## Final Exam, ECE 137A

## Wednesday March 18, 2015 7:30-10:30 PM

Name: $\qquad$
Closed Book Exam: Class Crib-Sheet and 3 pages ( 6 surfaces) of student notes permitted Do not open this exam until instructed to do so. Use any and all reasonable approximations (5\% accuracy), after stating \& justifying them.
Show your work:
Full credit will not be given for correct answers if supporting work is missing.
Good luck

| Time function | LaPlace Transform |
| :--- | :--- |
| $\delta(t)$ impulse | 1 |
| $U(t)$ unit step-function | $1 / s$ |
| $e^{-\alpha t} U(t)$ | $\frac{1}{s+\alpha}$ |
| $e^{-\alpha t} \cos \left(\omega_{d} t\right) U(t)$ | $\frac{s+\alpha}{(s+\alpha)^{2}+\omega_{d}^{2}}$ |
| $e^{-\alpha t} \sin \left(\omega_{d} t\right) U(t)$ | $\frac{\omega_{d}}{(s+\alpha)^{2}+\omega_{d}^{2}}$ |


| Part | Points <br> Received | Points <br> Possible | Part | Points <br> Received | Points <br> Possible |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :---: |
| 1a |  | 5 | 2c |  | 15 |  |
| 1b |  | 6 | 2d |  | 10 |  |
| 1c |  | 4 | 3 a |  | 7 |  |
| 1d |  | 10 | 3 b |  | 8 |  |
| 1e |  | 10 | 3 c |  | 7 |  |
| 2a |  | 10 | 3d |  | 8 |  |
| 2b |  | 10 |  |  |  |  |
| total |  | 100 |  |  |  |  |

## Problem 1, 35 points

This is an NOT an Op-Amp: Analyze under the assumption that the differential and common mode input voltages are at zero volts


All the transistors have the same (matched) $I_{S}$, have $\beta=250$, and $V_{A}=\infty$ Volts.
$V_{C E(s a t)}=0.5 \mathrm{~V} . V_{b e}$ is roughly 0.7 V , but use $V_{b e}=(k T / q) \ln \left(I_{E} / I_{S}\right)$ when necessary and appropriate. The supplies are +2.5 Volts and -2 Volts.

Q1ab,2ab,4a,5a,6a are to be biased at $100 \mu \mathrm{~A}$ collector current.
Q4B,5B,6B are to be biased at 2 mA collector current.
Q8ab are to be biased at $500 \mu \mathrm{~A}$ collector current.
Q9ab are to be biased at $125 \mu \mathrm{~A}$ collector current.
$R_{L}=250 \Omega \mathrm{R} 1 \mathrm{a}=\mathrm{R} 1 \mathrm{~b}=500$ Ohms

Part a, 5 points
DC bias---to simplify , assume $\beta=\infty$ for the DC analysis only.


On the circuit diagram above, label the DC voltages at ALL nodes, the DC currents through ALL resistors, and the DC collector currents of all transistors.

Part b, 6 points
DC bias:
Find the value of all resistors.
R3= $\qquad$ R4ba= $\qquad$ R6a= $\qquad$ Rref= $\qquad$ $\mathrm{R} 8 \mathrm{a}=$ $\qquad$ $\mathrm{R} 8 \mathrm{~b}=$ R9a= R9b= $\qquad$

## Part c, 4 points

Find the transconductance of the transistors below:
gm1a= $\qquad$ $\mathrm{gm} 1 \mathrm{~b}=$ gm5a $=\quad$ gm7a $=$
gm7b= $\qquad$ gm9a= $\qquad$ $\mathrm{gm} 9 \mathrm{~b}=$
$\qquad$

## Part d, 10 points.

Find the following, using the actual value of $\beta$, i.e. $\beta=250$

|  | Voltage Gain | Input impedance |
| :--- | :--- | :--- |
| Transistor combination <br> Q2a,2b,1a,1b |  |  |
| Q5a |  |  |
| Q7a or 7b |  |  |
| Q9a or 9b |  |  |
| Overall differential <br> Vout/Vin |  |  |

## Part e, 10 points

Maximum peak-peak output voltage (show all your work)

|  | magnitude and sign of <br> maximum output signal <br> swing due to cutoff | magnitude and sign of <br> maximum output signal <br> swing due to saturation |
| :--- | :--- | :--- |
| Transistor Q4a |  |  |
| Transistor Q5a |  |  |
| Transistor Q7a |  |  |
| Transistor Q7b |  |  |
| Transistor Q9a |  |  |
| Transistor Q9b |  |  |

Be warned: In some cases a limit is not relevant at all. Mark those answers "not relevant". But, give a 1-sentence statement below as to why it is not relevant. Q7ab and Qab form a push pull stage, so be careful about your answers here.

## Problem 2, 35 points

This is an Op-Amp---analyze the bias under the assumption that DC output voltage is zero volts, that the positive input $\mathrm{Vi}+$ is zero volts, and that we must determine the DC value of the negative input voltage (Vi-) necessary to obtain this.


The NMOSFETs and the PMOSFETS have a 0.20 V threshold, a 22 nm gate length, $300 \mathrm{~cm}^{2} / \mathrm{Vs}$ mobility, a $10^{7} \mathrm{~cm} / \mathrm{s}$ saturation drift velocity, and $1 / \lambda=3$ Volts. The gate oxide thickness is 1.0 nm and the dielectric constant is 3.8 . This gives
$\mu c_{o x} W_{g} / 2 L_{g}=15 \mathrm{~mA} / \mathrm{V}^{2} \cdot\left(W_{g} / 1 \mu \mathrm{~m}\right)$ and
$v_{s a t} c_{o x} W_{g}=3.36 \mathrm{~mA} / \mathrm{V} \cdot\left(W_{g} / 1 \mu \mathrm{~m}\right) \quad$ (both are a bit unrealistic for a real technology).
and $v_{s a t} L_{g} / \mu=0.113 \mathrm{~V}$
$V_{D D}=+0.75 \mathrm{~V},-V_{S S}=-0.75 \mathrm{~V}$,

Part a, 10 points
DC bias.
Approximation: ignore the term $\left(1+\lambda V_{D S}\right)$ in DC bias analysis.
Analyze the bias under the assumption that DC output voltage is zero volts, that the positive input Vi+ is zero volts, and that we must determine the DC value of the negative input voltage (Vi- ) necessary to obtain this.
Q1ab,2ab are to be biased at $25 \mu \mathrm{~A}$ drain current.
Q4ab,5ab,6ab,7ab are to be biased at $100 \mu \mathrm{~A}$ drain current
All transistors are to operate with $\left|V_{g s}\right|=0.25 \mathrm{~V}$.
Find the gate widths of all transistors.

Find:
Wg1a= $\qquad$ $\mathrm{Wg} 1 \mathrm{~b}=$ $\qquad$ Wg2a= $\qquad$ $\mathrm{Wg} 2 \mathrm{~b}=$ $\qquad$ $\mathrm{Wg} 3=$ $\qquad$
$\mathrm{Wg} 4 \mathrm{a}=$ $\qquad$ $\mathrm{Wg} 4 \mathrm{~b}=$ $\qquad$ Wg5a= $\qquad$ $\mathrm{Wg} 5 \mathrm{~b}=$ $\qquad$
Wg6a= $\qquad$ $\mathrm{Wg} 6 \mathrm{~b}=$ $\qquad$ Wg7a= $\qquad$ $\mathrm{Wg} 7 \mathrm{~b}=$ $\qquad$
Rref= $\qquad$

Part b, 10 points
DC bias


On the circuit diagram above, label the DC voltages at ALL nodes the drain currents of ALL transistors, and the gate widths of ALL transistors

## Part c, 15 points.

You will now compute the op-amp differential gain. You must consider the $\left(1+\lambda V_{D S}\right)$ term in the FET IV characteristics when you do this.

The capacitors C1-C4 are all zero Ohms AC impedance. (They would not be present in a real design; they are added here to simplify the exam).

Find the following

|  | Voltage Gain | Input impedance |
| :--- | :--- | :--- |
| Transistor combination <br> Q2a,2b,1a,1b |  |  |
| Q5a |  |  |
| Overall differential <br> Vout/Vin |  |  |

(Alternative----if you very skilled, you might be able to compute the combined gain of Q2a,2b, 1a, 1b and Q5a, all together, in a single step using Norton or Thevenin methods. If you do so, first, don't ask for hints on how to do this and, second, do please also calculate the input impedance of Q5a. )

## Part d, 10 points

Maximum peak-peak output voltage at the positive output Vo+ (show all your work)

|  | magnitude and sign of <br> maximum output signal <br> swing due to cutoff | magnitude and sign of <br> maximum output signal <br> swing due to: <br> knee voltage (saturation) |
| :--- | :--- | :--- |
| Transistor Q5a |  |  |
| Transistor Q6a |  |  |
| Transistor Q2a |  |  |
| Transistor Q2b |  |  |

Be warned: in some cases a limit is not relevant. Mark those answers "not relevant". But, give a 1 -sentence statement why below.

Problem 3, 30 points


## Part a, 7 points

Draw a small-signal equivalent circuit of the circuit.

## Part b, 8 points

gm $=10 \mathrm{mS} . \mathrm{L}=1 \mu \mathrm{H} . \mathrm{R}=1000$ Ohms
Find, by nodal analysis, a small-signal expression for Vout/Vin. Be sure to give the answer with ${ }^{* *}$ correct units** and in ratio-of-polynomials form, i.e.
$\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=K \cdot \frac{1+b_{1} s+b_{2} s^{2}+\ldots}{1+a_{1} s+a_{2} s^{2}+\ldots}$ or (as appropriate) $\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=K \cdot(s \tau)^{n} \cdot \frac{1+b_{1} s+b_{2} s^{2}+\ldots}{1+a_{1} s+a_{2} s^{2}+\ldots}$
Note that an expression like
$\frac{V_{\text {out }}(s)}{V_{\text {gen }}(s)}=\frac{1}{1+\left(3 \cdot 10^{-6}\right) s}$ is dimensionally wrong; $\frac{1}{1+\left(3 \cdot 10^{-6} \text { seconds }\right) s}$ is dimensionally correct

Vout(s)/Vin(s)=

## Part c, 7 points

Find any/all pole and zero frequencies of the transfer function, in Hz :
$\qquad$ , $\qquad$ , $\qquad$ ,

Draw a clean Bode Plot of Vout/Vin,
LABEL AXES, LABEL all relevant gains and pole or zero frequencies, Label Slopes


## Part d, 8 points

$\operatorname{Vin}(\mathrm{t})$ is a 0.1 V amplitude step-function.
Find $\operatorname{Vout}(\mathrm{t})=$ $\qquad$
Plot it below. Label axes, show initial and final values, show time constants.


