m}\right]\cdot(V_{gs}-V_{th})^2$

(2) [$I_D = 37.5\mu A\left(\frac{W_g}{1\mu m}\right) \rightarrow W_g = 1\mu m \cdot \frac{I_D}{37.5\mu A}$

(1.33) [Q1a, 2a, 1b, 2b: $I_D = 25\mu A \rightarrow W_g = 0.666\mu m$

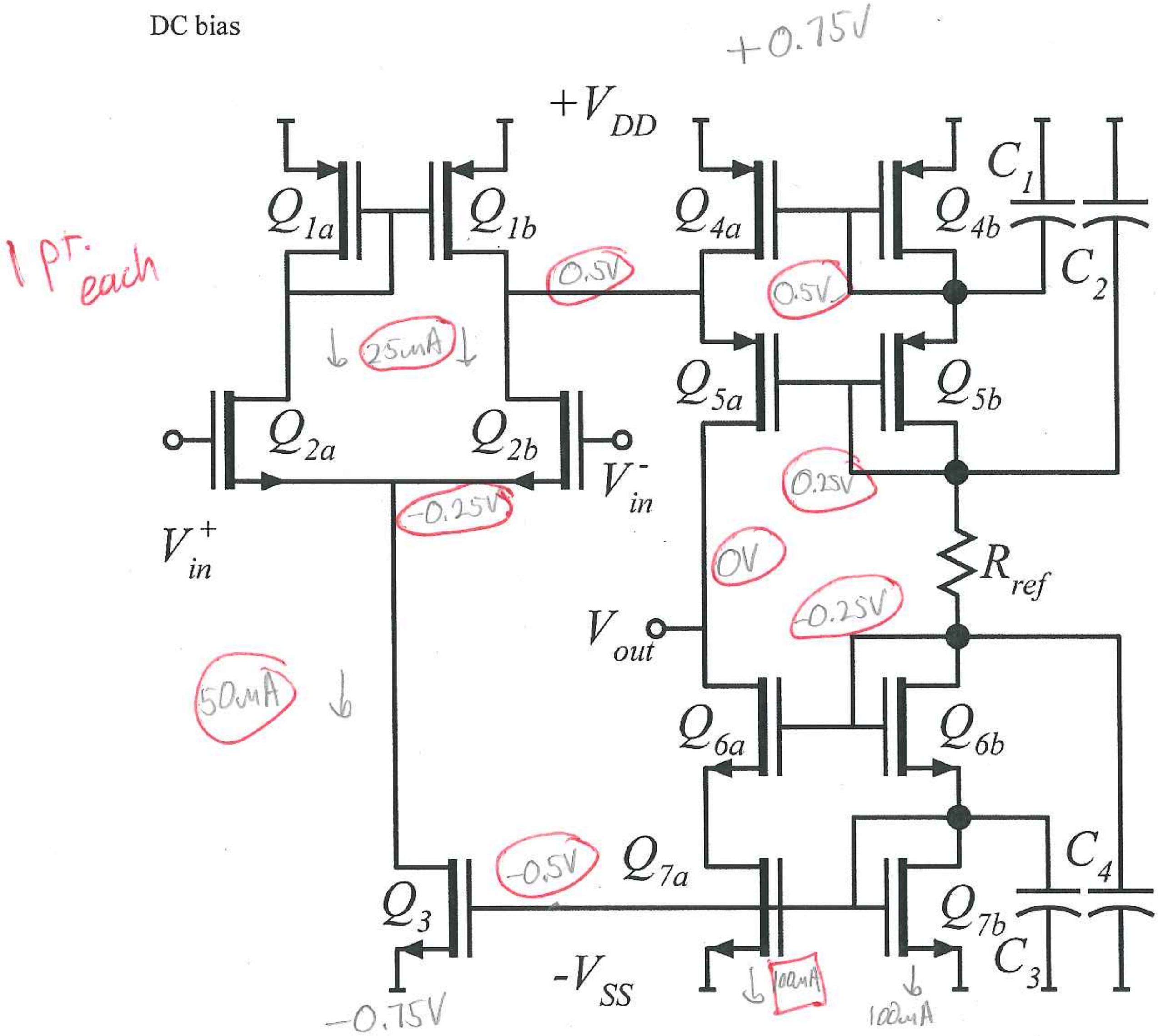
(1.33) [Q3: $I_d = 50\mu A \rightarrow W_g = 1.33\mu m$

(1.33) [Q4ab, 5ab, 6ab, 7ab: $I_D = 100\mu A \rightarrow W_g = 2.66\mu m$

b

Part b, 10 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes** the drain currents of **ALL transistors,** and the gate widths of **ALL transistors**

Part c, 15 points.

You will now compute the op-amp differential gain. ***You must consider the*** $(1+\lambda V_{DS})$ ***term in the FET IV characteristics when you do this.***

The capacitors C1-C4 are all zero Ohms AC impedance. (They would not be present in a real design; they are added here to simplify the exam).

Find the following

	Voltage Gain	Input impedance
Transistor combination Q2a,2b,1a,1b	21.3	∞
Q5a	129	33kΩ
Overall differential Vout/Vin	2746	∞

(Alternative----if you very skilled, you might be able to compute the combined gain of Q2a,2b,1a,1b and Q5a, all together, in a single step using Norton or Thevenin methods. If you do so, first, don't ask for hints on how to do this and, second, do please also calculate the input impedance of Q5a.)

(1) Since $I_D = \frac{15mA}{V^2}(V_{gs}-V_{th})^2 \cdot \frac{W_g}{1\mu m}$, so $g_m = \frac{\delta I_D}{\delta V_{gs}} = \frac{30mA}{V^2}(V_{gs}-V_{th})\left(\frac{W_g}{1\mu m}\right)$

$g_m = 1.5mS \cdot \frac{W_g}{1\mu m}$

(1) Q1ab,2ab: $g_m = 1.0mS$

Q3: $g_m = 2.0mS$

Q4ab,5ab,6ab,7ab: $g_m = 4.0mS$

1pt

$R_{DS} = \frac{3.25V}{100\mu A} = 32.5k\Omega$

← $R_{Leq} = R_{out,6drain}$

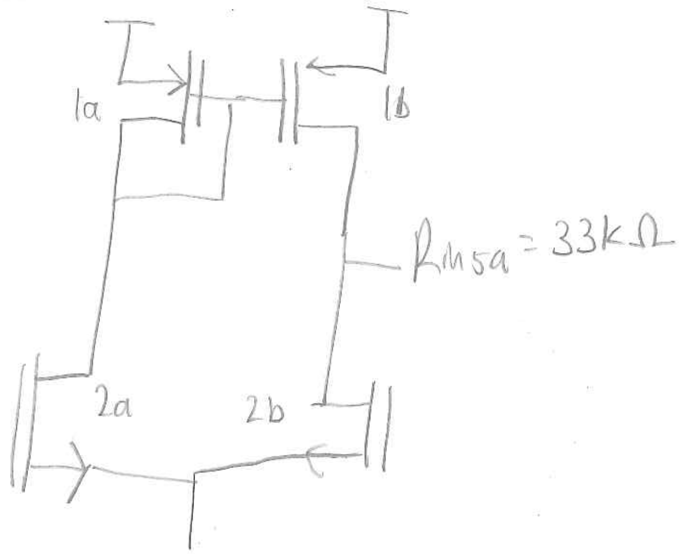
(1) Q5a: common gate: $R_{out,drain6a} = R_{DS_{6a}}(1 + g_{m_{6a}} R_{DS7a}) = 4.26M\Omega = R_{Leq5a}$

2pts

(1) $R_{in5a} = \frac{1}{g_{m5a}}\left(\frac{R_{Leq}+R_{DS}}{R_{DS}}\right) = 33k\Omega$

(1) $A_{v5a} = \frac{R_{Leq}}{R_{in5a}} = 129$

b



$Q_{1ab, 2ab}$

(1) [all gms = 1.0mS

(1) [all Ros ≈ $\frac{3V}{25uA}$ = 120kΩ

(2) [Stage transconductance = gm1ab = 1mS

(1) [$R_{L eq} = R_{in5a} \| R_{os1b} \| R_{os2b}$ = 33kΩ || 120kΩ || 120kΩ
= 21.3kΩ

(2) [gain = 1mS · 21.3kΩ = **21.3**

b

Part d, 10 points

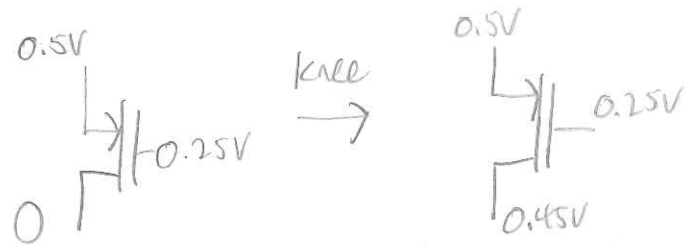
Maximum peak-peak output voltage at the positive output Vo+ (***show all your work***)

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to: ***knee voltage*** (saturation)
Transistor Q5a	− kilovolts	+0.45V
Transistor Q6a	cannot cut off	−0.45V
Transistor Q2a	+ Huge → irrelevant	cannot be reached → irrelevant
Transistor Q2b	− Huge → irrelevant	− irrelevant

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant". But, give a 1-sentence statement why below.

(2) [μ-limited, so $V_{ds,sat} = V_{gs} - V_{th} = 50mV$ for all Fets

(1) [Q5a:



$\Delta V_{out} = +0.45V$

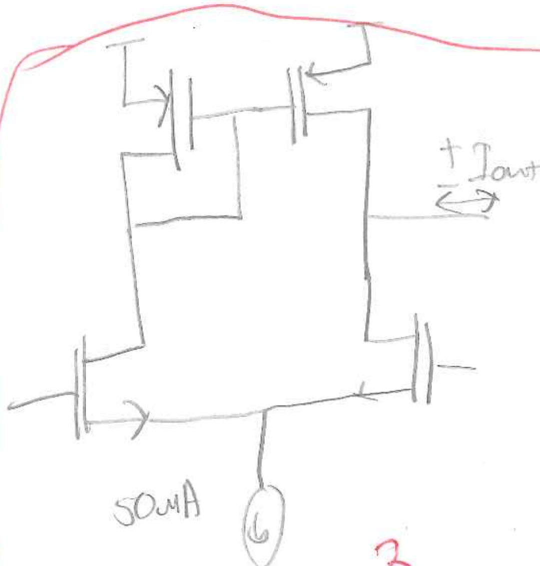
(1) [By symmetry, knee voltage of Q6a limits ΔV_{out} to $-0.45V$

(1) [Cutoff of Q5a · $\Delta I_d = 100uA$, $R_{L,eq} = 4.26M\Omega$

$\Delta V_{out} =$ kilovolts

b

Cutoff of Q2a & Q2b



As 2a and 2b are driven, one full-on and the other cut off, Iout varies by ± 50µA

This drives ± 50µA into effective load of 4.26MΩ; (± 50µA)(4.26MΩ) = ±213V

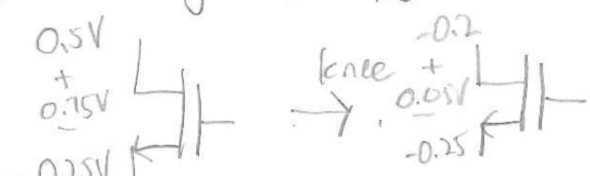
Huge → Irrelevent

3

1 Knee voltage of Q2a – cannot be reached until past Q2b cutoff

irrelevent

1 Knee voltage of Q2b



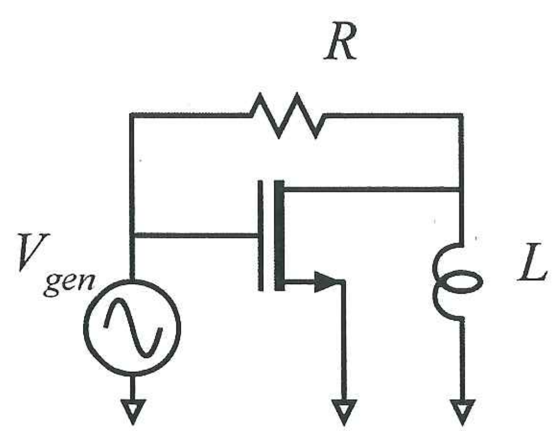
$\Delta V_{drain} = -0.7V$

$\Delta V_{out} = (-0.7V) A_{v5a} = -$ large number

irrelevent

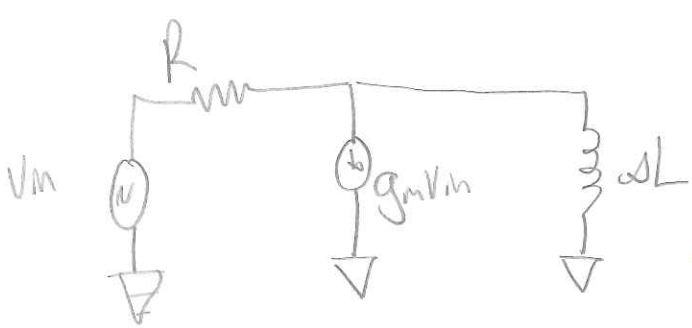
b

Problem 3, 30 points

	You will be working on the circuit to the left Ignore DC bias analysis. You don't need it. The transistor has transconductance gm. Its output resistance Rds is infinity...so you don't need to include this element in the circuit diagram !

Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.



(-3) if control voltage not shown

b

Part b, 8 points

gm=10 mS. L=1 μH . R= 1000 Ohms

Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with **correct units** and in ratio-of-polynomials form, i.e.

$$\frac{V_{out}(s)}{V_{gen}(s)} = K \cdot \frac{1+b_1 s+b_2 s^2+\ldots}{1+a_1 s+a_2 s^2+\ldots} \text{ or (as appropriate) } \frac{V_{out}(s)}{V_{gen}(s)} = K \cdot (s\tau)^n \cdot \frac{1+b_1 s+b_2 s^2+\ldots}{1+a_1 s+a_2 s^2+\ldots}$$

Note that an expression like

$$\frac{V_{out}(s)}{V_{gen}(s)} = \frac{1}{1+(3\cdot 10^{-6})s} \text{ is dimensionally wrong; } \frac{1}{1+(3\cdot 10^{-6}\text{seconds})s} \text{ is dimensionally correct}$$

Vout(s)/Vin(s)= ________________

$$\sum I @ V_{out} = 0 \Rightarrow V_{out}\left(\frac{1}{sL} + \frac{1}{R}\right) + V_{in}\left[g_m - \frac{1}{R}\right] = 0$$

4 pts for some version of this

$$\frac{V_o}{V_{in}} = -\left[g_m R - 1\right] \frac{sL/R}{1 + sL/R} \qquad \frac{L}{R} = 10^{-9}\text{sec} = 1\text{ns}$$

$$\frac{V_o}{V_{in}} = -9\left[\frac{s\cdot 1\text{ns}}{1 + s\cdot 1\text{ns}}\right]$$ 4 pts

b

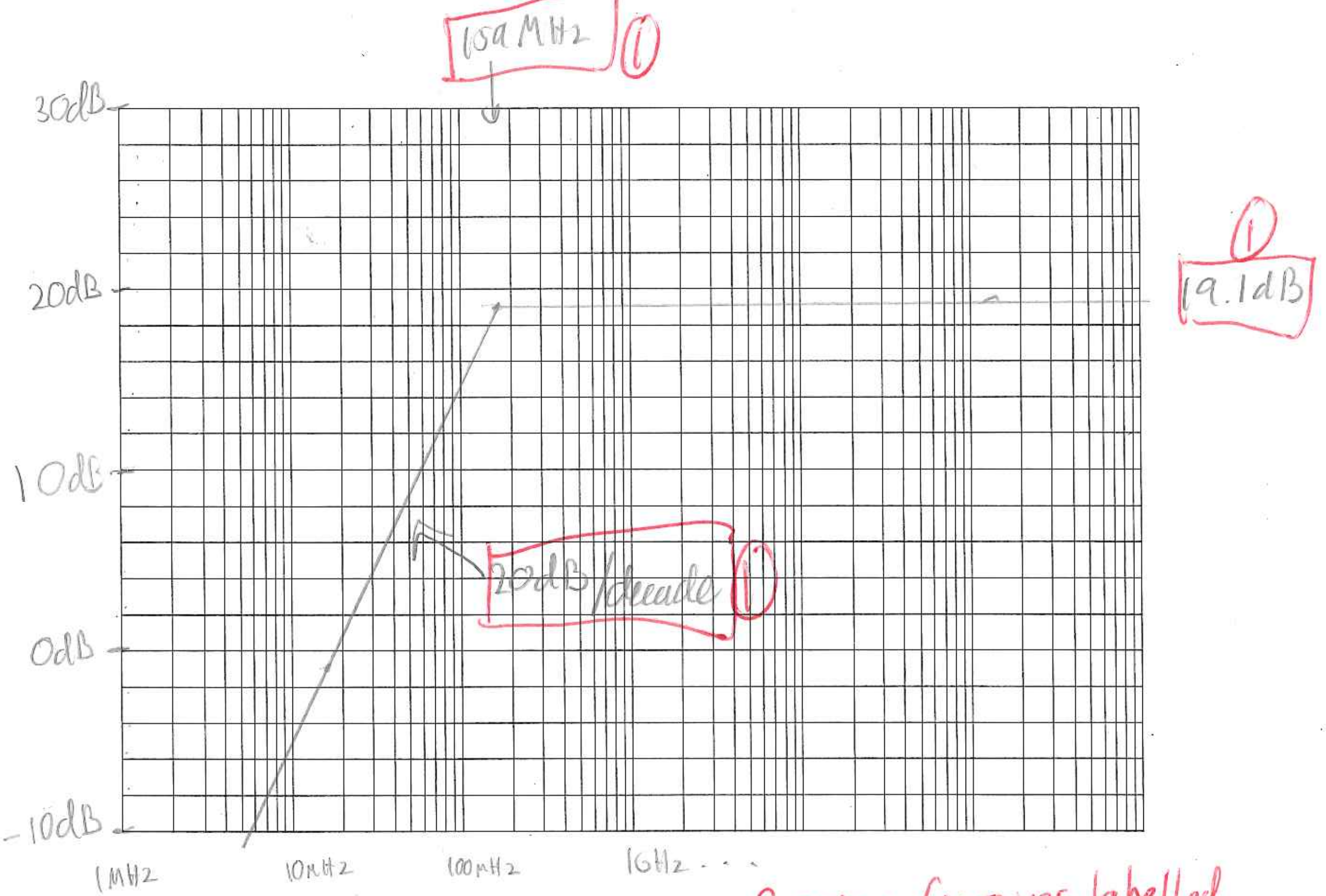
Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz:

1 0 at DC, pole at 159 MHz, , 1

Draw a clean Bode Plot of Vout/Vin,

LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes



2 points for axes labelled Sensibly

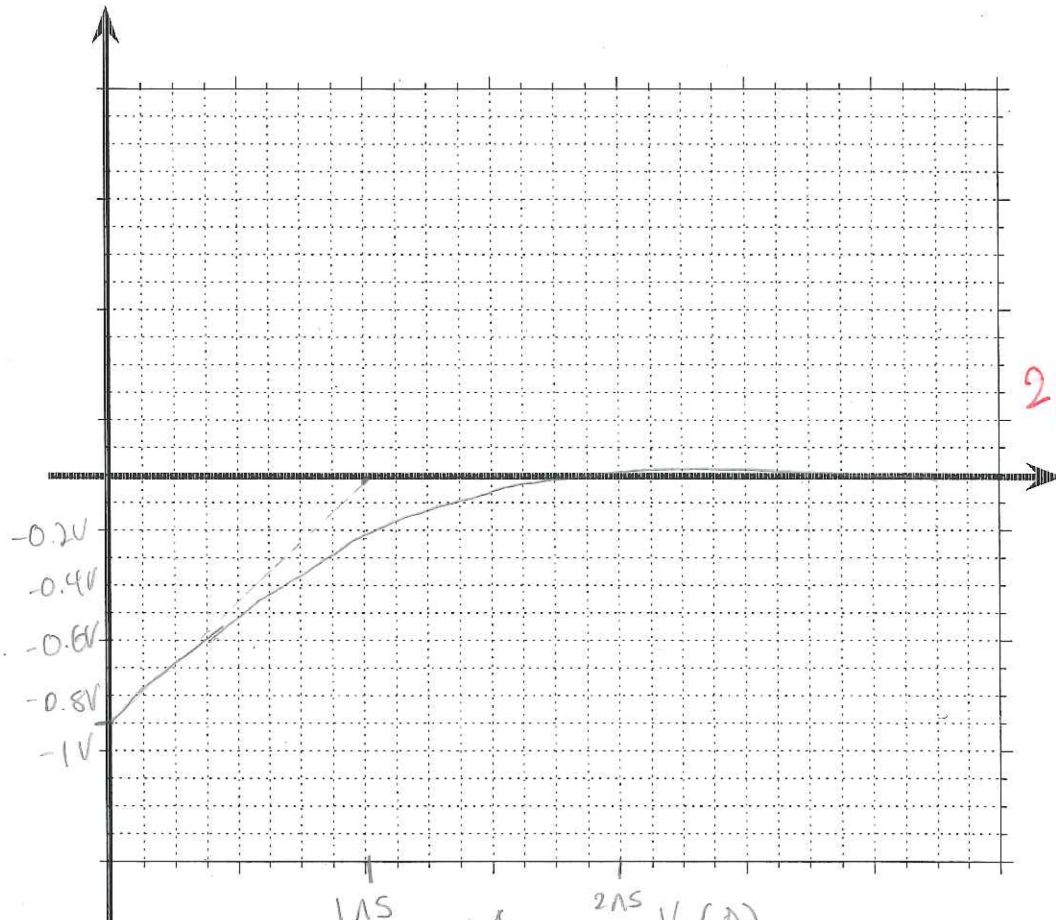
b

Part d, 8 points

Vin(t) is a 0.1 V amplitude step-function.

Find Vout(t)=______________________________

Plot it below. Label axes, show initial and final values, show time constants.



2 pts for reasonable graph

$$H(s) = -9 \cdot \frac{s \cdot 1ns}{1 + s1ns} = \frac{V_o(s)}{V_i(s)}$$

② $\left(V_{in}(s) = \frac{0.1V}{s}\right)$ so $\left(V_{out}(s) = -0.9V \cdot \frac{1ns}{1 + s1ns}\right)$ 2

$$V_{out}(t) = -0.9V \cdot u(t) \cdot e^{-t/1ns}$$

b