## Final Exam, ECE 137A

## Wednesday March 22, 2017, 7:30-10:30pm

Name: $\qquad$
Closed Book Exam:
Class Crib-Sheet and 3 pages ( 6 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5\% accuracy), after stating \& justifying them.
Show your work:
Full credit will not be given for correct answers if supporting work is missing. Good luck

| Time function | LaPlace Transform |
| :--- | :--- |
| $\delta(t)$ impulse | 1 |
| $U(t)$ unit step-function | $1 / s$ |
| $e^{-\alpha t} U(t)$ | $\frac{1}{s+\alpha}$ |
| $e^{-\alpha t} \cos \left(\omega_{d} t\right) U(t)$ | $\frac{s+\alpha}{(s+\alpha)^{2}+\omega_{d}^{2}}$ |
| $e^{-\alpha t} \sin \left(\omega_{d} t\right) U(t)$ | $\frac{\omega_{d}}{(s+\alpha)^{2}+\omega_{d}^{2}}$ |


| Part | Points <br> Received | Points <br> Possible | Part | Points <br> Received | Points <br> Possible |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 1a |  | 5 | 2c |  | 15 |  |
| 1b |  | 6 | 2d |  | 10 |  |
| 1c |  | 4 | 3 a |  | 7 |  |
| 1d |  | 10 | 3 b | 8 |  |  |
| 1e |  | 10 | 3c |  | 7 |  |
| 2a |  | 10 | $3 d$ | 8 |  |  |
| 2b |  | 10 |  |  |  |  |
| total |  | 100 |  |  |  |  |

## Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts


All the transistors have the same (matched) $I_{S}$, have $\beta=100$, and $V_{A}=\infty$ Volts .
$V_{C E(s a t)}=0.5 \mathrm{~V}$.
$V_{b e}$ is roughly 0.7 V , but use $V_{b e}=(k T / q) \ln \left(I_{E} / I_{S}\right)$ when necessary and appropriate.
The supplies are +3 Volts and -3 Volts.
$\mathrm{R} 1=\mathrm{R} 1 \mathrm{~A}, \mathrm{R} 2=\mathrm{R} 2 \mathrm{~A}, \mathrm{R} 3=\mathrm{R} 3 \mathrm{~A}, \mathrm{R} 6=\mathrm{R} 6 \mathrm{~A}, \mathrm{R} 7=\mathrm{R} 7 \mathrm{~A}$.
The voltage drops across R1 and R1A are both 300 mV .
Q1, Q1A, Q6, Q6A, Q7, Q7A, Q5, Q5A, Q8, Q8A : $I_{C}=1 \mathrm{~mA}$.
Q9, Q9A: $I_{\mathrm{C}}=0.5 \mathrm{~mA}$.
$R L=100$ Ohms, $R 6=R 6 A=1000$ Ohms. $R 4=R 4 A=R 5=R 5 A=0$ Ohms.

Part a, 5 points
DC bias---to simplify , assume $\beta=\infty$ for the DC analysis only.


On the circuit diagram above, label the DC voltages at ALL nodes, the DC currents through ALL resistors, and the DC collector currents of all transistors.

Part b, 6 points
DC bias:
Find the value of all resistors.

| R1/R1A = | $\mathrm{R} 2 / \mathrm{R} 2 \mathrm{~A}=$ | $\mathrm{R} 3 / \mathrm{R} 3 \mathrm{~A}=$ | $\mathrm{R} 4 / \mathrm{R} 4 \mathrm{~A}=$ |
| :---: | :---: | :---: | :---: |
|  | R6/R6A = | R7/R7A = | $\mathrm{R} 8=$ |

## Part c, 4 points

find the following

| device | $\mathrm{Q} 1 / 1 \mathrm{~A}$ | $2 / 2 \mathrm{~A}$ | $3 / 3 \mathrm{~A}$ | $4 / 4 \mathrm{a}$ | $5 / 5 \mathrm{~A}$ | $6 / 6 \mathrm{~A}$ | $7 / 7 \mathrm{~A}$ | $8 / 8 \mathrm{~A}$ | $9 / 9 \mathrm{~A}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| gm, S |  |  |  |  |  |  |  |  |  |
| Rce, $\Omega$ |  |  |  |  |  |  |  |  |  |

Part d, 10 points.
The circuit is $100 \%$ symmetric, and can be represented by the simpler small-signal diagram below;


Find the following, using the actual value of $\beta$, i.e. $\beta=100$

|  | Voltage Gain | Input impedance |
| :--- | :--- | :--- |
| Q9 |  |  |
| Q8 |  |  |
| Q5 |  |  |
| Q7 |  |  |
| Q6 |  |  |
| Overall differential <br> Vout/Vin |  |  |

Note: with some insight, you can find the combined gain of Q7/Q4/Q5 in a single step. If would would like to do so, omit the separate answers for Q5 and Q7 in the table above, and instead fill in the table below,

|  | Voltage Gain | Input impedance |
| :--- | :--- | :--- |
| Q7/Q4/Q5 combination. |  |  |

## Part e, 10 points

Maximum peak-peak output voltage (show all your work)
For this, you must use the full circuit diagram, not the half circuit diagram.

|  | magnitude and sign of <br> maximum output signal <br> swing due to cutoff | magnitude and sign of <br> maximum output signal <br> swing due to saturation |
| :--- | :--- | :--- |
| Transistor Q9 |  |  |
| Transistor Q9A |  |  |
| Transistor Q8 |  |  |
| Transistor Q8A |  |  |
| Transistor Q5 |  |  |
| Transistor Q5A |  |  |
| Transistor Q7 |  |  |
| Transistor Q7A |  |  |

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant. Q9/9A form a push pull stage, so be careful about your answer there. Hint: There is, effectively, another push-pull stage in the circuit, which will affect two of the other answers.

## Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input $\mathrm{Vi}+$ is zero volts, and that we must determine the DC value of the negative input voltage ( $\mathrm{Vi}-$ ) necessary to obtain this.


The NMOSFETs and the PMOSFETS have a 0.20 V threshold, a 22 nm gate length, 200 $\mathrm{cm}^{2} / \mathrm{Vs}$ mobility, a $10^{7} \mathrm{~cm} / \mathrm{s}$ injection velocity, and $1 / \lambda=6$ Volts. The gate oxide thickness is 1.0 nm and the dielectric constant is 3.8. This gives
$\mu c_{o x} W_{g} / 2 L_{g}=15.3 \mathrm{~mA} / \mathrm{V}^{2} \cdot\left(W_{g} / 1 \mu \mathrm{~m}\right)$ and $v_{\text {sat }} c_{o x} W_{g}=3.36 \mathrm{~mA} / \mathrm{V} \cdot\left(W_{g} / 1 \mu \mathrm{~m}\right) \quad$ (both are a bit unrealistic for a real technology). and $v_{\text {sat }} L_{g} / \mu=0.110 \mathrm{~V}$
$V_{D D}=+1.5 \mathrm{~V},-V_{S S}=-1.5 \mathrm{~V}, \quad R_{\mathrm{L}}=20 \mathrm{kOhm}$

Part a, 10 points
DC bias.
Approximation: ignore the term $\left(1+\lambda V_{D S}\right)$ in DC bias analysis.
Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi- ) necessary to obtain this.

All transisistors except Q6, Q13 have $\left|V_{\mathrm{gs}}\right|=0.3 \mathrm{~V}$. Q6 and Q13 have Vgs=0.35V.
All transisistors except Q8, Q15, Q17 have $\left|I_{D}\right|=0.156 \mathrm{~mA}$.
Q8 and Q15 have $\left|I_{D}\right|=1.56 \mathrm{~mA}$.
You can figure out $\left|I_{D}\right|$ for Q17.
Find the gate widths of all transistors, plus R1 and R2.

Find:
Wal= $\qquad$ $\mathrm{Wg} 2=$ $\qquad$ $\mathrm{Wg} 3=\quad \mathrm{Wg} 4=$ $\qquad$
Wg5= $\qquad$ Wg6 = $\qquad$ $\mathrm{Wg} 7=$ $\qquad$ $\mathrm{Wg} 8=$
Wg9= $\qquad$ $\mathrm{Wg} 10=$ $\qquad$ $\mathrm{Wg} 11=$ $\qquad$ $\mathrm{Wg} 12=$ $\qquad$
Wg13= $\qquad$ $\mathrm{Wg} 14=$ $\qquad$ Wg15= Wg16= $\qquad$
Wg17= Wg18= $\qquad$ Wg19
$\mathrm{R} 1=$ $\qquad$ $\mathrm{R} 1=$ $\qquad$

Part b, 10 points
DC bias


On the circuit diagram above, label the DC voltages at ALL nodes, the drain currents of ALL transistors, and the gate widths of ALL transistors

Part c, 15 points.
You will now compute the op-amp differential gain. You must consider the $\left(1+\lambda V_{D S}\right)$ term in the FET IV characteristics when you do this.

Find the following

|  | Voltage Gain | Input impedance |
| :--- | :--- | :--- |
| Transistor combination <br> Q3,4,10,11 |  |  |
| Q5,7 |  |  |
| Q9 or Q12. |  |  |
| Q8 or Q15 |  |  |
| Overall differential <br> Vout/Vin |  |  |

Notes:

1) You can analye Q5 and Q7 as separate stages, or as a combined stage using Norton/Thevenin methods. Don't ask for hints as to how to do this.
2) For $\mathrm{Q} 9 / 12$ and for $\mathrm{Q} 8 / 15$, you can assume that Q 9 and Q 12 are on for the positive signal swing and Q8 and Q15 are on for the negative signal swing. More accurately, you can assume, for the signal swing near zero volts, that all are on. If you take the latter approach (and do it correctly), you will receive a couple of extra credit points. One hint (don't ask for any other hints): use symmetry.

## Part d, 10 points

Maximum peak-peak output voltage at the positive output Vo+ (show all your work)

|  | magnitude and sign of <br> maximum output signal <br> swing due to cutoff | magnitude and sign of <br> maximum output signal <br> swing due to: <br> knee voltage (saturation) |
| :--- | :--- | :--- |
| Transistor Q8 |  |  |
| Transistor Q15 |  |  |
| Transistor Q9 |  |  |
| Transistor Q12 |  |  |
| Transistor Q2 |  |  |
| Transistor Q19 |  |  |
| Transistor Q7 |  |  |
| Transistor Q14 |  |  |

Be warned: in some cases a limit is not relevant. Mark those answers 'not relevant"..

Problem 3, 30 points


## Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.

## Part b, 8 points

$\mathrm{gm}=10 \mathrm{mS} . \mathrm{C}=1 \mathrm{nF} . \mathrm{R}=1000$ Ohms
Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with ${ }^{* *}$ correct units** and in ratio-of-polynomials form, i.e.
$\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=K \cdot \frac{1+b_{1} s+b_{2} s^{2}+\ldots}{1+a_{1} s+a_{2} s^{2}+\ldots}$ or (as appropriat e) $\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=K \cdot(s \tau)^{n} \cdot \frac{1+b_{1} s+b_{2} s^{2}+\ldots}{1+a_{1} s+a_{2} s^{2}+\ldots}$
Note that an expression like
$\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=\frac{1}{1+\left(3 \cdot 10^{-6}\right) s}$ is dimensiona lly wrong; $\frac{1}{1+\left(3 \cdot 10^{-6} \text { seconds }\right) s}$ is dimensiona lly correct
$\qquad$

## Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz :

Draw a clean Bode Plot of Vout/Vin,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes


## Part d, 8 points

$\operatorname{Vin}(\mathrm{t})$ is a 0.1 V amplitude step-function.
Find $\operatorname{Vout}(\mathrm{t})=$ $\qquad$
Plot it below. Label axes, show initial and final values, show time constants.


