Final Exam, ECE 137A

Wednesday March 22, 2017, 7:30 - 10:30pm

Name: solution

Closed Book Exam:
Class Crib-Sheet and 3 pages (6 surfaces) of student notes permitted
Do not open this exam until instructed to do so. Use any and all reasonable approximations (5% accuracy), ***after stating & justifying them***.
Show your work:
Full credit will not be given for correct answers if supporting work is missing.
Good luck

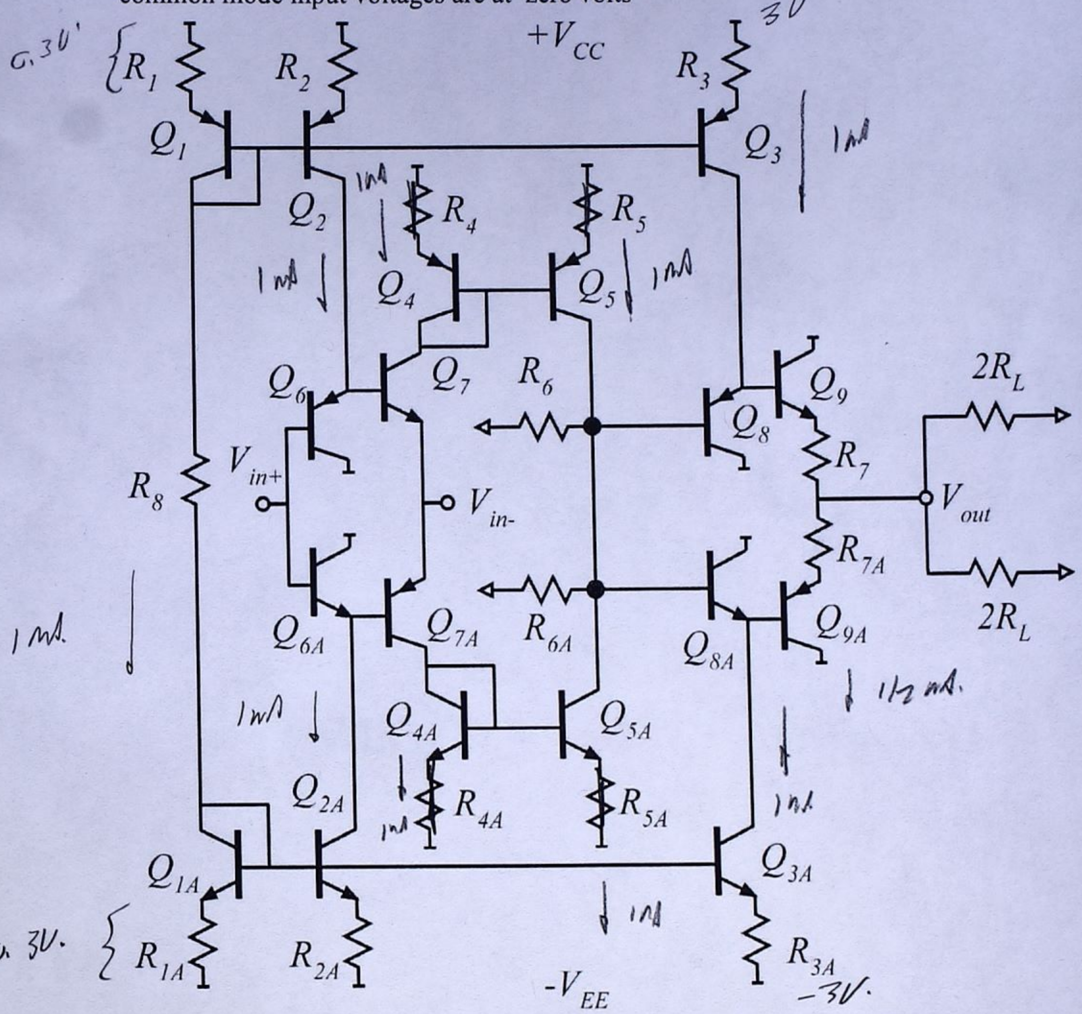
Time function	LaPlace Transform
$\delta(t)$ impulse	1
$U(t)$ unit step-function	$1/s$
$e^{-\alpha t}U(t)$	$\frac{1}{s+\alpha}$
$e^{-\alpha t}\cos(\omega_d t)U(t)$	$\frac{s+\alpha}{(s+\alpha)^2+\omega_d^2}$
$e^{-\alpha t}\sin(\omega_d t)U(t)$	$\frac{\omega_d}{(s+\alpha)^2+\omega_d^2}$

Part	Points Received	Points Possible	Part	Points Received	Points Possible
1a		5	2c		15
1b		6	2d		10
1c		4	3a		7
1d		10	3b		8
1e		10	3c		7
2a		10	3d		8
2b		10			
total		100			

b

Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts



All the transistors have the same (matched) I_S, have $\beta = 100$, and $V_A = \infty$ Volts.
$V_{CE(sat)} = 0.5\text{V}$.
V_{be} is roughly 0.7 V, but use $V_{be} = (kT/q)\ln(I_E/I_S)$ when necessary and appropriate.
The supplies are +3 Volts and -3 Volts.
R1=R1A, R2=R2A, R3=R3A, R6=R6A, R7=R7A.
The voltage drops across R1 and R1A are both 300mV.
Q1, Q1A, Q6, Q6A, Q7, Q7A, Q5, Q5A, Q8, Q8A : I_C =1mA.
Q9, Q9A: I_C =0.5 mA.
RL=100 Ohms, R6=R6A=1000 Ohms. R4= R4A=R5=R5A=0 Ohms.

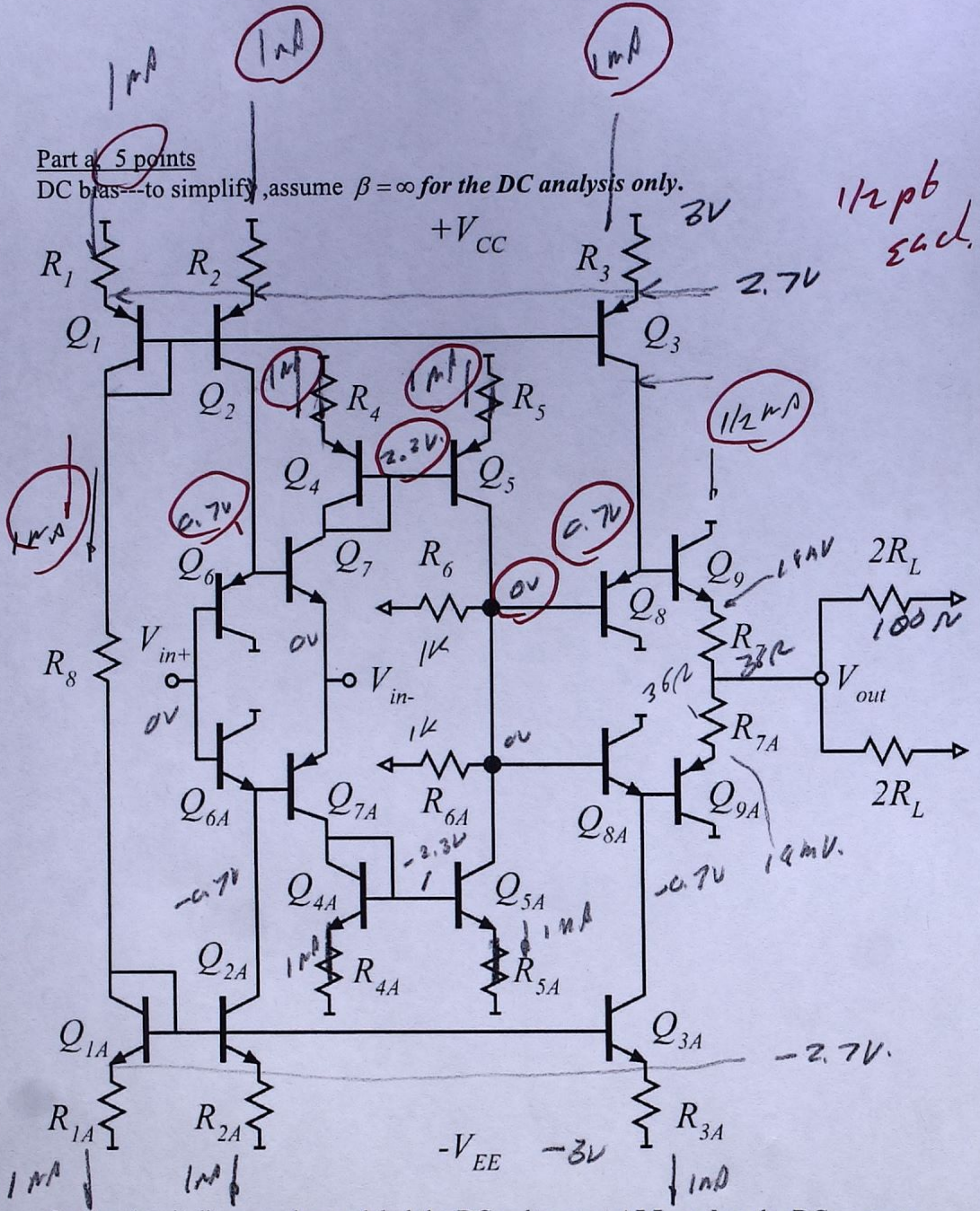
$$R_7 = \frac{kT}{q(1/2\,mA)} \ln 2 = \underline{\underline{36\Omega}}$$

b

Part a: 5 points

DC bias---to simplify ,assume $\beta = \infty$ ***for the DC analysis only.***

1/2 pt each.



On the circuit diagram above, label the DC voltages at **ALL nodes**, the DC currents through **ALL resistors**, and the DC collector currents of **all transistors**.

Part b, 6 points

DC bias:

Find the value of all resistors.

R1/R1A=300 R2/R2A = 300 R3/R3A = 300 R4/R4A = 0

R5/R5A = 0 R6/R6A = 1kΩ R7/R7A = 36 R8 = 5.4kΩ

2 [$R_1/1A/2/2A/3/3A$ $\quad R = \frac{0.3V}{1mA} = 300\Omega.$

2 [$R7/7A$ $\quad V_{be8} + V_{be8A} - V_{be9} - V_{be9A} = 2 \cdot R_7 \cdot 0.5mA$

$$2 \cdot \frac{kT}{q} \cdot \ln(2) = 2 \cdot R_7 \cdot 0.5mA$$

$$R_7 = 36\Omega$$

2 [R_8: Vdrop is $(3V - 0.3V) \cdot 2$, current is 1mA

$$R = \frac{2(2.7V)}{1mA} = 5.4k\Omega$$

Part c, 4 points

find the following

device	Q1/1A	2/2A	3/3A	4/4a	5/5A	6/6A	7/7A	8/8A	9/9A
gm, S									
Rce, Ω									

1 $$R_{ce} = \frac{\overset{\infty V}{V_A} + V_{CE}}{I_c} \rightarrow \infty\Omega$$

1.5 all Xs except Q9/9A.

$I_c = 1mA \rightarrow 1/g_m = 26\Omega$

$\rightarrow g_m = 38.5mS$

1.5 9/9A

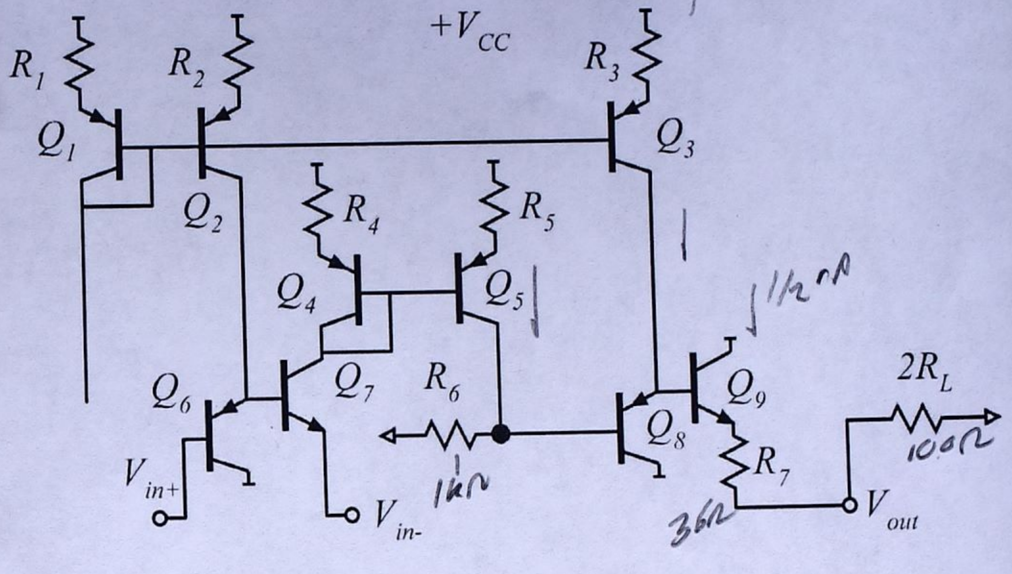
$I_c = 1/2\,mA \rightarrow 1/g_m = 52\Omega$

$\hookrightarrow g_m = 19.2mS$

All 1mA

Part d, 10 points.

The circuit is 100% symmetric, and can be represented by the simpler small-signal diagram below;



Find the following, ***using the actual value of β, i.e. β=100***

	Voltage Gain	Input impedance
Q9	0.53	15.8 kΩ
Q8	1	1.88 MΩ
Q5	- 3.805	2.6 kΩ
Q7	-1	2.6kΩ
Q6	1	260kΩ
Overall differential Vout/Vin	20.38	260 kΩ

Note: with some insight, you can find the combined gain of Q7/Q4/Q5 in a single step. If would would like to do so, omit the separate answers for Q5 and Q7 in the table above, and instead fill in the table below,

	Voltage Gain	Input impedance
Q7/Q4/Q5 combination.		

b

Q9

1 [$R_7 / 2R_L$ divider

$V_{out}/V_{E9} = \frac{100\Omega}{136\Omega} = 0.735$

$Q9 \; A_v = \frac{136\Omega}{136\Omega + 1/g_m} = \frac{136\Omega}{\underbrace{136\Omega + 52\Omega}_{188\Omega}} = 0.72$ } 0.53

1/2 [$R_i = \beta(188\Omega) = 18.8\,k\Omega$

Q8 1/2 [$R_{Leq} = R_{i9} = 18.8\,k\Omega$

1/2 [$A_v = \frac{18.8k\Omega}{18.8k\Omega + 26\Omega} \cong 1$ to 1% precis..

1/2 [$R_{i8} = \beta(18.8k\Omega) = 1.88\,M\Omega.$

Q5 1/2 [$R_{Leq5} = 1k\Omega // 1.88M\Omega \cong 1k\Omega$ to 1% precision

1/2 [$A_{v5} = -g_m R_{Leq5} = -1k\Omega/26\Omega = -38.5.$

1/2 [$R_{i5} = \beta/g_m = 2.6k\Omega$

Q7 1/2 [$R_{Leq} = 1/g_{m4} // R_{i5} = 1/g_{m4}$ to 1%

$= 26\Omega$

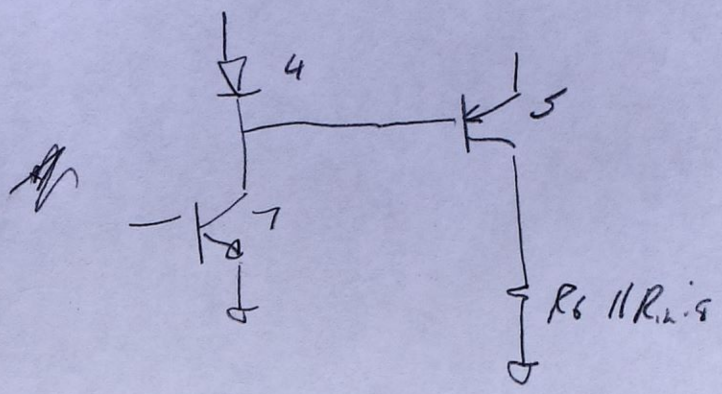
1 [$A_{v5} = -g_m R_{Leq} = -26/26. = -1.$

1 [$R_{i7} = \beta/g_m = 2.6k\Omega$

Q6/ 1 ~~Ac~~ [$R_{Leq} = R_{i7} = 2.6\,k\Omega$ 1 [$A_v = \frac{2.6k\Omega}{2.6k\Omega + 26\Omega} = 0.99 \cong 1.$

1 [$R_{i6} \cong \beta R_{Leq} = 260k\Omega.$

Alternative for Q5/7



$A_v = g_{m7} \cdot R_6 || R_{in8}$

$= \frac{1000\Omega}{26\Omega} = -38.5$

Note QxA answers are the negative of Qx answers.

Part e, 10 points

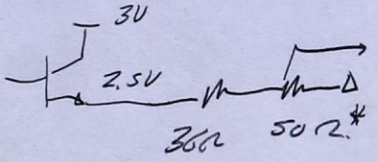
Maximum peak-peak output voltage (***show all your work***)
For this, you must use the full circuit diagram, not the half circuit diagram.

	magnitude and sign of maximum output signal swing due to *cutoff*	magnitude and sign of maximum output signal swing due to ***saturation***
Transistor Q9	1.45V	N/A
Transistor Q9A	-1.45V.	N/A
Transistor Q8	-2.03	+5V
Transistor Q8A	+2.03.	-5V.
Transistor Q5	+1.45V	N/A.
Transistor Q5A	-1.45V.	N/A
Transistor Q7	N/A	N/A
Transistor Q7A	N/A	N/A

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant. Q9/9A form a push pull stage, so be careful about your answer there. ***Hint***: There is, effectively, another push-pull stage in the circuit, which will affect two of the other answers.

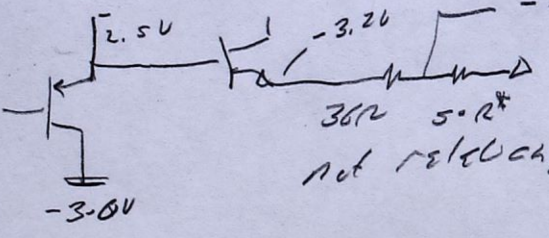
1 [Q9 cutoff - not relevant - push-pull.

1 [Q9 saturation.



$2.5V \cdot \frac{50\Omega}{86\Omega} = 1.45V$

1 [Q8 saturation

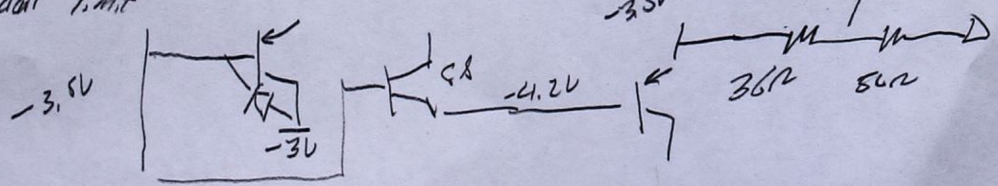


$-3.2V \cdot \frac{50\Omega}{86\Omega} = \ldots$

not relevant, as Q9 will be off

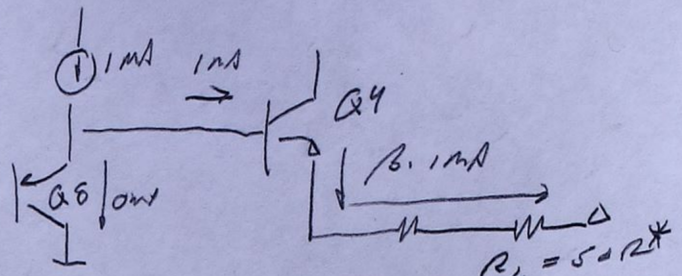
1 [Actual limit

$-3.5V \cdot \frac{50\Omega}{86\Omega} = -2.03V.$



[note - we will accept use of Rc = 100Ω throughout this part]

1 Cutoff Q8



$V_{out} = \beta \cdot 1mA \cdot R_L = 100mA \cdot 50\Omega = +5V.$

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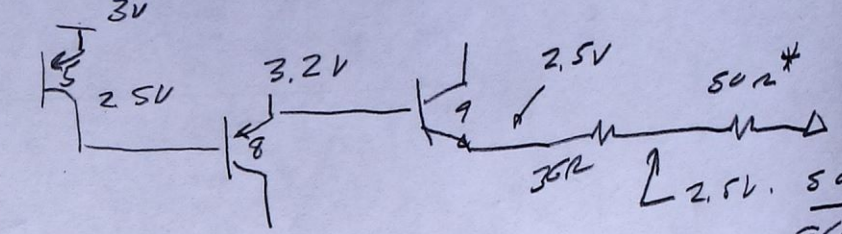
2 Sat Q5.



$2.5V \cdot \frac{50\Omega}{86\Omega}$

$= 1.45V.$

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1 Cutoff Q5: not relevant, as Q5/5A form a push-pull pair

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1 Sat Q7: not relevant, as when $V_{CE7} = 0.5V$ V_{BE5} would be 2.5V, giving a very large I_C for Q5.

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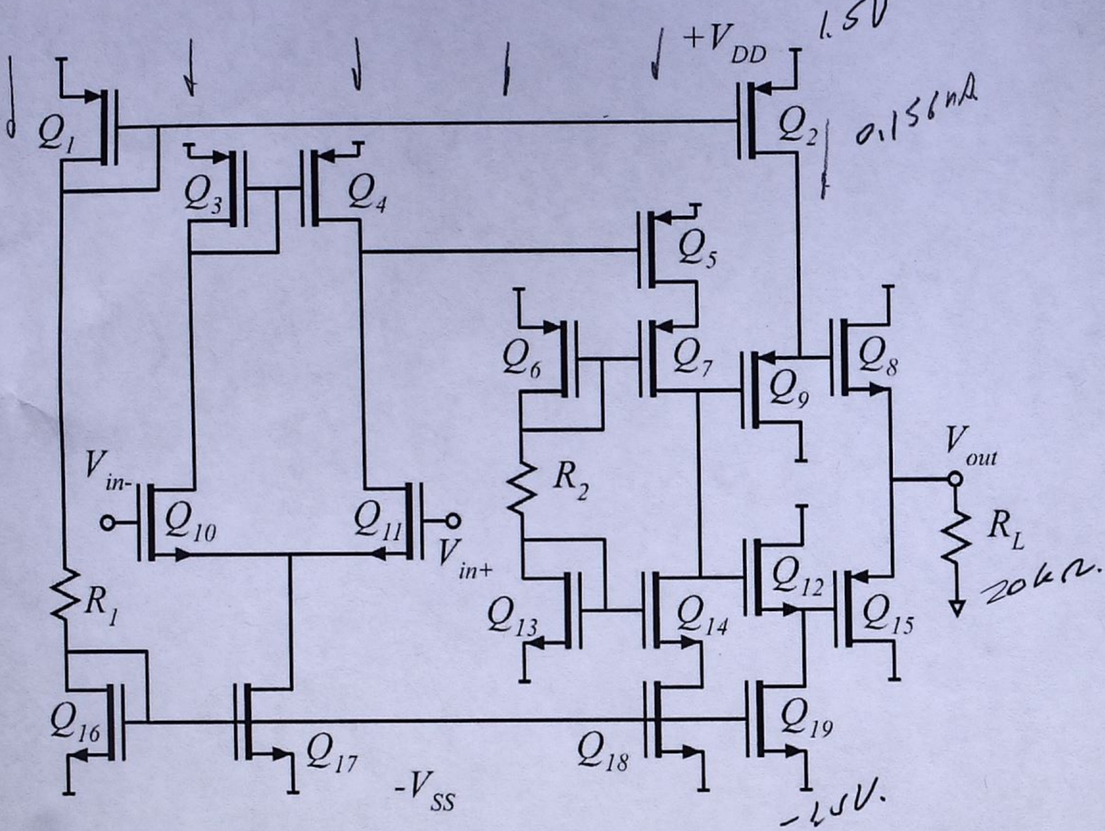
1 Cutoff Q7 - not relevant, as Q7/7A and Q5/5A form a push pull pair

Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.



The NMOSFETs and the PMOSFETS have a 0.20 V threshold, a 22nm gate length, 200 cm^2/Vs mobility, a 10^7cm/s injection velocity, and $1/\lambda=6$ Volts. The gate oxide thickness is 1.0nm and the dielectric constant is 3.8. This gives

$\mu c_{ox} W_g / 2L_g = \underline{15.3\ \text{mA/V}^2} \cdot (W_g / 1\mu\text{m})$ and

$v_{sat} c_{ox} W_g = 3.36\ \text{mA/V} \cdot (W_g / 1\mu\text{m})$ (both are a bit unrealistic for a real technology).

and $v_{sat} L_g / \mu = 0.110$ V

$V_{DD} = +1.5$ V, $-V_{SS} = -1.5$ V, R_L=20 kOhm

b

Part a, 10 points

DC bias.

Approximation: ignore the term $(1+\lambda V_{DS})$ ***in DC bias analysis.***

Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.

All transisistors ***except Q6, Q13*** have $|V_{gs}|$=0.3V.
Q6 and Q13 have Vgs=0.35V.
All transisistors ***except Q8, Q15, Q17*** have $|I_D|$=0.156mA.
Q8 and Q15 have $|I_D|$=1.56mA.
You can figure out $|I_D|$ for Q17.

Find the gate widths of all transistors, plus R1 and R2.

Find:
Wg1=______ Wg2 =______ Wg3=______ Wg4 =______
Wg5=______ Wg6 =______ Wg7=______ Wg8 =______
Wg9=______ Wg10=______ Wg11=______ Wg12 =______
Wg13=______ Wg14 =______ Wg15=______ Wg16=______
Wg17=______ Wg18=______ Wg19______
R1=______ R1=______

1 [$|V_{gs}| = 0.3V \rightarrow$ less than $V_{th} + \Delta V \rightarrow$ mobility limit.

2 [$0.156mA = 15.3\frac{mA}{V^2} \cdot \frac{W_g}{1\mu m}(0.3V - 0.2V)^2$

$\rightarrow W_g = 1.03\mu m \cong \underline{\underline{1\mu m}}$

for all Tr except Q17, Q8, Q15, 6, 13.

1 [Q 8,15 $\rightarrow$ $W_g = 10\mu m$ (10:1 more I_D, same V_{gs})

2 [Q17: $0.312mA = 15.3\frac{mA}{V^2}\frac{W_g}{1\mu m}(0.3V - 0.2V)$

$\rightarrow W_g = 2\mu m$

1 [R_1 // $I_1 = 0.156mA$, $V = 2(1.5V - 0.3V) \Rightarrow R = 15.4 k\Omega$.

1 [R_2 // $I = 0.156mA$, $V = 2(1.5V - 0.35V)$

$\Rightarrow R = \underline{\underline{14.7k\Omega}}$

2

Q6, Q13

$V_{gs} = 0.35V$, $I_D = 0.156mA$

$\llcorner$ more than $V_{th} + \Delta V = 0.2V + 0.11V$.

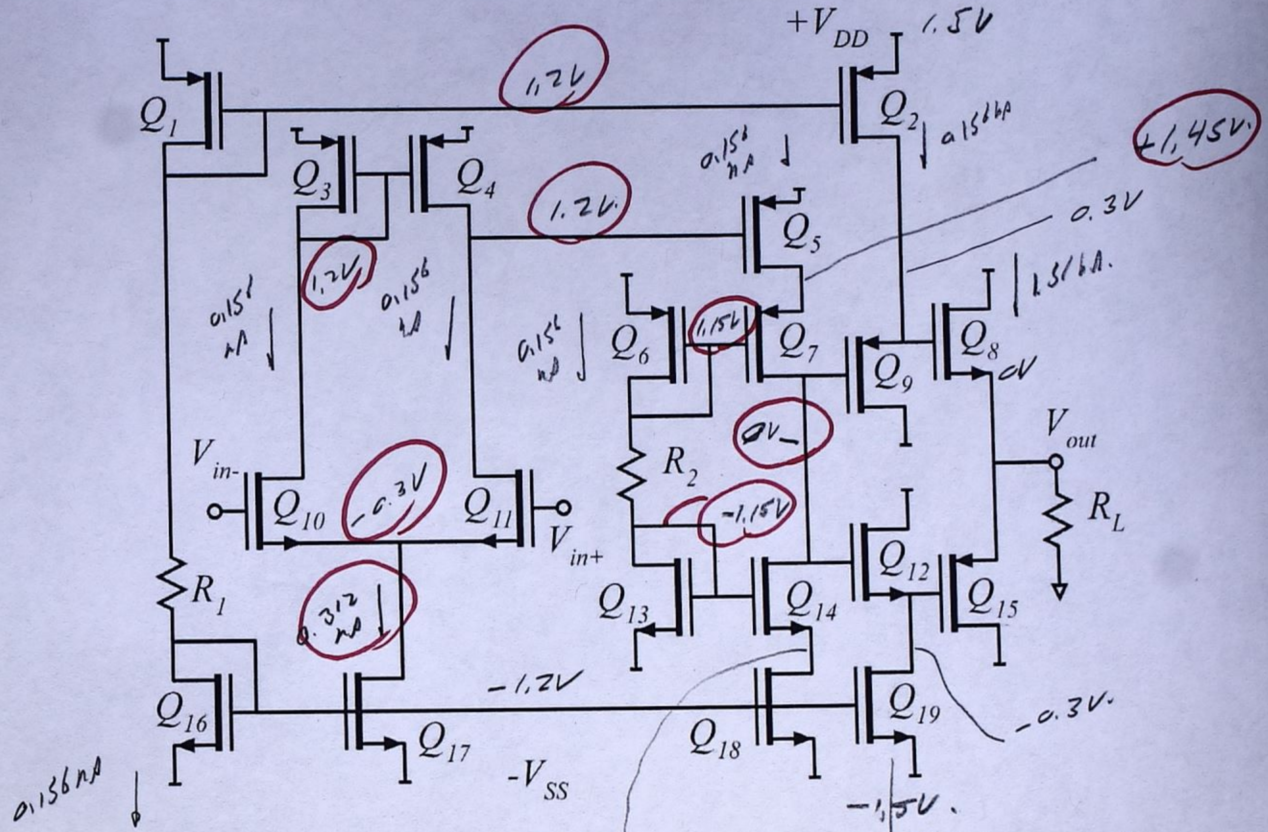
$$I_D = 0.156mA = 3.36\frac{mA}{V}\underbrace{\left(0.35V - 0.2V - \frac{0.11V}{2}\right)}_{0.095V} \cdot \frac{W_9}{1\mu m}$$

$$\Rightarrow W_9 = 0.490\mu m.$$

b

Part b, 10 points

DC bias



On the circuit diagram above, label the DC voltages at **ALL nodes**, the drain currents of **ALL transistors,** and the gate widths of **ALL transistors**

-1.45V.

0.156mA

Part c, 15 points.

You will now compute the op-amp differential gain. ***You must consider the*** $(1+\lambda V_{DS})$ ***term in the FET IV characteristics when you do this.***

Find the following

	Voltage Gain	Input impedance
Transistor combination Q3,4,10,11	58.9	$\infty\ \Omega$
Q5,7	6940	∞
Q9 or Q12.	0.98	$\infty\ \Omega$
Q8 or Q15	0.99	$\infty\ \Omega$
Overall differential Vout/Vin	396,000	$\infty\ \Omega$.

Notes:

1) You can analye Q5 and Q7 as separate stages, or as a combined stage using Norton/Thevenin methods. Don't ask for hints as to how to do this.

2) For Q9/12 and for Q8/15, you can assume that Q9 and Q12 are on for the positive signal swing and Q8 and Q15 are on for the negative signal swing. More accurately, you can assume, for the signal swing near zero volts, that all are on. If you take the latter approach (and do it correctly), you will receive a couple of extra credit points. One hint (don't ask for any other hints): use symmetry.

2 $$g_m = 2\overbrace{(V_{gs} - V_{tn})}^{0.1V.}\,\frac{15.3\,mA}{V^2}\cdot \underset{\substack{\uparrow\\ 1\mu m \text{ or } 10\mu m}}{W/L}$$

$= 3.06\ mS$ for all Tx except Q8, Q15.

$= 30.6\ mS$ for Q8, Q15.

1 $R_{DS} = 1/\lambda I_D = 6V/0.156mA = 38.5k\Omega$ all Tx except Q8, 15

$= 3.85\ k\Omega$ Q8, 15.

Q8 1 $R_{Leq} = R_L \,//\, R_{DS8} = 20k\Omega // 3.85k\Omega$

$= 3.22k\Omega$

1 $$A_v = \frac{3.22k\Omega}{3.22k\Omega + \underbrace{1/g_m}_{32.6\Omega}} = 0.990.$$

1 $R_{in8} = \infty$

Q9: 1 $R_{Leq} = R_{DS9} \,//\, R_{DS2} = \frac{38.5k\Omega}{2} = 19.25k\Omega$

1 $$A_v = \frac{19.25\,k\Omega}{19.25k\Omega + \underbrace{1/g_m}_{326\Omega}} = 0.983.$$

1 $R_{in} = \infty$

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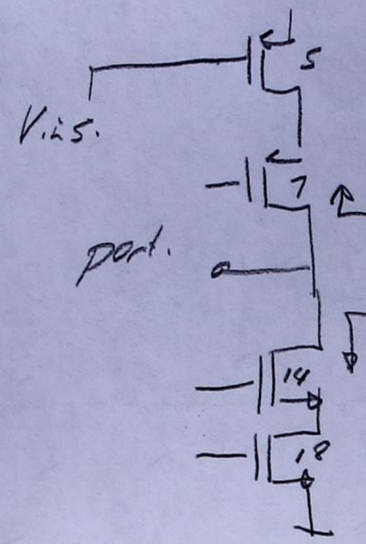
0 wal cr 2 stges

Q5/7/14/15 use Norton method.



R = (same calculation) = 4.57 MΩ.

$R = R_{DS14}(1 + g_{m14} R_{DS18})$
= 4.57MΩ

1

Total impedance @ node = $\frac{4.57M\Omega}{2}$
= 2.28 MΩ

1

short-circuit current = $g_{m5} \cdot V_{in5} \cdot \frac{R_{DS5}}{R_{DS5} + 1/g_{m7}}$ 1

1 gain of Q5/7 = $-\underbrace{g_{m5} \cdot 2.28M\Omega}_{\approx 7000} \cdot \underbrace{\frac{R_{DS5}}{R_{DS5} + 1/g_{m7}}}_{0.99}$ = -6940

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gain of Q3/4/10/11

1 $R_{Leq} = R_{DS4} \,//\, R_{DS11} = \frac{38.5k\Omega}{2} = 19.25\,k\Omega$.

1 $A_v = g_{m10} R_{Leq} = 19.25k\Omega\,(3.06\,mS) = 58.9$

b

Part d, 10 points

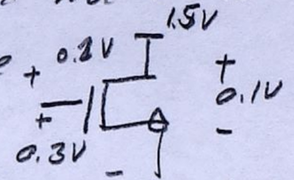
Maximum peak-peak output voltage at the positive output Vo+ (***show all your work***)

	magnitude and sign of maximum output signal swing due to *cutoff*	magnitude and sign of maximum output signal swing due to: ***knee voltage*** (saturation)
Transistor Q8	N/A	1.4V
Transistor Q15	N/A	−1.4V
Transistor Q9	+6V or large/ignore	−1.7V
Transistor Q12	+6V or large/ignore	+1.7V.
Transistor Q2	N/A	+1.1V
Transistor Q19	N/A	−1.1V.
Transistor Q7	− 350V or N/A.	1.35V
Transistor Q14	N/A.	−1.35V.

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant"..

1 [Q8, 15 – cutoff not relevant – push pull.

1 [Q8 knee voltage



knee @ V_{DS} = 0.1V.

V_{out} = 1.5V − 0.1V = 1.4V]

1/2 [Q15 knee voltage – identical calculation

1/2 [Q9 cutoff ΔI_D = 0,156mA but R_{eq} is R_{DS2} → large.

ΔV = 0,156mA · R_{DS2} – +6V → large – ignore]

1/2 [Q12 cutoff – identical calculation.

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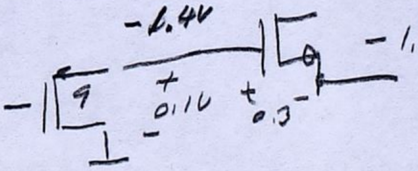
1 [Q9 knee voltage.

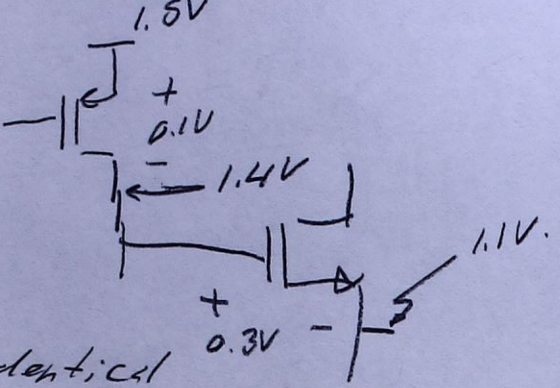


−1.7V – beyond supply

1/2 [Q12 knee voltage $-1.5V$ identical calculation → +1.7V.

b

1 [Q2 knee voltage



1/2 [Q19 knee – identical calculation → −1.1V.

1/2 [Q2/19 cutoff – not relevant – Id does not vary.

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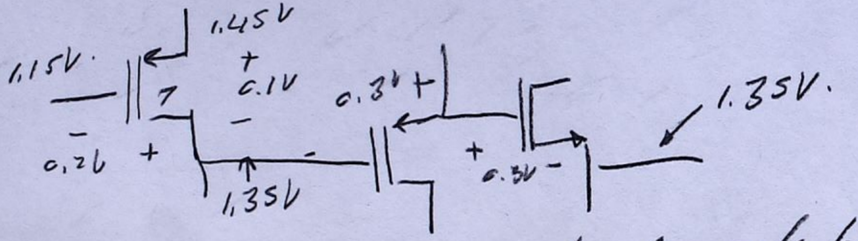
1 [Q7 knee Voltage



1/2 [Q14 knee voltage – identical calculation

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1 [Q7 Cutoff

$\Delta I = 0.156\,mA$, $R_{eq} \parallel R_{out7} = 2.28\,M\Omega$

$\Delta V = \text{product} = -355V$ → Very large, not relevant

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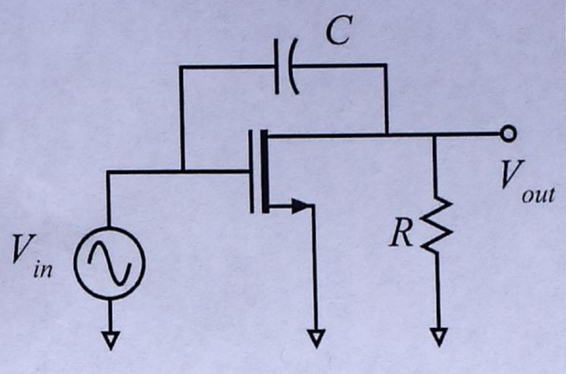
1/2 [Q14 cutoff – not relevant

b

Problem 3, 30 points



You will be working on the circuit to the left

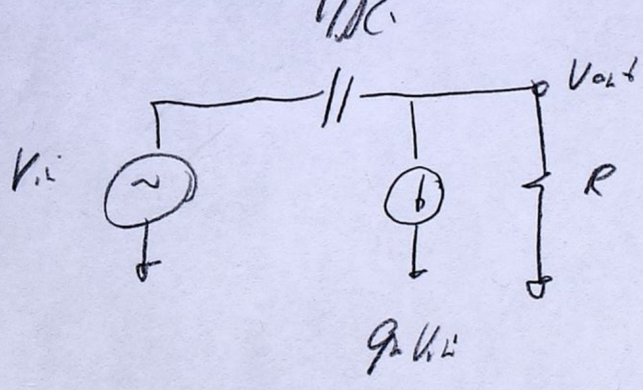
Ignore DC bias analysis. You don't need it.

The transistor has transconductance gm.

Its output resistance Rds is infinity...so you don't need to include this element in the circuit diagram !

Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.



all elements must be labelled

↑
control must be identified

Part b, 8 points

gm=10 mS. C=1 nF. R= 1000 Ohms

Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with **correct units** and in ratio-of-polynomials form, i.e.

$$\frac{V_{out}(s)}{V_{gen}(s)} = K \cdot \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots} \text{ or (as appropriat e) } \frac{V_{out}(s)}{V_{gen}(s)} = K \cdot (s\tau)^n \cdot \frac{1+b_1 s + b_2 s^2 + \ldots}{1+a_1 s + a_2 s^2 + \ldots}$$

Note that an expression like

$$\frac{V_{out}(s)}{V_{gen}(s)} = \frac{1}{1+(3\cdot 10^{-6})s} \text{ is dimensiona lly wrong; } \frac{1}{1+(3\cdot 10^{-6}\text{ seconds })s} \text{ is dimensiona lly correct}$$

Vout(s)/Vin(s)= ____________________

$\Sigma I = 0$ @ Vout.

$$4 \left[V_{out}(1/R) + g_m V_{in} + (V_{out} - V_{in})(sC) = 0. \right]$$

$$V_{out}(1/R + sC) = V_{in}(+sC - g_m)$$

$$V_o/V_i = \frac{sC - g_m}{sC + 1/R} = \frac{-(g_m - sC)}{1/R + sC} = -g_m R \frac{1 - sC/g_m}{1 + sCR}$$

4.

$$\boxed{\frac{V_o(s)}{V_{in}(s)} = -g_m R \cdot \frac{1 - sC/g_m}{1 + sCR}}$$

$$= -10 \cdot \frac{1 - s\tau_z}{1 + s\tau_p}$$

100ns

1µs

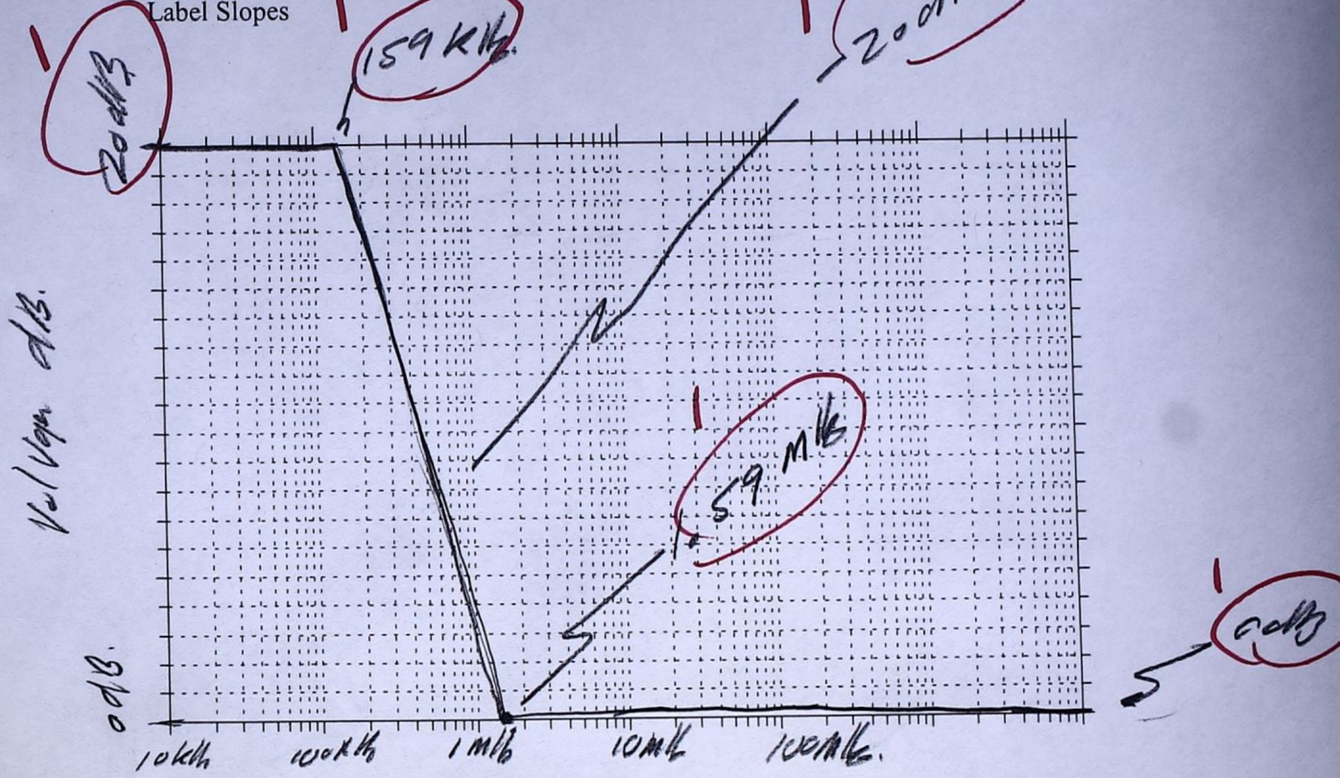
points deducted if these are not correctly located

Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz:

_______________, _______________, _______________, _______________

Draw a clean Bode Plot of Vout/Vin,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies,
Label Slopes



1 $\left[f_p = \frac{1}{2\pi RC} = 159 kHz. \right.$

1 $\left[f_z = \frac{g_m}{2\pi C} = \underline{1.59 MHz} \right.$

$g_m R = 10 mS \cdot 1000\Omega = 10 \rightarrow 20 dB.$

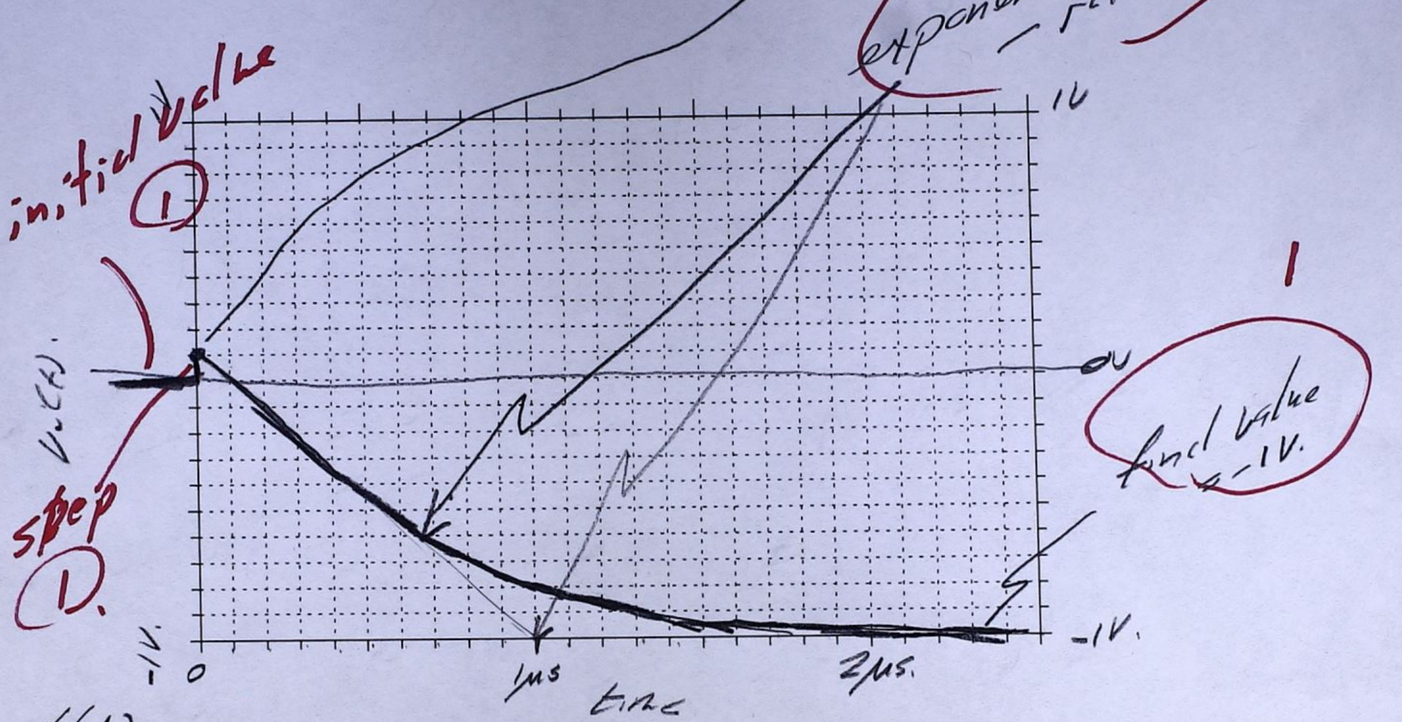
b

Part d, 8 points

Vin(t) is a 0.1 V amplitude step-function.

Find Vout(t)=_______________________________

Plot it below. Label axes, show initial and final values, show time constants.



$V_i(s) = 0.1V/s$

$$V_o(s) = \frac{0.1V}{s}(-g_m R_L)\frac{1-s\tau_z}{1+s\tau_p} = \frac{0.1V}{s}(-g_m R_L)\left[1 - \frac{s(\tau_z+\tau_p)}{1+s\tau_p}\right]$$

$$= \frac{0.1V}{s}(-g_m R_L) + (g_m R_L)\,0.1V \cdot \frac{\tau_z+\tau_p}{\tau_p}\;\frac{s\tau_p}{s+s\tau_p}$$

$$\underline{= -\frac{1V}{s} + 1.1V\frac{s\tau_p}{1+s\tau_p}} \rightarrow \underline{V_o(t) = -1V \cdot u(t) + 1.1V\,u(t)\,e^{-t/1\mu s.}}$$

2

1

b