Final Exam, ECE 137A

Wednesday March 18, 2015 7:30-10:30 PM

Name: Solution a

Closed Book Exam: Class Crib-Sheet and 3 pages (6 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), ***after stating & justifying them.***
Show your work:
Full credit will not be given for correct answers if supporting work is missing.

Good luck

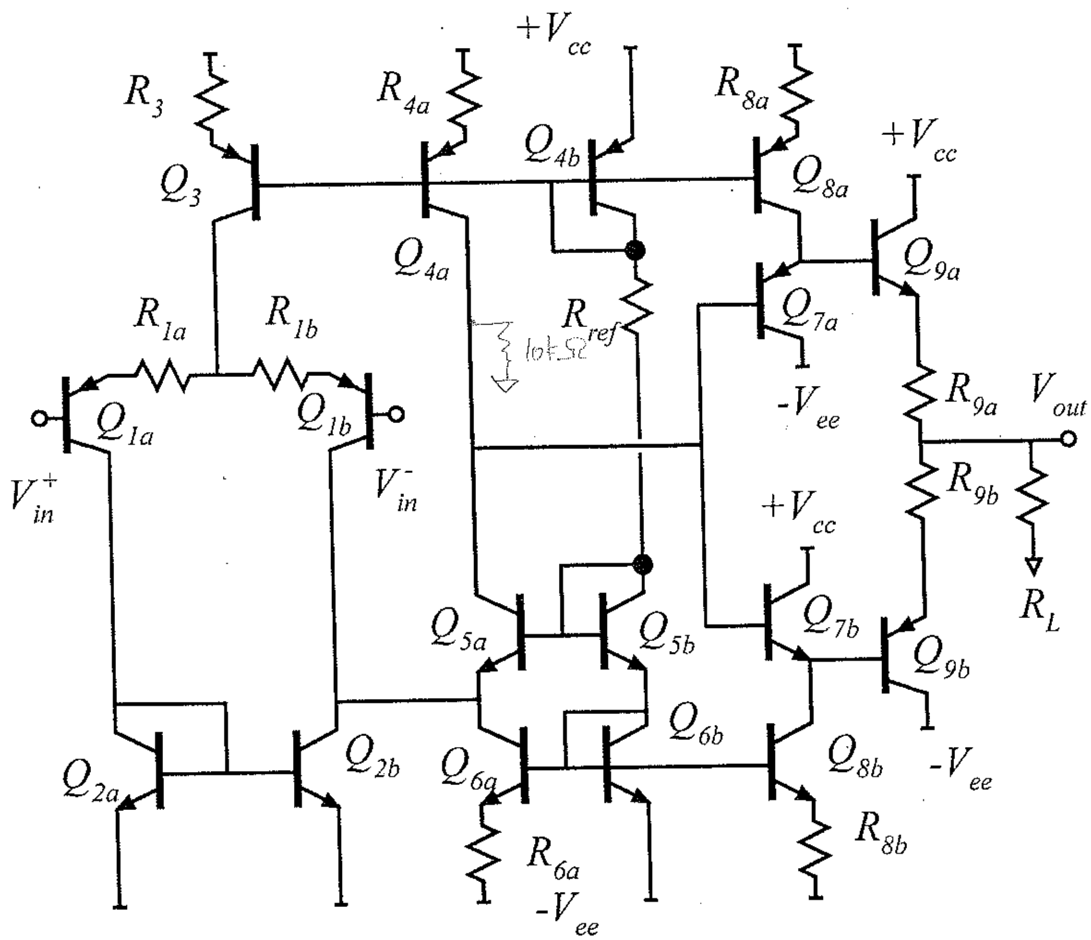
Time function	LaPlace Transform
$\delta(t)$ impulse	1
$U(t)$ unit step-function	$1/s$
$e^{-\alpha t}U(t)$	$\frac{1}{s+\alpha}$
$e^{-\alpha t}\cos(\omega_d t)U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		5	2c		15
1b		6	2d		10
1c		4	3a		7
1d		10	3b		8
1e		10	3c		7
2a		10	3d		8
2b		10			
total		100			

a

Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts

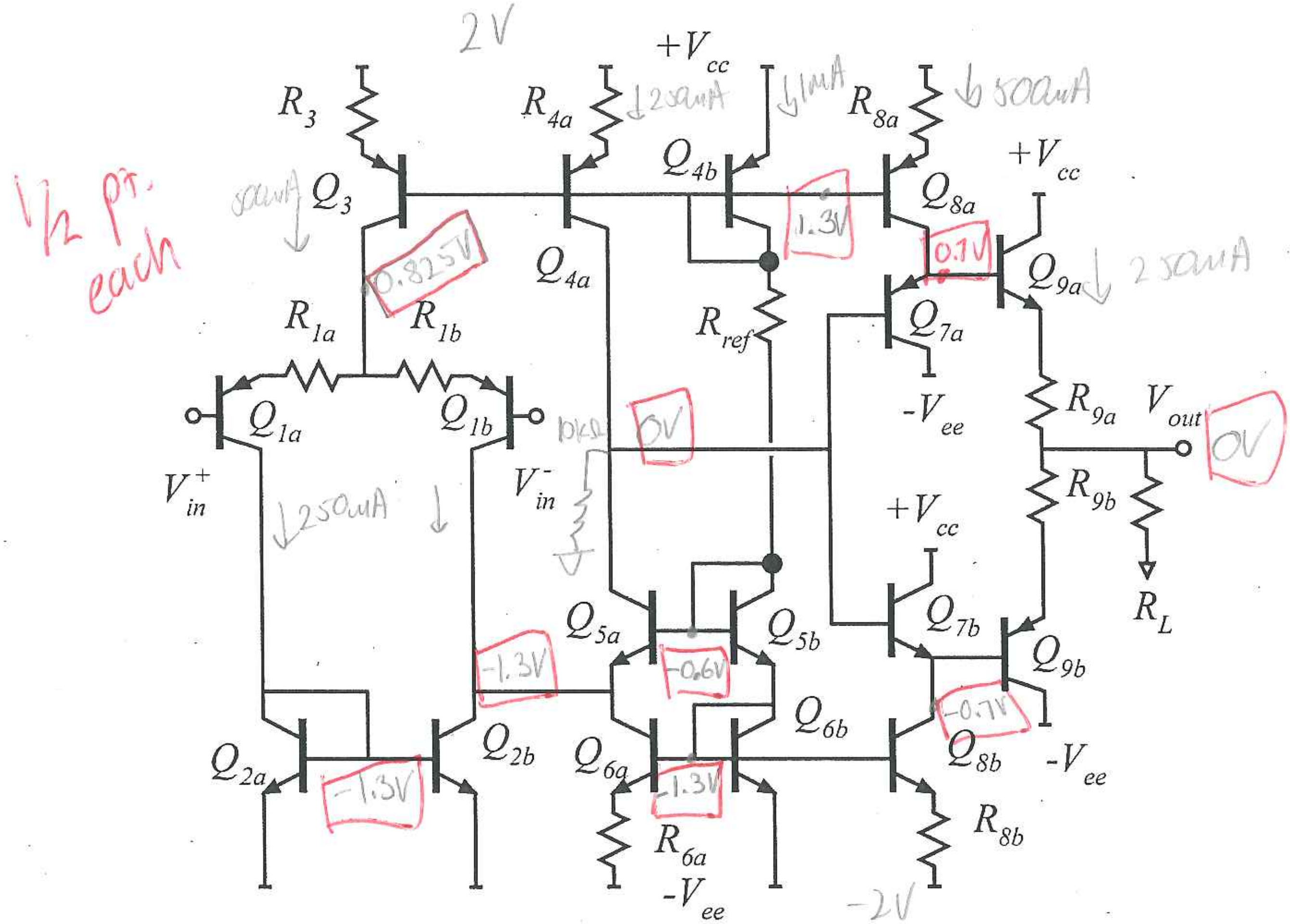


All the transistors have the same (matched) I_S, have $\beta = 100$, and $V_A = \infty$ Volts. $V_{CE(sat)} = 0.5\text{V}$. V_{be} is roughly 0.7 V, but use $V_{be} = (kT/q)\ln(I_E/I_S)$ when necessary and appropriate. The supplies are +2 Volts and -2 Volts.

Q1ab,2ab,4a,5a,6a are to be biased at 250 μA collector current.
Q4B,5B,6B are to be biased at 1 mA collector current.
Q8ab are to be biased at 500 μA collector current.
Q9ab are to be biased at 250 μA collector current.
$R_L = 250\Omega$ R1a=R1b=500 Ohms

Part a, 5 points

DC bias---to simplify ,assume $\beta = \infty$ ***for the DC analysis only.***



On the circuit diagram above, label the DC voltages at **ALL nodes**, the DC currents through **ALL resistors,** and the DC collector currents of **all transistors.**

① $\left[V_{be6b} - V_{be6a} = \frac{kT}{q}\ln\left[\frac{1mA}{250\mu A}\right] = 36mV \rightarrow R_{6a} = \frac{36mV}{250\mu A} = \boxed{144\Omega}\right.$

① $\left[V_{be6b} - V_{be8b} = \frac{kT}{q}\ln\left[\frac{1mA}{500\mu A}\right] = 18mV \rightarrow R_{8b} = \frac{18mV}{500\mu A} = \boxed{36\Omega}\right.$

① $\left[V_{be7b} - V_{be9b} = \frac{kT}{q}\ln\left[\frac{500\mu A}{250\mu A}\right] = 18mV \rightarrow R_{9b} = \frac{18mV}{250\mu A} = \boxed{72\Omega}\right.$

by same arguments, $R_{8a} = \boxed{36\Omega}$, $R_{9a} = \boxed{72\Omega}$

a

(i) [$V_{be4b} - V_{be4a} = \frac{kT}{q} \ln\left[\frac{1mA}{250\mu A}\right] = 36mV \rightarrow R_{4a} = \frac{36mV}{250\mu A} = \boxed{144\Omega}$

(i) [$V_{be4b} - V_{be3} = \frac{kT}{q} \ln\left[\frac{1mA}{500\mu A}\right] = 18mV \rightarrow R_3 = \frac{18mV}{500\mu A} = \boxed{36\Omega}$

(i) [$R_{ref} = \frac{1.3V - (-0.6V)}{1mA} = \boxed{1900\Omega}$

a

Part b, 6 points

DC bias:

Find the value of all resistors.

R3=36Ω R4ba=144Ω R6a=144Ω Rref=1.9kΩ R8a=36Ω R8b=36Ω
R9a=72Ω R9b=72Ω

a

Part c, 4 points

Find the transconductance of the transistors below:
gm1a=9.61mS gm1b=9.61mS gm5a=9.61mS gm7a=19.2mS
gm7b=19.2mS gm9a=9.61mS gm9b=9.61mS

(1) [$Q_{1a,b}$: $I_C = 250\mu A$, $\frac{1}{g_m} = 104\Omega \rightarrow$ 9.61mS

(1) [Q_{5a}: $I_C = 250\mu A$, $\frac{1}{g_m} = 104\Omega \rightarrow$ 9.61mS

(1) [$Q_{7a,b}$: $I_C = 500\mu A$, $\frac{1}{g_m} = 52\Omega \rightarrow$ 19.23mS

(1) [$Q_{9a,b}$: $I_C = 250\mu A$, $\frac{1}{g_m} = 104\Omega \rightarrow$ 9.61mS

a

Part d, 10 points.

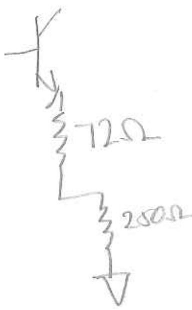
Find the following, ***using the actual value of*** β***, i.e.*** β***=100***

1 pt. each

	Voltage Gain	Input impedance
Transistor combination Q2a,2b,1a,1b	0.172	70.8kΩ
Q5a	95.67	104.52Ω
Q7a or 7b	1.0	4.26MΩ
Q9a or 9b	0.587	42.6kΩ
Overall differential Vout/Vin	9.66	70.8kΩ

* Push-pull → analyze with one off and one on.

Q9a:



Voltage Divider gain:

$$\frac{250}{250+72} = 0.776$$

Emitter-Follower gain:

$R_{L,eq} = 250\Omega + 72\Omega = 322\Omega$

$$A_v = \frac{322\Omega}{322\Omega + 109\Omega} = 0.756$$

overall: $(0.776)(0.756) = \boxed{0.587}$

$R_{in9a} = \beta(109\Omega + 72\Omega + 250\Omega) = \boxed{42.6k\Omega}$

a

Q_{7b}: $R_{Leq7b} = R_{in8b} = 42.6k\Omega$ (neglect $R_{4,6,8b}$)

$\frac{1}{g_{m7b}} = 52\Omega$

$A_V = \frac{R_{Leq}}{R_{Leq} + 1/g_m} \cong$ **1.0**

$R_{in7b} = \beta[42.6k\Omega] =$ **4.26MΩ**

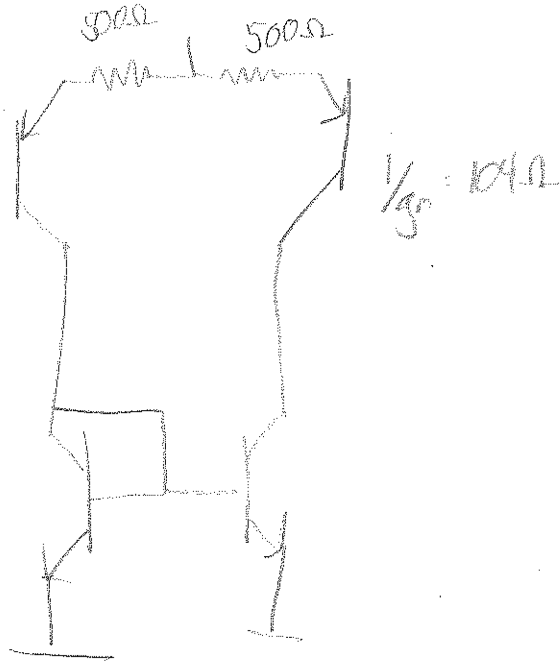
Q_{5a}: $R_{Leq5a} = 10k\Omega \| R_{in7a} \cong 10k\Omega$

Impedance presented to base of Q_{5a} is that of diodes Q_{6b} & Q_{6b}, each 26Ω for a total of 52Ω = R_{base}

$R_{in5a} = \frac{1}{g_{m5a}} + \frac{R_{base\,5a}}{\beta} = 104\Omega + \frac{52\Omega}{100} =$ **104.52Ω**

$A_{V5a} = \frac{10k\Omega}{104.52\Omega} =$ **95.6755**

a



overall transconductance = $\frac{1}{\frac{1}{g_{m1a}} + R_{1a}} = (604\Omega)^{-1}$

gain = $\frac{104\Omega}{604\Omega}$ = **0.172**

input impedance per side is $\beta[604\Omega + 104] =$ **70.8kΩ**

a

Part e, 10 points

Maximum peak-peak output voltage (***show all your work***)

	magnitude and sign of maximum output signal swing due to *cutoff*	magnitude and sign of maximum output signal swing due to ***saturation***
Transistor Q4a	+0.88V	Not Relevant
Transistor Q5a	−0.47V	+1.47V
Transistor Q7a	+12.5V	−1.29V
Transistor Q7b	−12.5V	+1.29V
Transistor Q9a	Not Relevant	+1.16V
Transistor Q9b	Not Relevant	−1.16V

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant. Q7ab and Q9ab form a push pull stage, so be careful about your answers here.

(1) Qab:

Voltage divide ↓

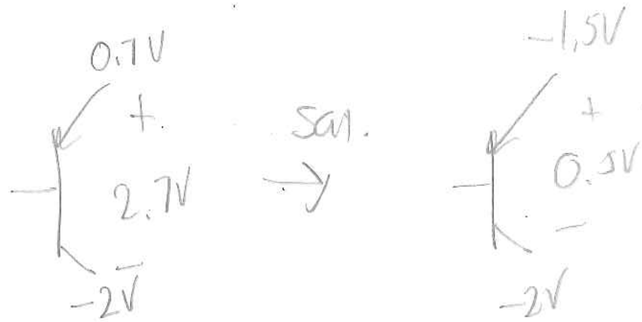
$\Delta V_{emitter} = -1.5V$ $\quad \Delta V_{out} = (-1.5V)(0.776) = \boxed{-1.16V}$

(1) [Cutoff: not relevant because Qaa takes over push-pull

(2) [Qaa: analysis exactly same but signs are reversed.

a

(1) Q_{7a}:



$\Delta V_{emitter} = -2.2V$

$\Delta V_{out} = (-2.2V)(A_{v9a}) = (-2.2V)(0.587) = \boxed{-1.29V}$

(1) Cutoff: $R_{L7}eq = R_{in9a} = 42.6k\Omega$

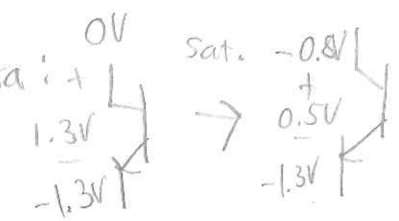
$I_{C,q} = 0.5mA$

$\Delta V_{emitter} = +21.3V$

$\Delta V_{out} = 21.3V \times A_{v9a} = \boxed{12.5V}$ (irrelevant)

(1) Q_{7b}: Mirror-symmetric w/ Q_{7a}, so answers the same w/ sign reversed

(1) Q_{5a}:



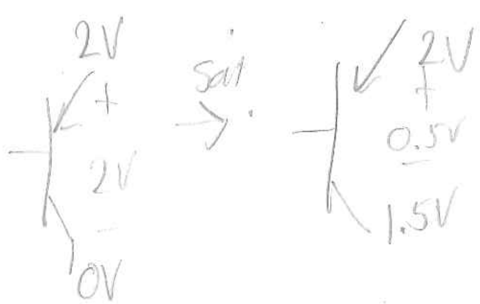
$\Delta V_{collector} = -0.8V$

$\Delta V_{out} = -0.8V \cdot A_{v7} \cdot A_{v9} = \boxed{-0.47V}$

(1) Cutoff: $\Delta V_{collector} = 250\mu A \cdot 10k\Omega = +2.5V$

$\Delta V_{out} = +2.5V \cdot A_{v7} \cdot A_{v9} = \boxed{+1.47V}$

(1) Q_{4a}:



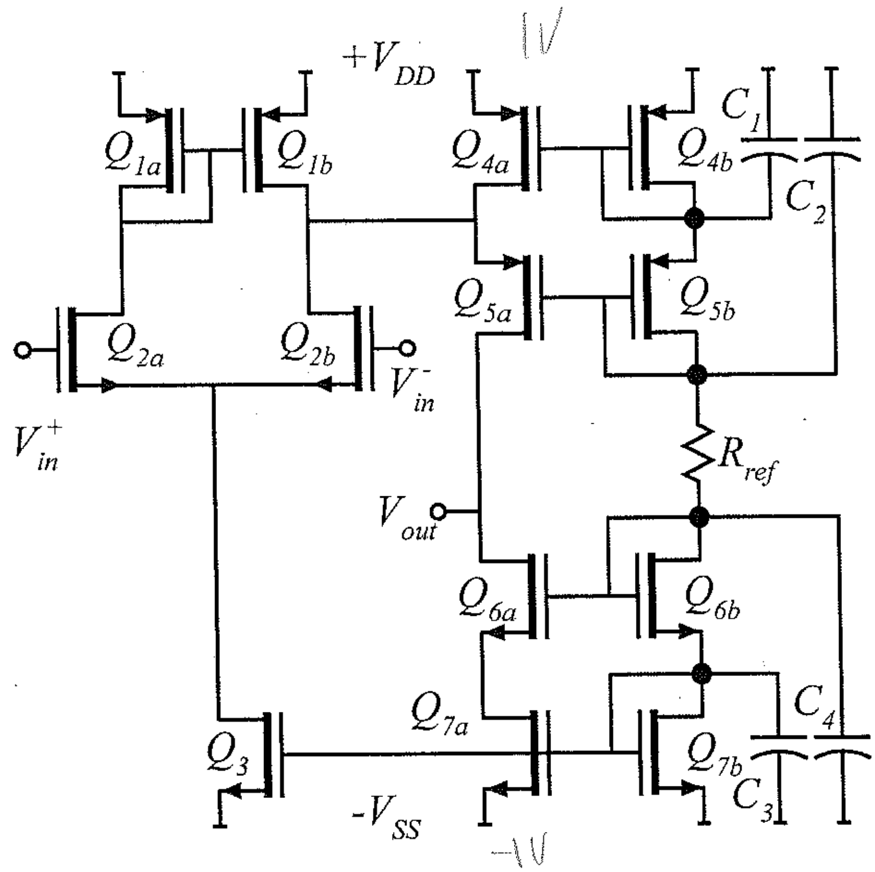
$\Delta V_{collector} = +1.5V$

$\Delta V_{out} = 1.5V \cdot A_{v7} \times A_{v9} = \boxed{+0.88V}$

a

Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.



The NMOSFETs and the PMOSFETS have a 0.20 V threshold, a 22nm gate length, 300 cm^2/Vs mobility, a 10^7cm/s saturation drift velocity, and $1/\lambda$=3 Volts. The gate oxide thickness is 1.0nm and the dielectric constant is 3.8. This gives

$\mu c_{ox} W_g / 2L_g = 15\ \mathrm{mA/V^2} \cdot (W_g / 1\mu m)$ and

$v_{sat} c_{ox} W_g = 3.36\ \mathrm{mA/V} \cdot (W_g / 1\mu m)$ (both are a bit unrealistic for a real technology).

and $v_{sat} L_g / \mu = 0.113$ V

V_{DD} =+1 V, $-V_{SS}$ = -1 V,

a

Part a, 10 points

DC bias.

Approximation: ignore the term $(1+\lambda V_{DS})$ in DC bias analysis.

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.

Q1ab,2ab are to be biased at 50 μA drain current.

Q4ab,5ab,6ab,7ab are to be biased at 200 μA drain current

All transistors are to operate with $|V_{gs}|$=0.30V.

Find the gate widths of all transistors.

Find:

Wg1a=______ Wg1b =______ Wg2a=______ Wg2b =______ Wg3=______

Wg4a=______ Wg4b =______ Wg5a=______ Wg5b =______

Wg6a=______ Wg6b =______ Wg7a=______ Wg7b =______

Rref=______

(2) $V_{gs} = 0.3V$ so $V_{gs} - V_{th} = 0.1V < \frac{v_{sat} L_g}{\mu}$ so mobility-limited

(2) $I_d = \left[\frac{15mA}{V^2}\right]\left[\frac{W_g}{1\mu m}\right](V_{gs}-V_{th})^2$

(2) $I_d = 150\mu A\left(\frac{W_g}{1\mu m}\right) \rightarrow W_g = 1\mu m \cdot \frac{I_D}{150\mu A}$

(1.33) Q1a, 2a, 1b, 2b: $I_D = 50\mu A \rightarrow$ $W_g = 0.333\mu m$

(1.33) Q3 : $I_D = 100\mu A \rightarrow$ $W_g = 0.666\mu m$

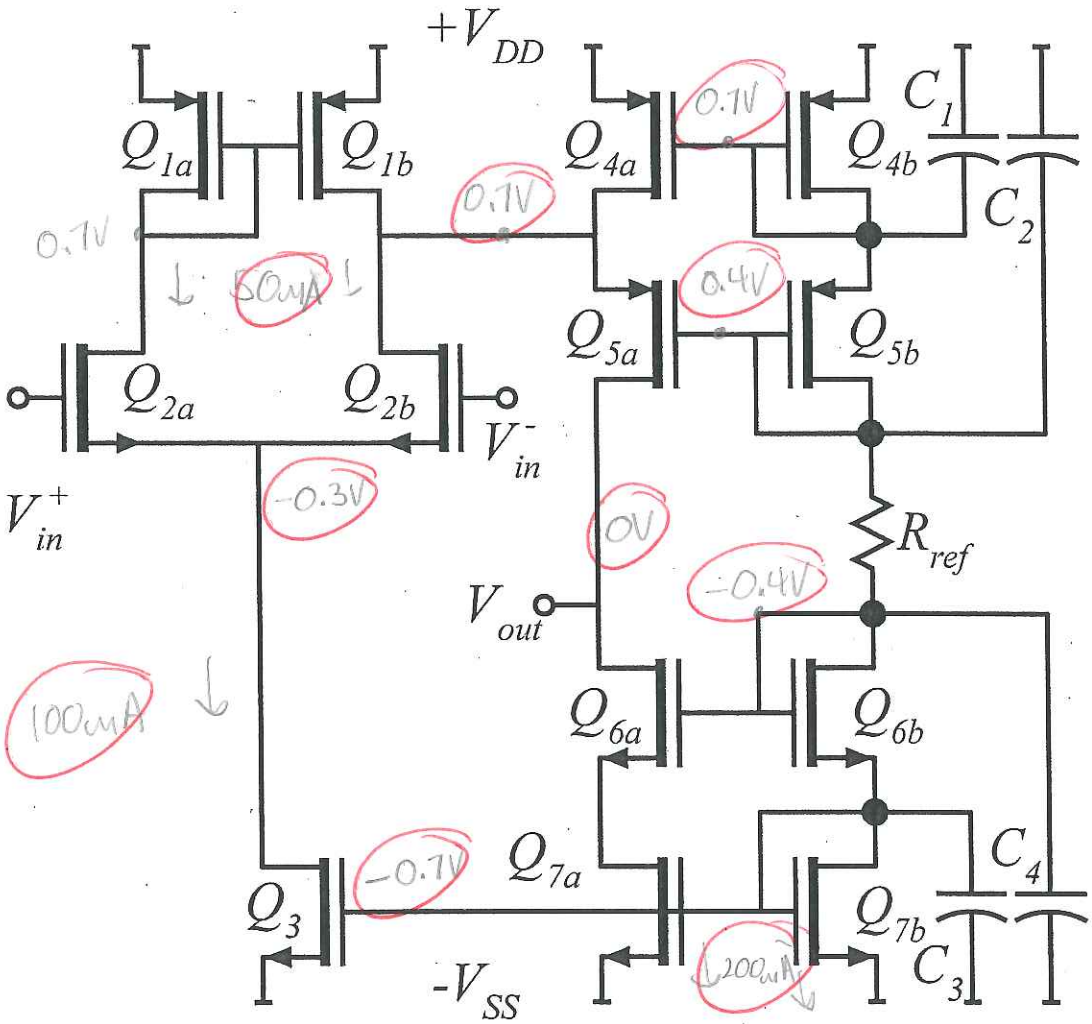
(1.33) Q4ab, 5ab, 6ab, 7ab: $I_d = 200\mu A \rightarrow$ $W_g = 1.33\mu m$

a

Part b, 10 points

DC bias

1 pt each



On the circuit diagram above, label the DC voltages at **ALL nodes** the drain currents of **ALL transistors,** and the gate widths of **ALL transistors**

a

Part c, 15 points.

You will now compute the op-amp differential gain. ***You must consider the*** $(1+\lambda V_{DS})$ ***term in the FET IV characteristics when you do this.***

The capacitors C1-C4 are all zero Ohms AC impedance. (They would not be present in a real design; they are added here to simplify the exam).

Find the following

	Voltage Gain	Input impedance
Transistor combination Q2a,2b,1a,1b	10.22	∞
Q5a	59	15.5kΩ
Overall differential Vout/Vin	603	∞

(Alternative----if you very skilled, you might be able to compute the combined gain of Q2a,2b,1a,1b and Q5a, all together, in a single step using Norton or Thevenin methods. If you do so, first, don't ask for hints on how to do this and, second, do please also calculate the input impedance of Q5a.)

(1) Since $I_D = \frac{15mA}{V^2}(V_{gs}-V_{th})^2 \frac{W_g}{1um}$, $g_m = \frac{30mA}{V^2}(V_{gs}-V_{th})\left(\frac{W_g}{1um}\right)$

$g_m = 3mS \cdot \frac{W_g}{1um}$

(1) Q1ab,2ab : $g_m = 1mS$

Q3 : $g_m = 2mS$

Q4ab, 5ab,6ab,7ab : $g_m = 4mS$

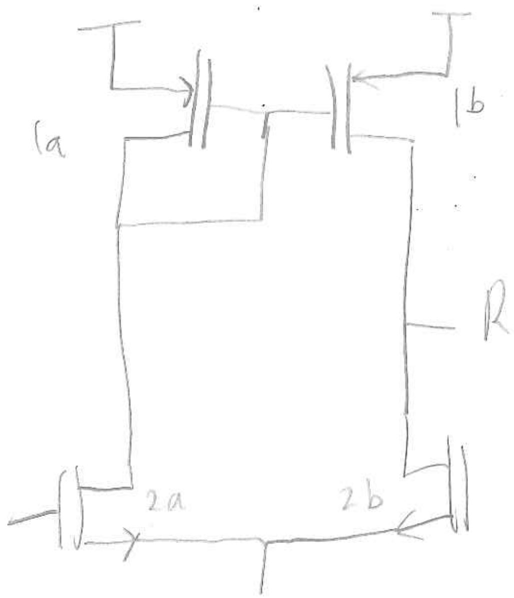
(1) $R_{DS} = \frac{1}{\lambda I_D}$, so $R_{DS5a,6a,7a} = \frac{3V}{200uA} = 15k\Omega$

(2) $R_{out,drain6a} = R_{DS6a}(1+g_{m6a}R_{DS7a}) = 915k\Omega$

(1) $R_{Leq} = R_{out,drain6a} = \boxed{915k\Omega}$

(1) $R_{in5a} = \frac{1}{g_{m5a}}\left(\frac{R_{Leq}+R_{DS}}{R_{DS}}\right) = \boxed{15.5k\Omega}$

(1) $A_{v5a} = \frac{R_{Leq}}{R_{in5a}} = \boxed{59}$



Q1ab , 2ab

① [all g_ms = 1mS

① [all $R_{DS} \approx \frac{3V}{50\mu A} = 60k\Omega$

② [Stage transconductance = g_{m1ab} = 1mS

① [$R_{L,eq} = R_{M5a} || R_{DS1b} || R_{DS1a} = 15.5k\Omega || 60k\Omega || 60k\Omega =$

$= \boxed{10.22k\Omega}$

② [gain = 1mS · 10.22kΩ = 10.22

a

Part d, 10 points

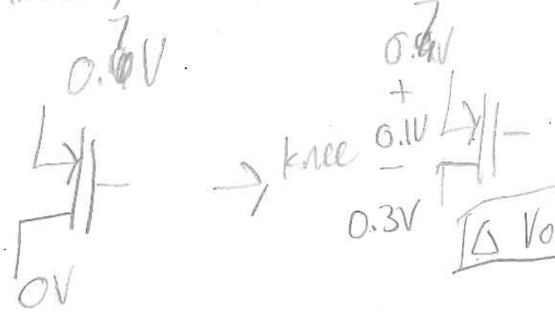
Maximum peak-peak output voltage at the positive output Vo+ (***show all your work***)

	magnitude and sign of maximum output signal swing due to ***cutoff***	magnitude and sign of maximum output signal swing due to: ***knee voltage*** (saturation)
Transistor Q5a	−183 V	+0.6V
Transistor Q6a	cannot cut off	−0.6V
Transistor Q2a	+ huge	Cannot be reached → irrelevant
Transistor Q2b	− huge	− irrelevant.

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant". But, give a 1-sentence statement why below.

(2) [μ-limited, so $V_{ds,sat} = V_{gs} - V_{th} = 0.1V$ for all Fets

(1) [Q5a:



$\Delta V_{out} = +0.6V$

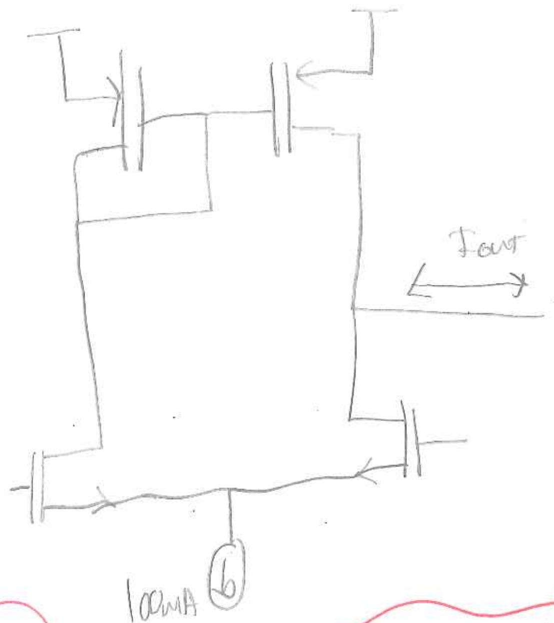
(1) [By symmetry, knee voltage of Q6a limits ΔV_{out} to −0.3V

(0) [Cutoff of Q5a, $\Delta I_d = 200\mu A$ $R_{eq} = 915k\Omega$

$\Delta V_{out} = -183V$ → irrelevant

a

cutoff of Q_{2a} & Q_{2b}

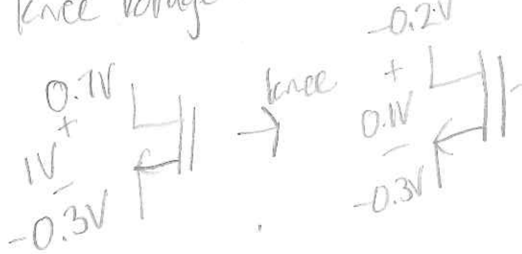


As 2a & 2b are driven, one full on and the other cut off
Iout varies by ± 100mA
This drives ± 100mA into effective load of 915kΩ
= ± 91.5V
Huge → irrelevant

3

1 knee voltage of Q_{2a} cannot be reached until past Q_{2b} cutoff
irrelevant

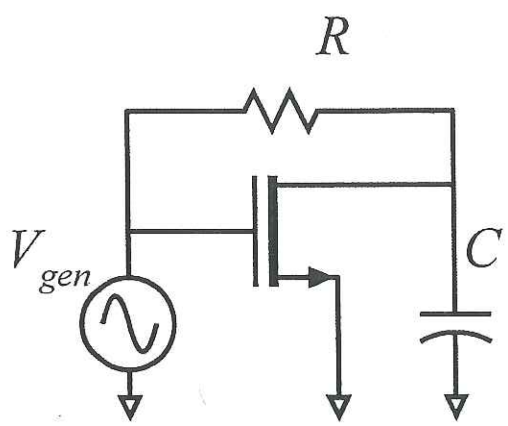
1 knee voltage of Q_{2b}



$\Delta V_{drain} = -0.9V$
$\Delta V_{out} = (-0.9V) A_{v5a}$ = − large number
irrelevant

a

Problem 3, 30 points



You will be working on the circuit to the left

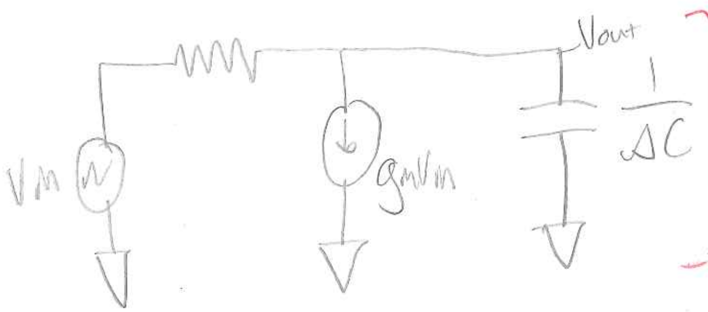
Ignore DC bias analysis. You don't need it.

The transistor has transconductance gm.

Its output resistance Rds is infinity...so you don't need to include this element in the circuit diagram !

<u>Part a, 7 points</u>

Draw a small-signal equivalent circuit of the circuit.



-3 if control voltage not shown

Part b, 8 points

gm=20 mS. C=1 μF . R= 1000 Ohms

Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with **correct units** and in ratio-of-polynomials form, i.e.

$$\frac{V_{out}(s)}{V_{gen}(s)} = K \cdot \frac{1+b_1 s + b_2 s^2 + ...}{1+a_1 s + a_2 s^2 + ...} \text{ or (as appropriate) } \frac{V_{out}(s)}{V_{gen}(s)} = K \cdot (s\tau)^n \cdot \frac{1+b_1 s + b_2 s^2 + ...}{1+a_1 s + a_2 s^2 + ...}$$

Note that an expression like

$$\frac{V_{out}(s)}{V_{gen}(s)} = \frac{1}{1+(3\cdot 10^{-6})s} \text{ is dimensionally wrong; } \frac{1}{1+(3\cdot 10^{-6}\text{seconds})s} \text{ is dimensionally correct}$$

Vout(s)/Vin(s)= ________________

ΣI @ $V_{out} = 0 \Rightarrow V_{out}\left(sC + \frac{1}{R}\right) + V_{in}\left(g_m - \frac{1}{R}\right) = 0$

4pts for some version of this

$$\frac{V_o}{V_{in}} = \frac{-\left(g_m - \frac{1}{R}\right)}{sC + \frac{1}{R}} = \frac{-[g_m R - 1]}{1 + sCR} \qquad RC = 1\text{mS} = 10^{-3}\text{sec}$$

$$\frac{V_{out}}{V_{in}} = -19\left[\frac{1}{1 + s \cdot 1\text{msec}}\right] \quad \text{4pts}$$

$2\pi f \cdot 1\text{mS} = 1 \Rightarrow f_{pole} = 159\text{ Hz}$

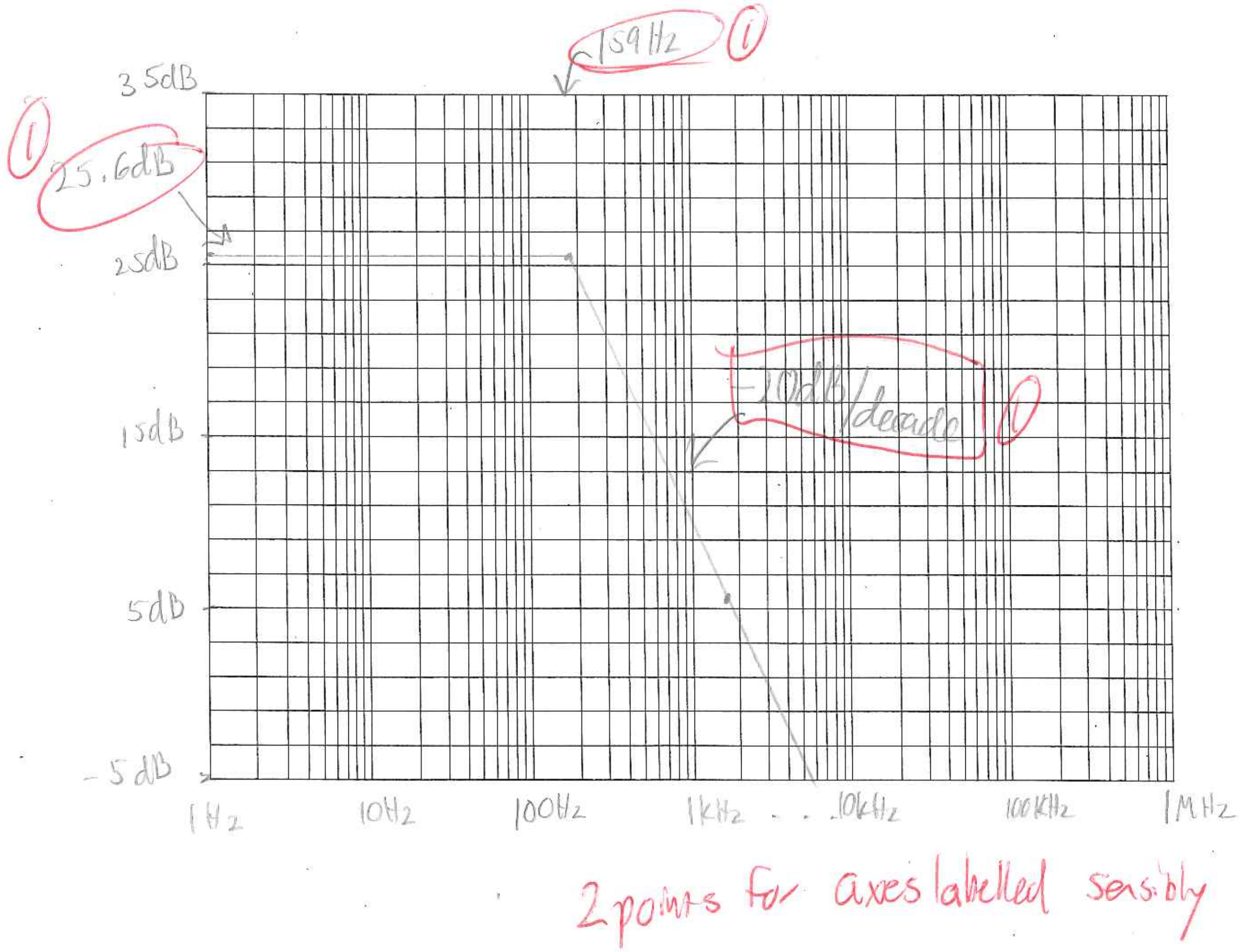
a

Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz:

______, pole at 159 Hz ______, ______, ______ (2)

Draw a clean Bode Plot of Vout/Vin,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes



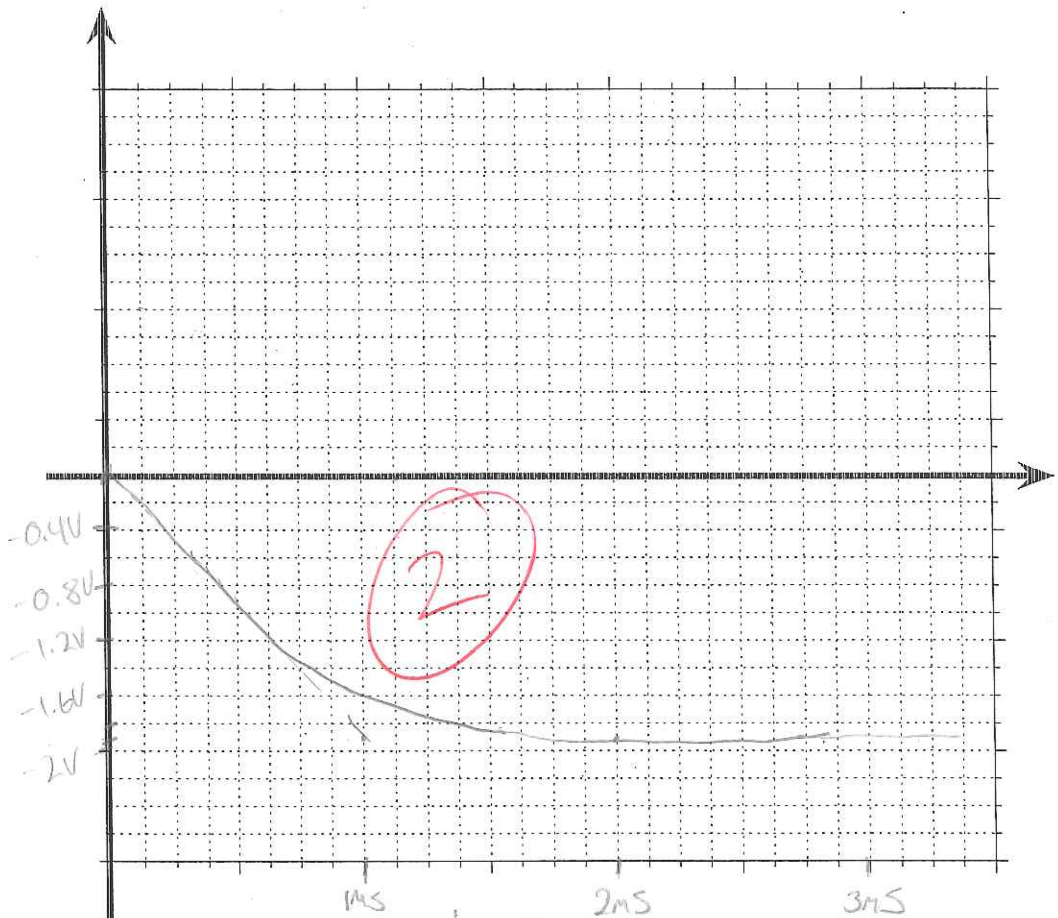
a

Part d, 8 points

Vin(t) is a 0.1 V amplitude step-function.

Find Vout(t)=______________________________

Plot it below. Label axes, show initial and final values, show time constants.



$H(s) = -19\left[\frac{1}{1+s\,1ms}\right]$

(2) $V_{in}(s) = \frac{0.1V}{s}$ so $V_{out}(s) = -1.9V \cdot \left(\frac{1}{s}\right)\left(\frac{1}{1+s\cdot 1ms}\right)$ (2)

$V_{out}(t) = \left[-1.9 + 1.9e^{\frac{-t}{1ms}}\right]u(t)$ (2)

a