ECE137A, notes set 2: MOSFETs

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Goals of this note set:

Rough physical sense of FET operation

FET current-voltage characteristics.

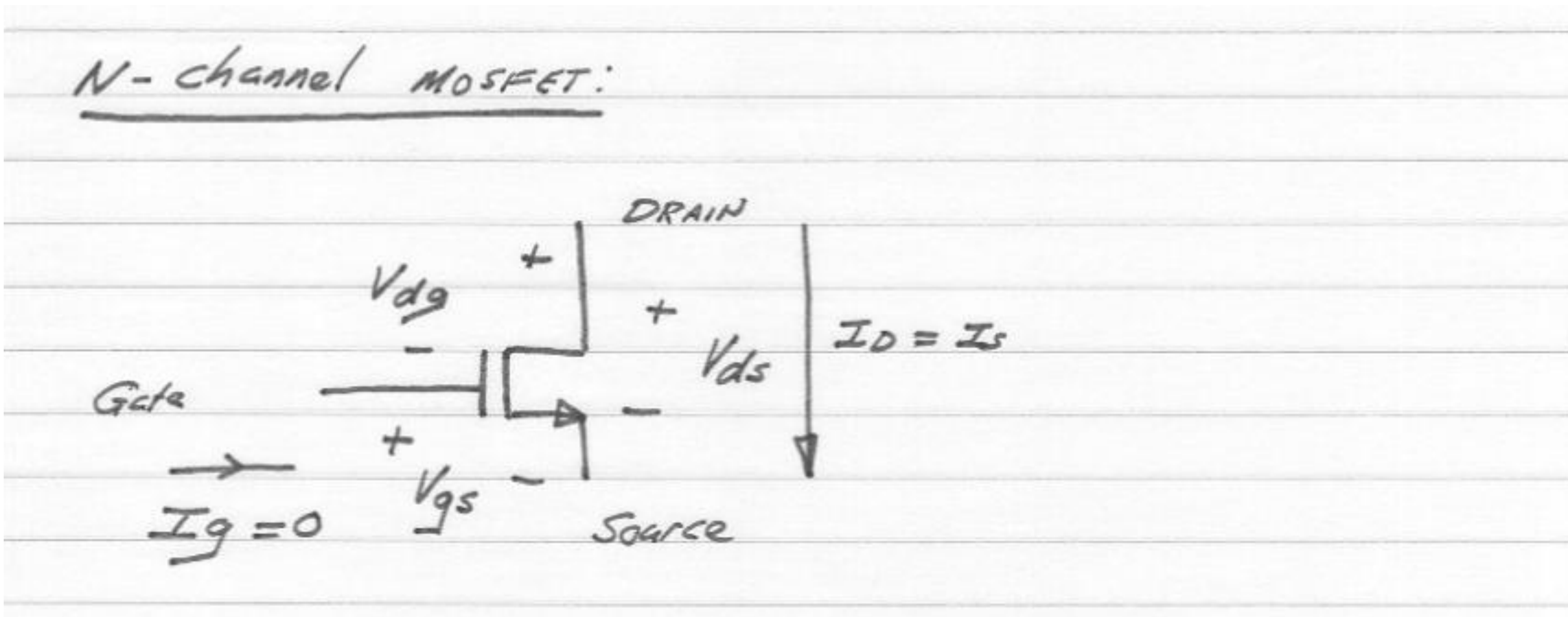
Rough mathematical models of MOSFETs

old-fashioned mobility-limited model.

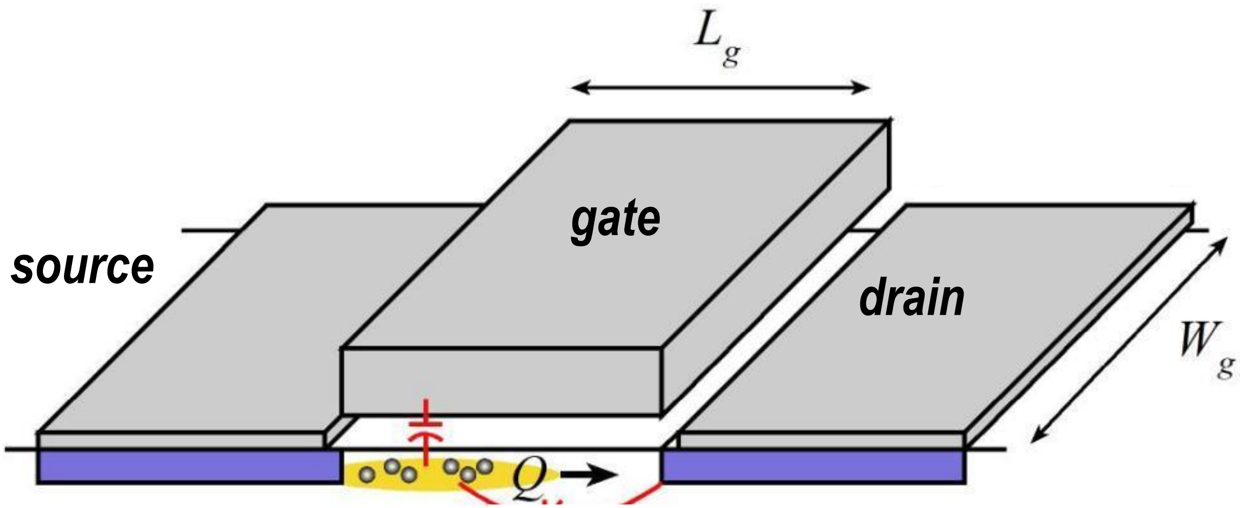
slightly less-old-fashioned velocity-limited model

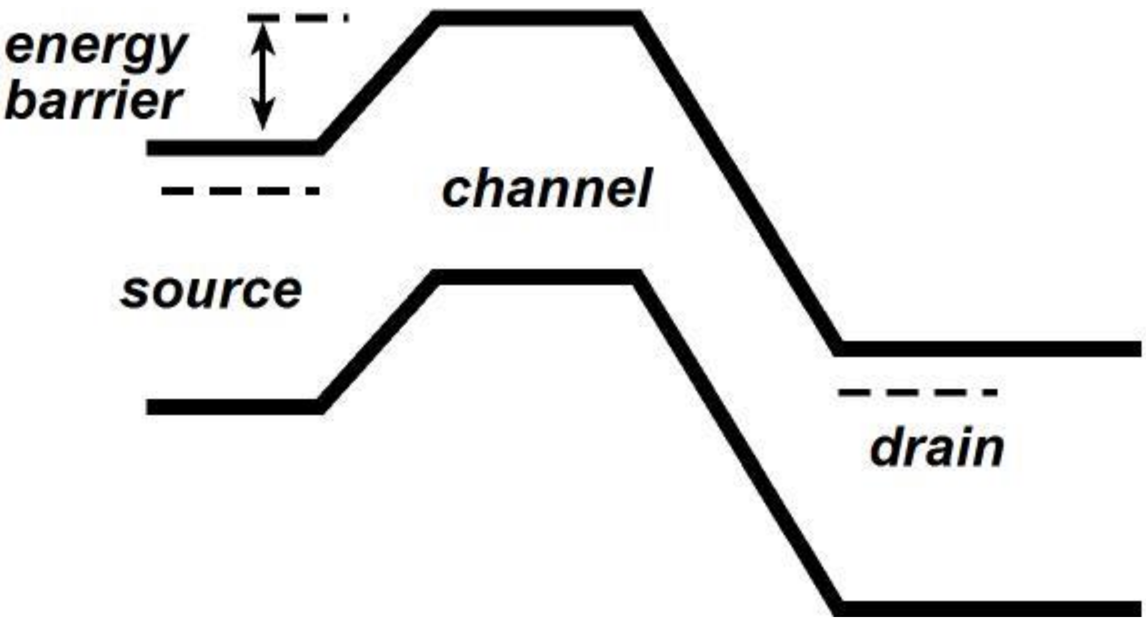
*We won't cover the ballistic injection velocity model

N-Channel MOSFET



Field-Effect Transistor Operation



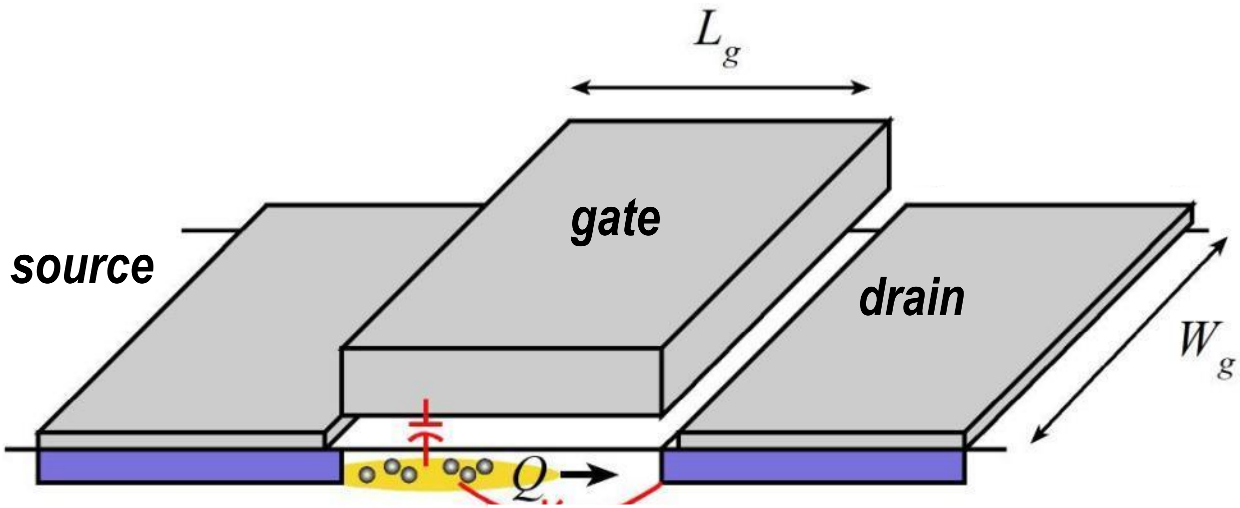


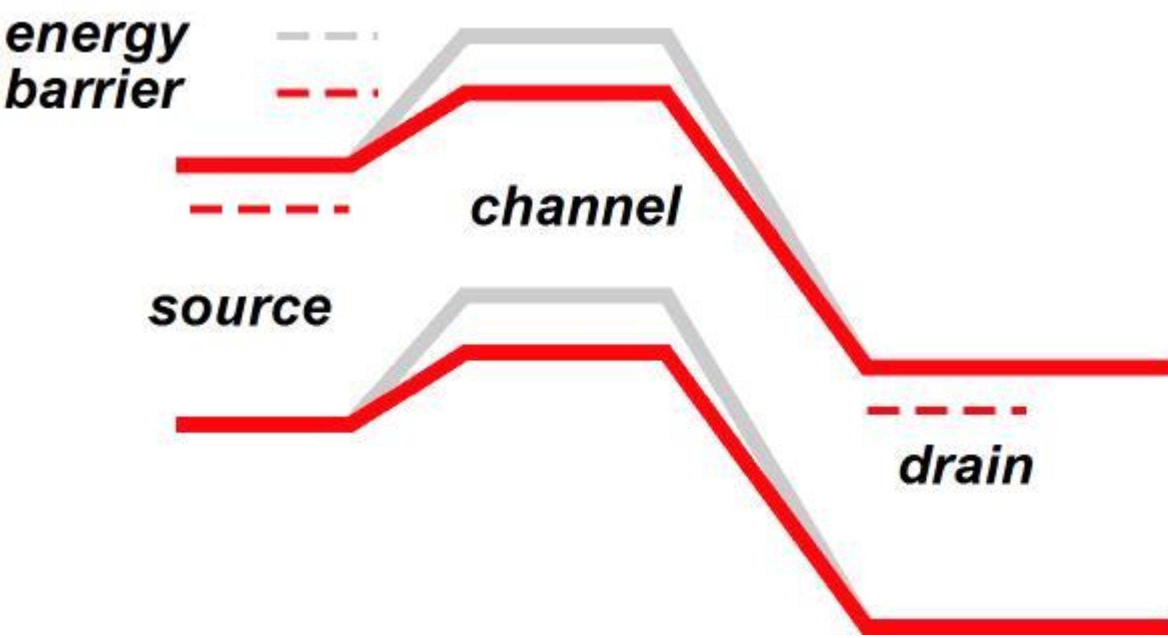
Positive Gate Voltage

→ reduced energy barrier

→ increased drain current

Field-Effect Transistor Operation





Positive Gate Voltage

→ reduced energy barrier

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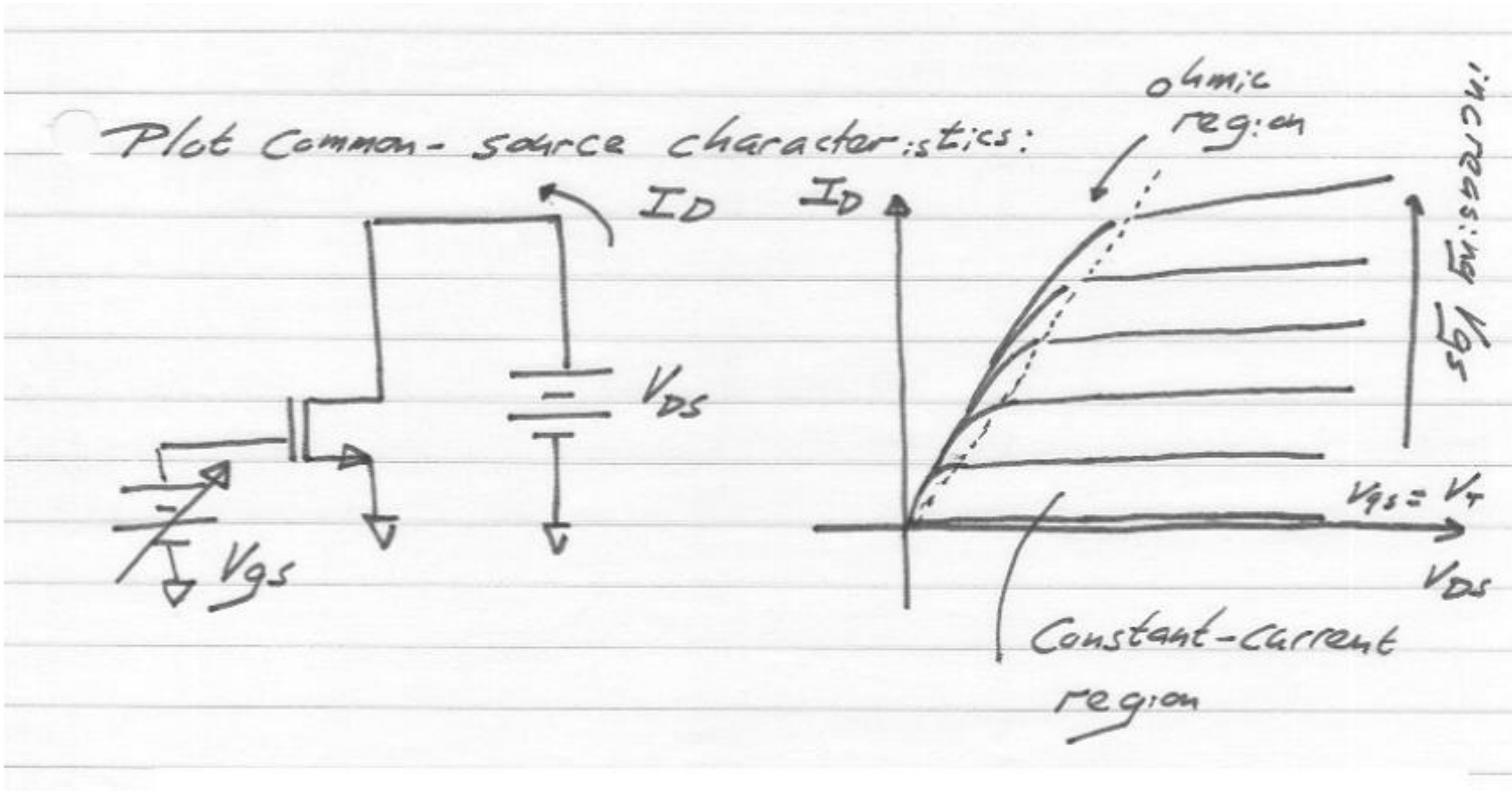
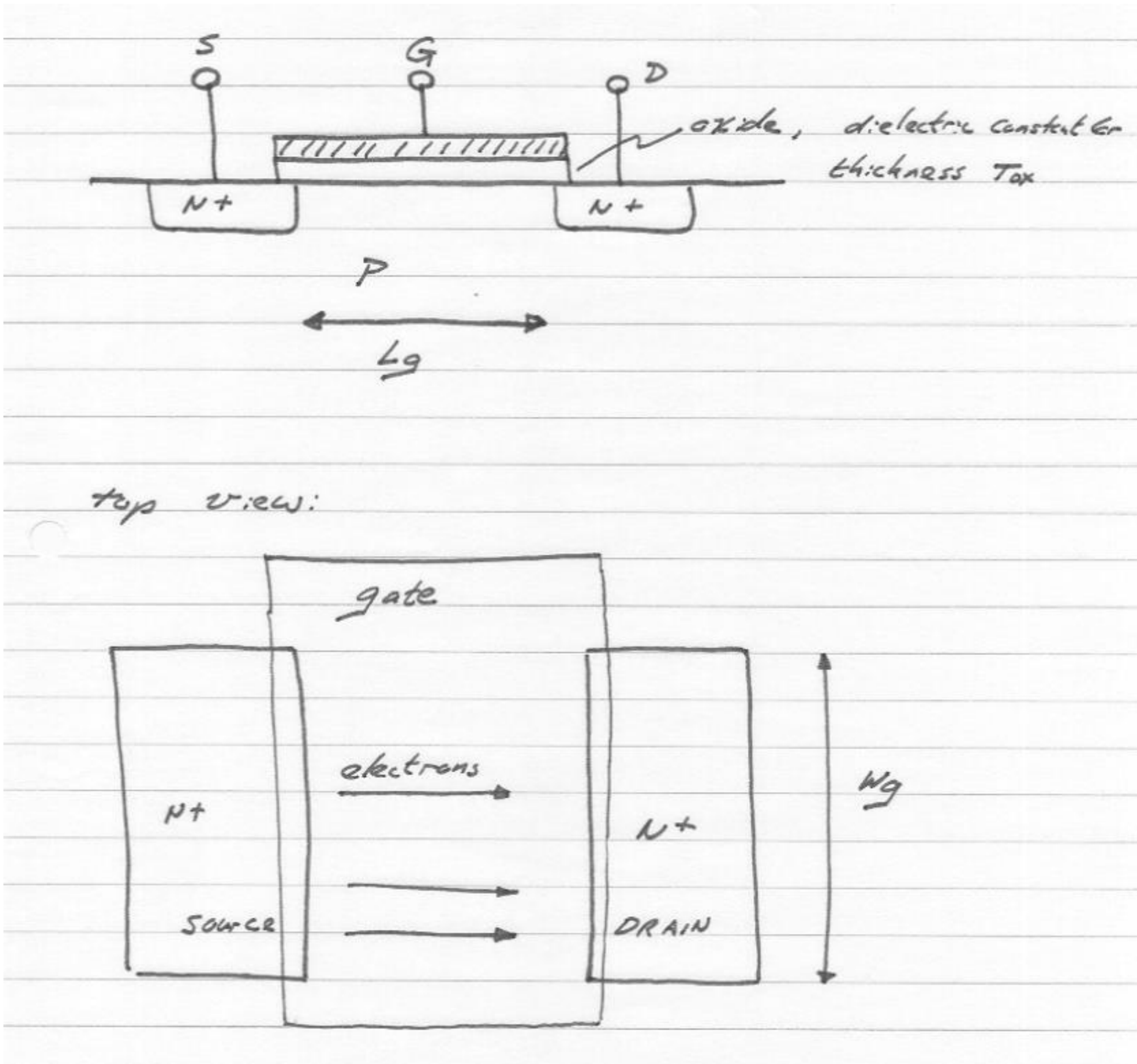
N-Channel MOSFET

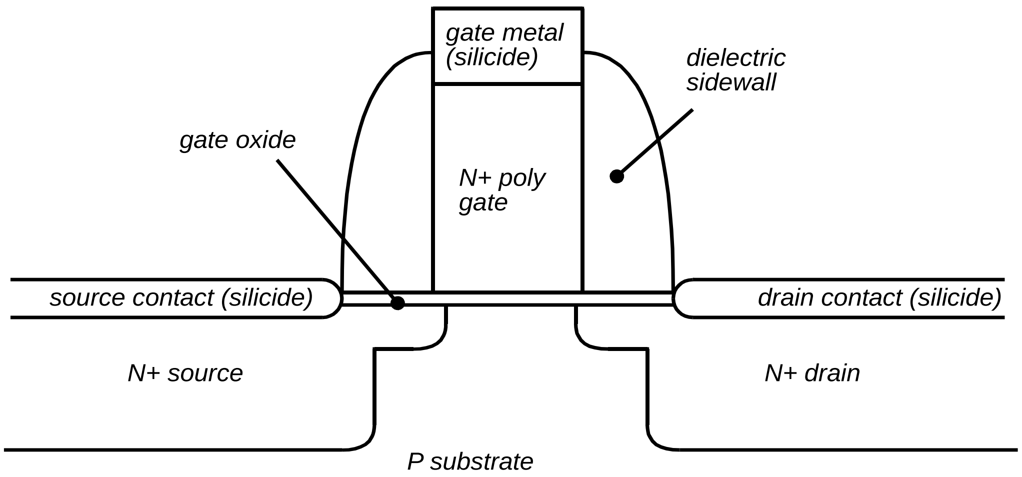


Physical Sketch

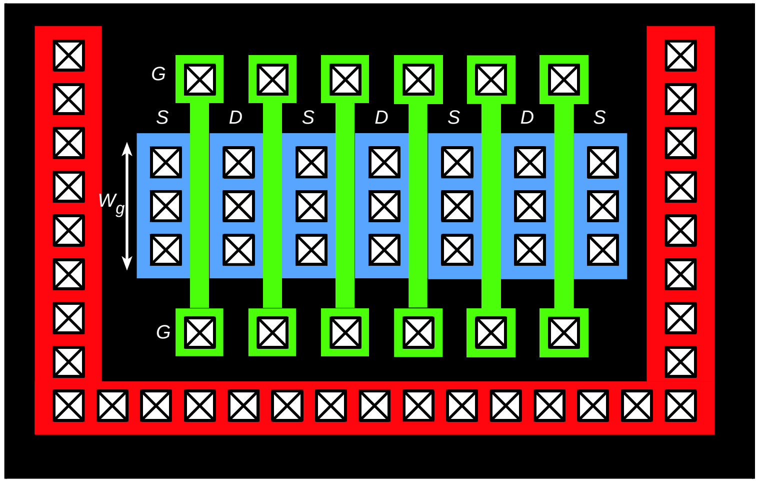


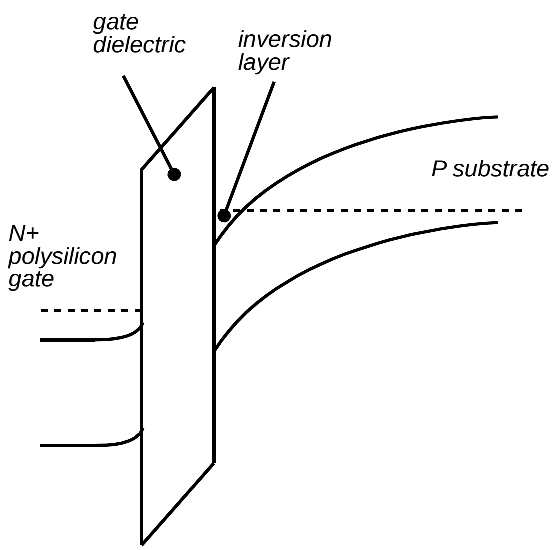
MOSFET Physical Structure: ~130nm node

Cross-Section



Layout

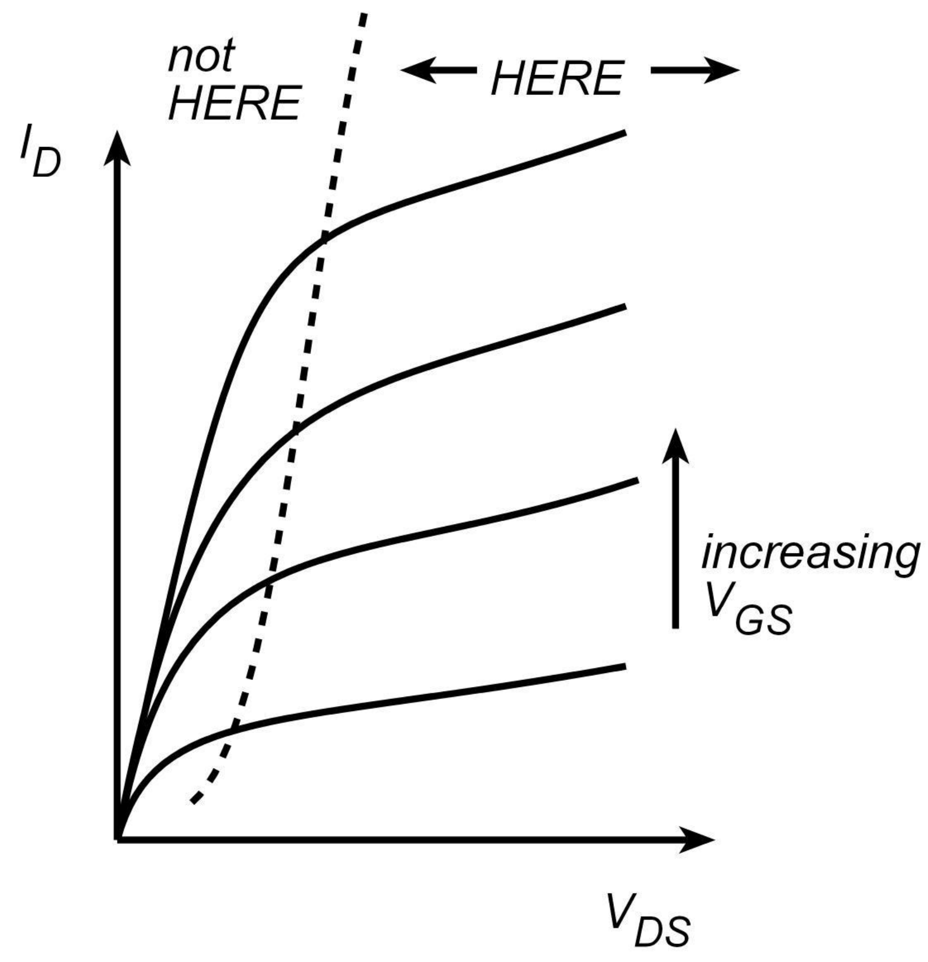




(6 FETs, each of gate width W_g , connected in parallel)

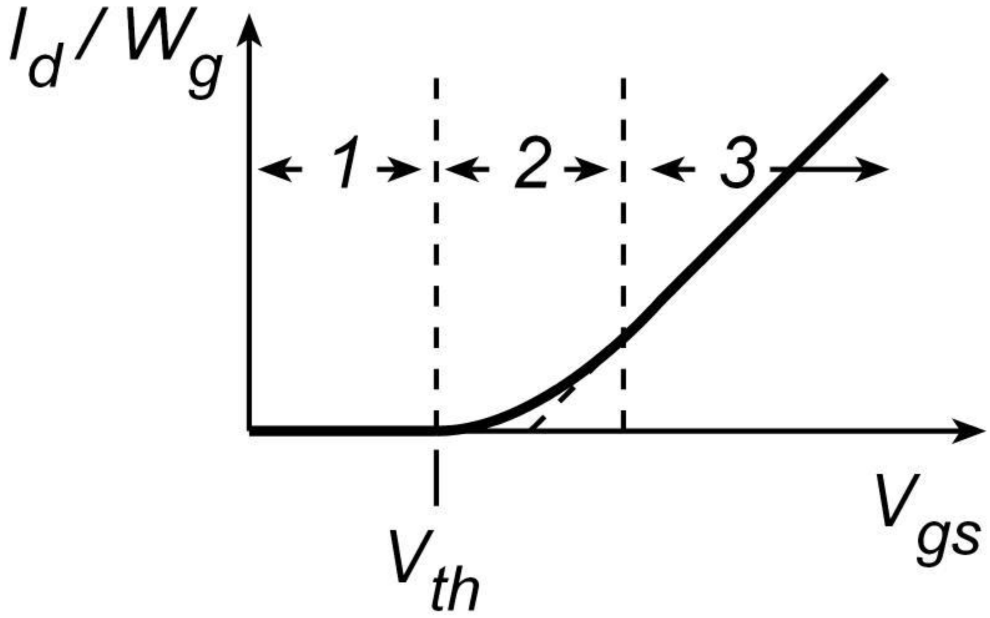
MOSFET I-V characteristics (approximate)

If we have drain voltages above the knee voltage :



MOSFET I-V characteristics (approximate)

Then we can plot I_D vs. V_{GS} :



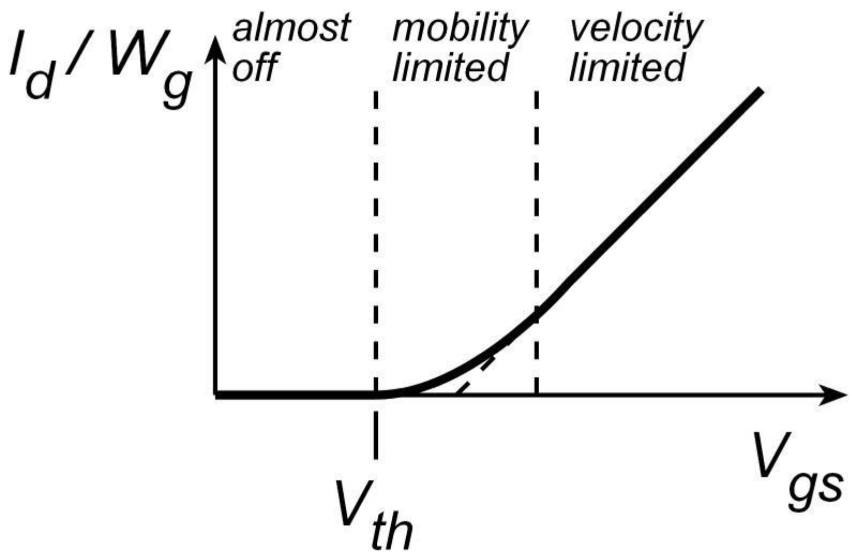
The **3 regions*** in the $I_D - V_{GS}$ curve :

1) Subthreshold = almost but not quite off

2) mobility-limited: $I_D \sim (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})$

3) velocity-limited: $I_D \sim (V_{gs} - V_{th} - \Delta V/2)^1 (1 + \lambda V_{DS})$

MOSFETs: Three Regions of Gate Voltage



$$\Delta V \approx v_{sat} L_g / \mu$$

When V_{gs} is less than threshold, transistor is (almost) off : "subthreshold"

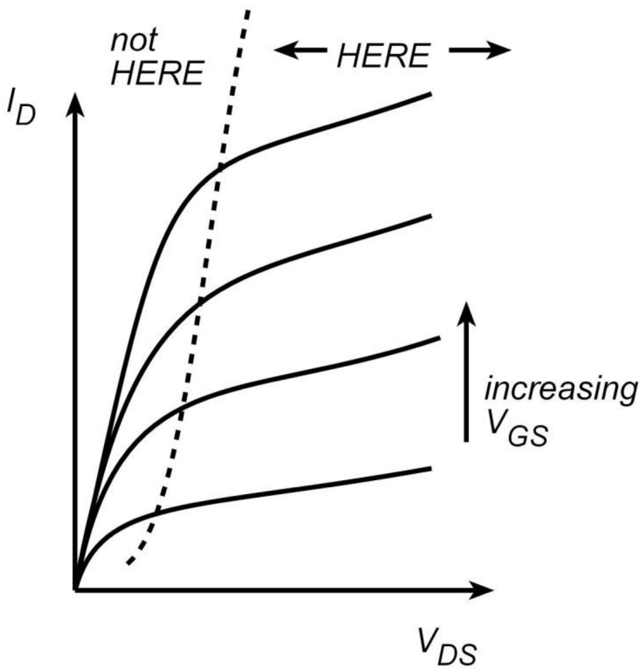
When V_{gs} is a little above threshold, current is mobility – limited

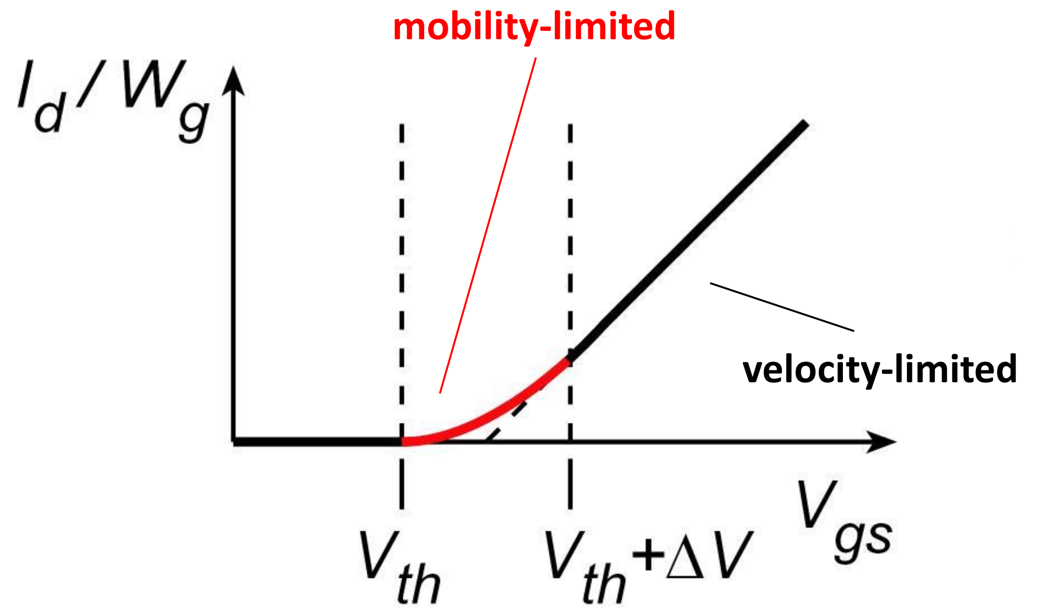
$$I_D \propto (V_{gs} - V_{th})^2$$

When V_{gs} is far above threshold, current is velocity – limited

$$I_D \propto (V_{gs} - V_{th} - \Delta V / 2)$$

MOSFET DC Characteristics: Mobility-Limited Case





mobility – limited current :

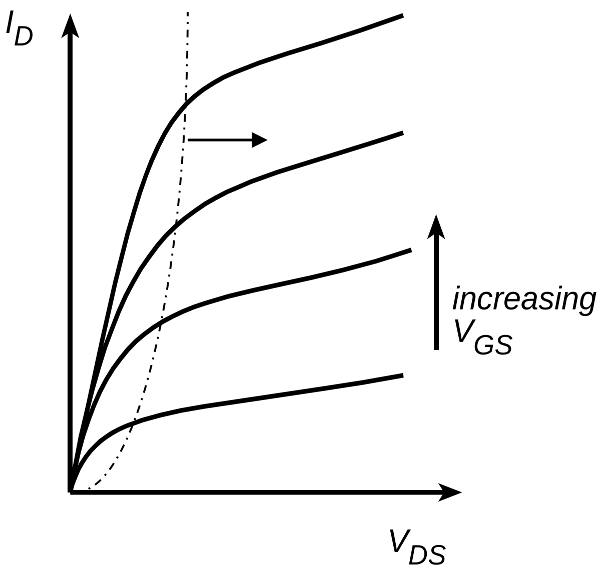
$$I_D \cong K_\mu (V_{gs} - V_{th})^2 (1 + \lambda V_{DS}); \text{ this is only approximate}$$

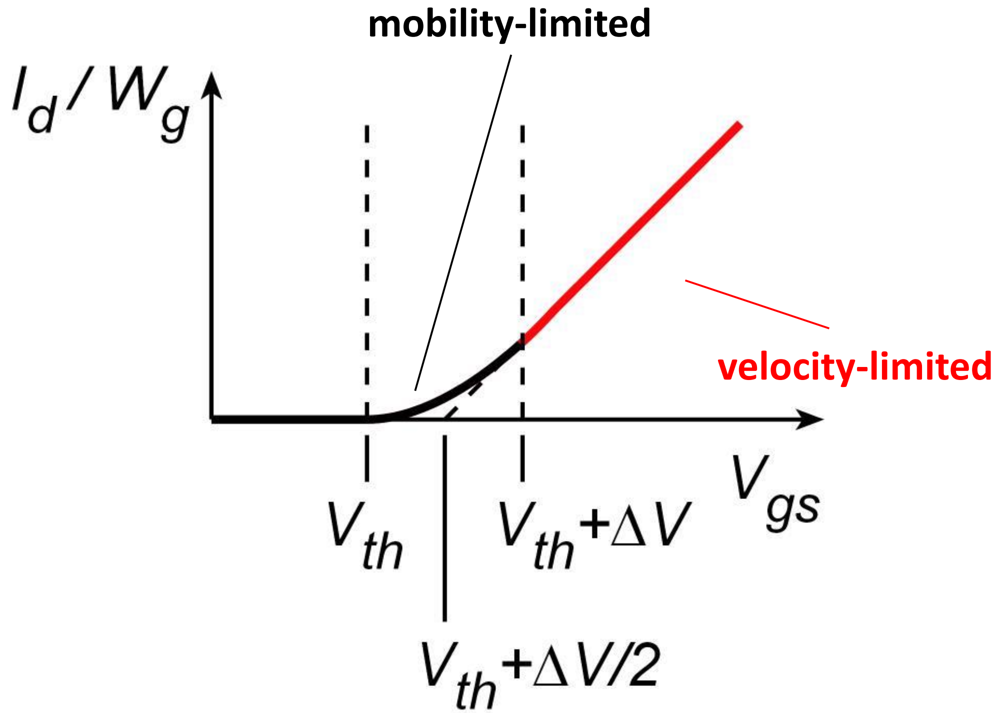
$$\text{for } V_{th} < V_{gs} < V_{th} + \Delta V \qquad \text{where } K_\mu \cong (\mu c_{gs} W_g / 2L_g)$$

$$\text{where } \Delta V \approx L_g v_{sat} / \mu$$

Applies for drain voltages larger than the knee voltage,

MOSFET DC Characteristics: Velocity-Limited Case





velocity – limited current

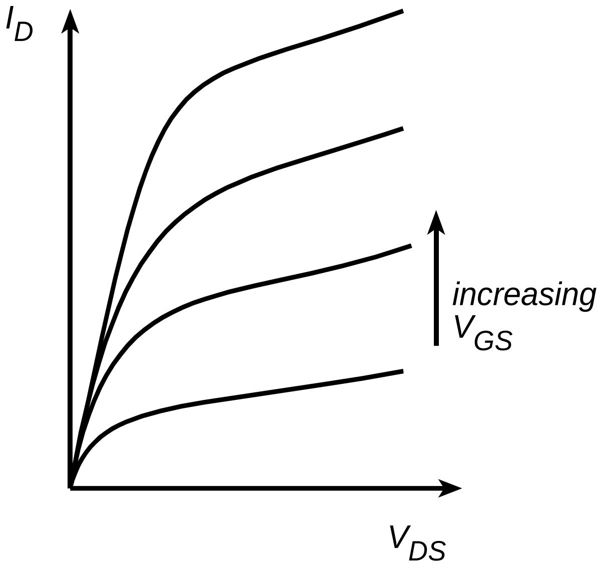
$$I_D \cong K_v (1+\lambda V_{DS})(V_{gs} - V_{th} - \Delta V/2)$$: this is only approximate

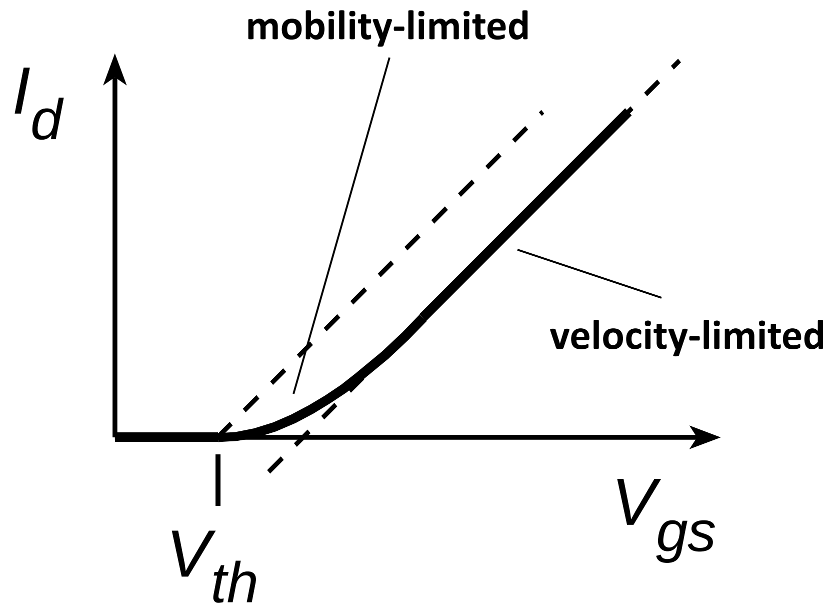
for $V_{gs} > V_{th} + \Delta V$
where $\Delta V \approx L_g v_{inj} / \mu$

Where $K_v \cong c_{gs} W_g v_{inj}$

Applies for drain voltages larger than the knee voltage,

DC Characteristics: *Somewhat* Better Approximation





Generalized Expression

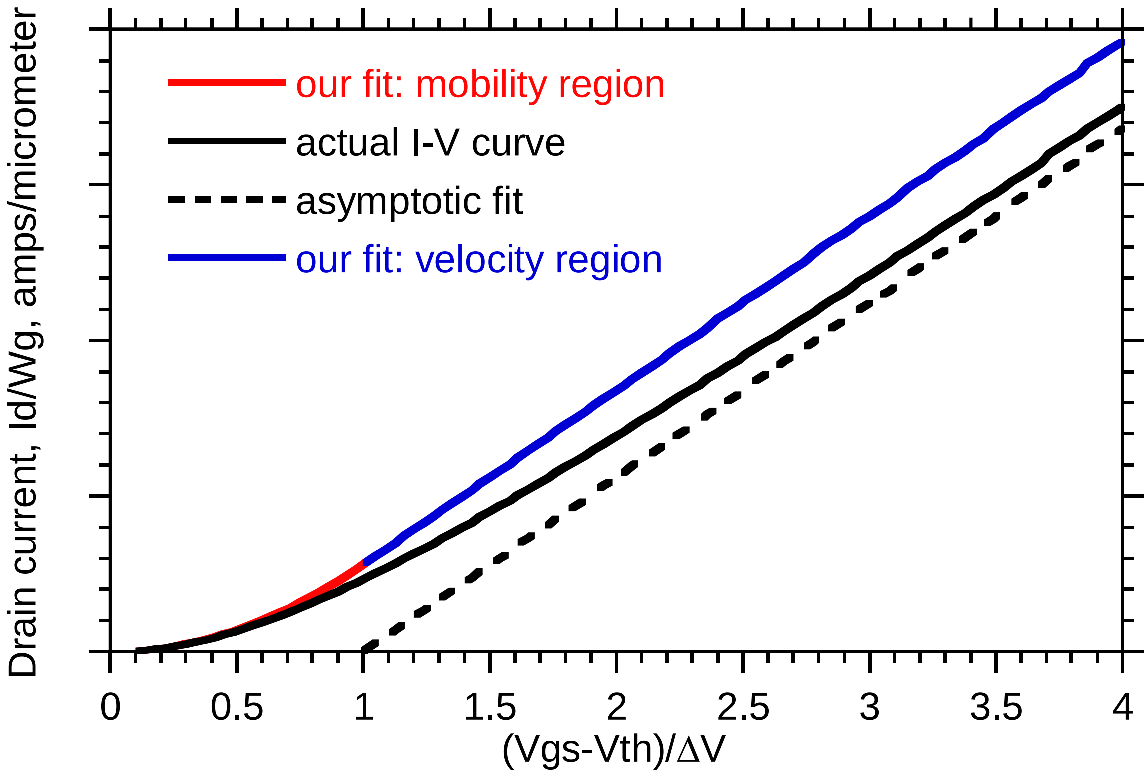
$$\left(\frac{I_D}{I_{D,2}}\right)^2+\left(\frac{I_D}{I_{D,1}}\right)=1$$

$$I_{D,1}=K_\mu(V_{gs}-V_{th})^2(1+\lambda V_{DS})$$

$$I_{D,2}=K_v(1+\lambda V_{DS})(V_{gs}-V_{th})$$

This expression is too complex for us to use in this class:
Instead use **as appropriate** either the velocity-limited or mobility limited expressions.

How well does our approximation work ?



Black: actual curve

Our fit: mobility region and velocity region

Observe: for very large $(V_{gs} - V_{th})$, a better fit would be $I_D \approx K_v(V_{gs} - V_{th} - \Delta V)$.

This is the dotted line.

No simple expression can fit perfectly at all V_{gs} !

Our simple (red/blue) model will suffice for this class.

Paramers and Typical #s (1)

v_{inj} = injection velocity from the source into the channel

$\sim 1.0 \cdot 10^7$ cm/s for N-MOSFETs

μ = carrier mobility at surface $\sim 150 - 200$ cm^2/(V-s) for N-MOSFET

In older technologies (~>35nm):

P-channel FETs have both v_{inj} and μ about half that of N-FETs

In newer technologies (~<35nm):

v_{inj} and μ are comparable for the PFET and NFET.

V_{th} threshold voltage ---usually 0.2-0.4 V for modern N-FETs

λ gives slope of output characteristics: $1/\lambda$ typically 3-20 V

Paramers and Typical #s (2)

$c_{gs} =$ gate-channel capacitance per unit area

$= \left(1/c_{ox} + 1/c_{semi}\right)^{-1}$ two capacitances in series

$c_{ox} = \varepsilon_r \varepsilon_0 / T_{ox}$ $(\varepsilon_r = 3.8$ for $SiO_2)$ This is the oxide capacitance

$T_{ox} =$ equivalent oxide thickness - about 1 nm = 10^{-9} m

$c_{semi} \approx 0.1$ F/m^2

This is the semiconductor surface capacitance, and arises because the semiconductor surface does not approximate that of a perfect conductor

(c_{semi} arises from two effects:

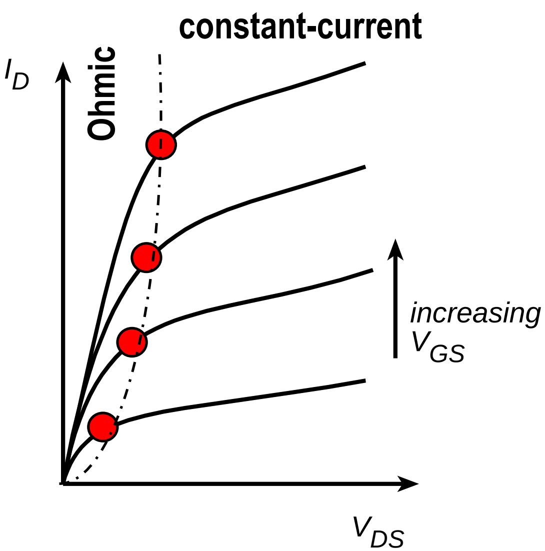
the finite # of available quantum states within the semiconductor,

and the nonzero depth of the wavefunction within the semiconductor)

Stern and Howard: Properties of Semiconductor Surface Inversion Layers in the Electric Quantum Limit, Phys. Rev. 163, 816 – 15 Nov. 1967, https://journals.aps.org/pr/abstract/10.1103/PhysRev.163.816 , doi = 10.1103/PhysRev.163.816

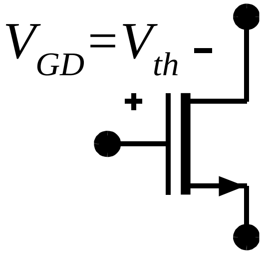
Knee Voltage: Mobility-Limited Case

The knee voltage defines the boundary between the Ohmic and constant - current regions

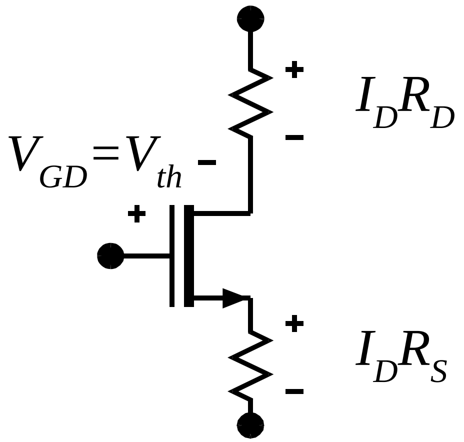


In the mobility - limited regime, the knee in curve occurs when

$$V_{dg} = V_{ds} - V_{gs} = -V_{th}$$

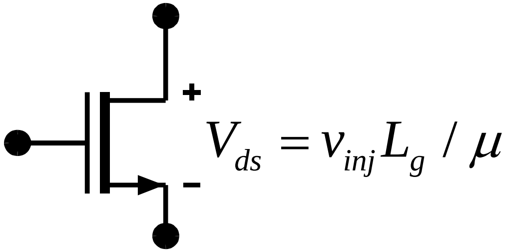


The Knee Voltage is further increased by voltage drops across the parasitic source & drain resistances.

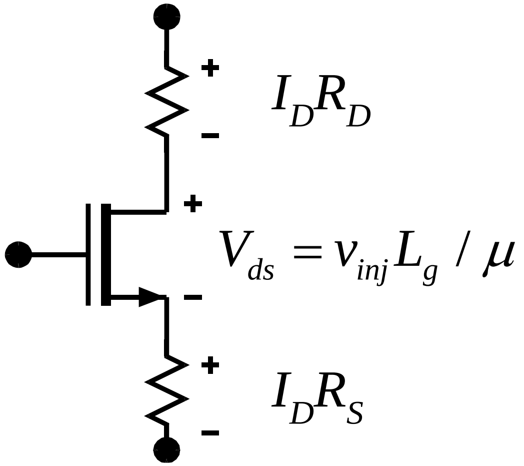


Knee Voltage: Velocity-Limited Case

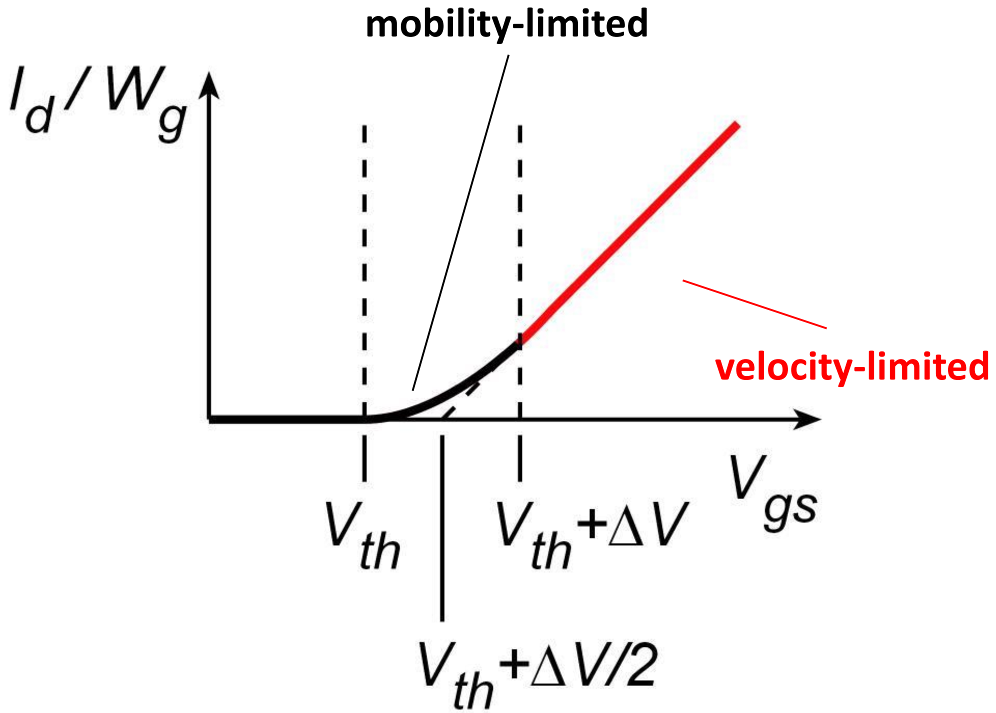
In the velocity-limited regime, the knee in curve occurs when $V_{ds} = v_{inj} L_g / \mu$



Again, the Knee Voltage is further increased by voltage drops across the parasitic source & drain resistances.



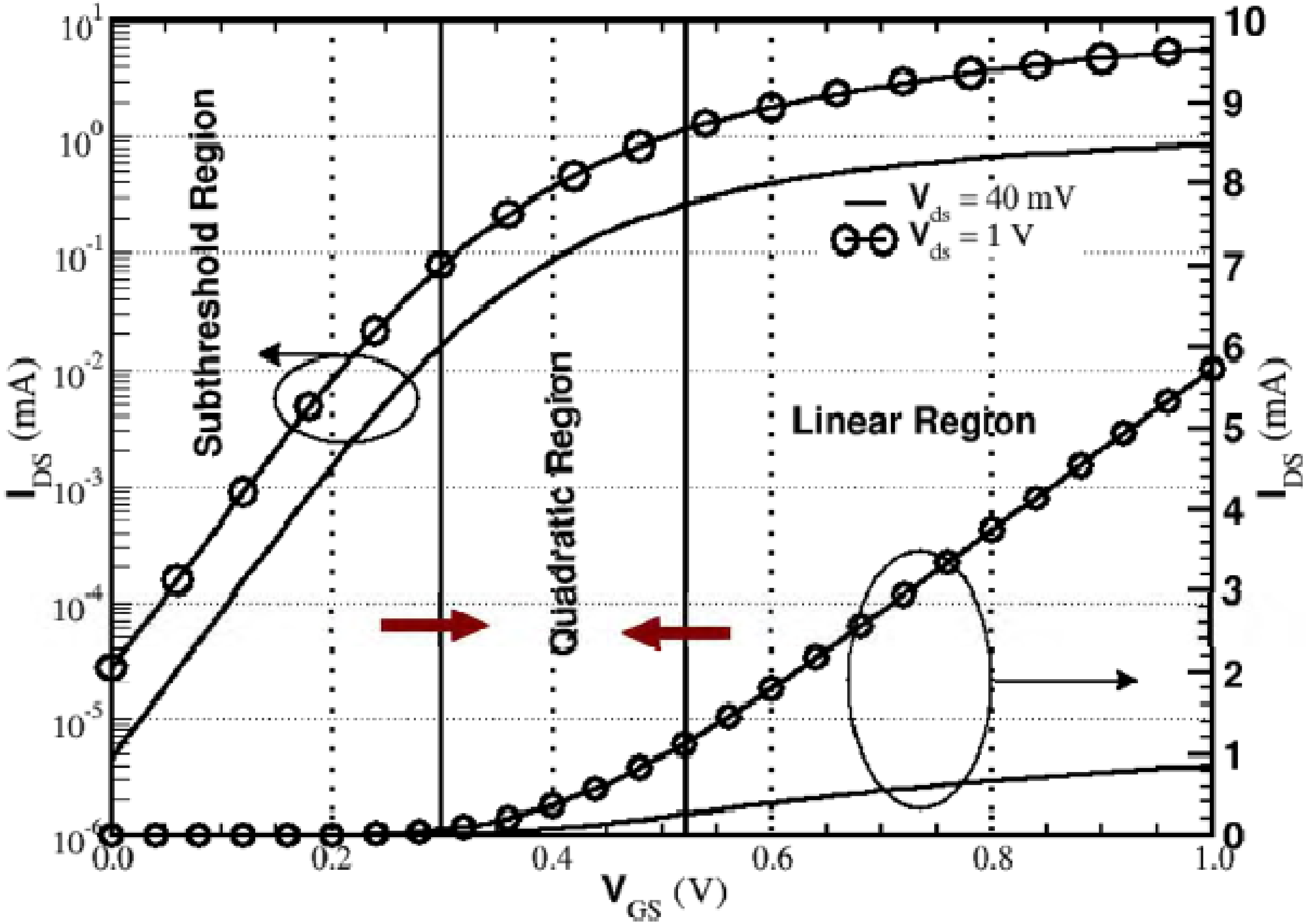
Which Model to use When ?



If $V_{gs} - V_{th} < \Delta V$ where $\Delta V \cong v_{inj} L_g / \mu$, use the mobility-limited model

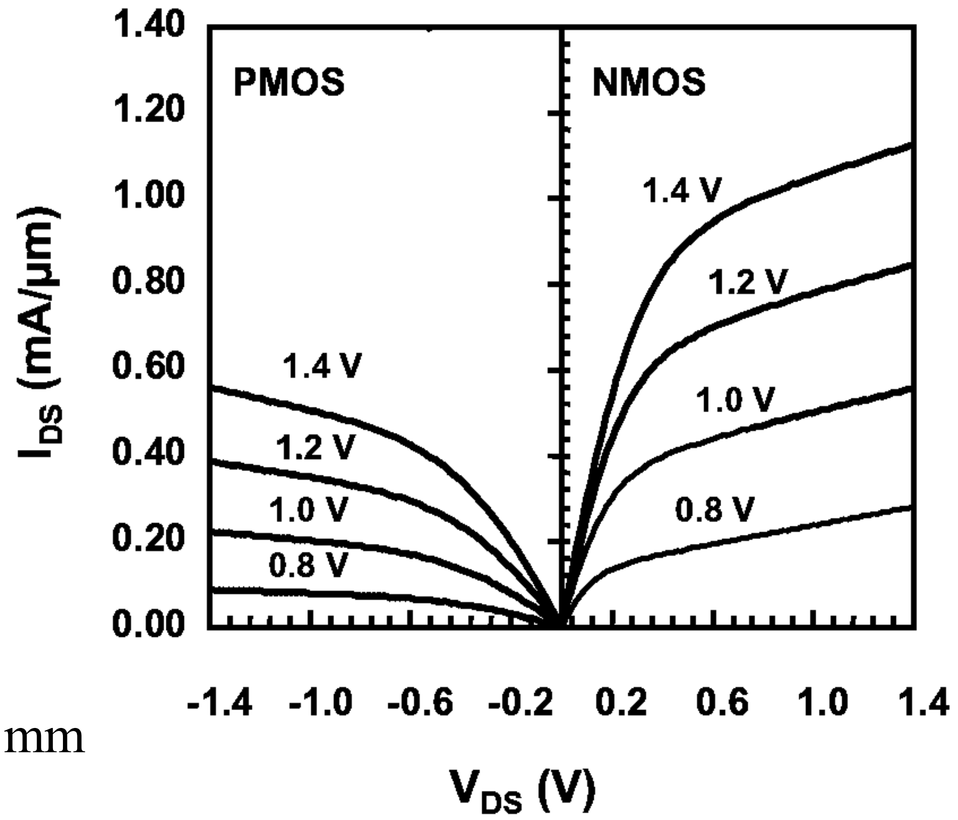
If $V_{gs} - V_{th} > \Delta V$, use the velocity limited model

Linear vs. Square-Law Characteristics: 90 nm



Sorin Voinigescu, CSICS RF & High Speed CMOS, Nov. 12, 2006

90 nm MOSFET DC Characteristics



N-channel

$$g_m / W_g = c_{gs} v_{inj} = 1.4 \text{ mS/µm} = 1.4 \text{ S/mm}$$

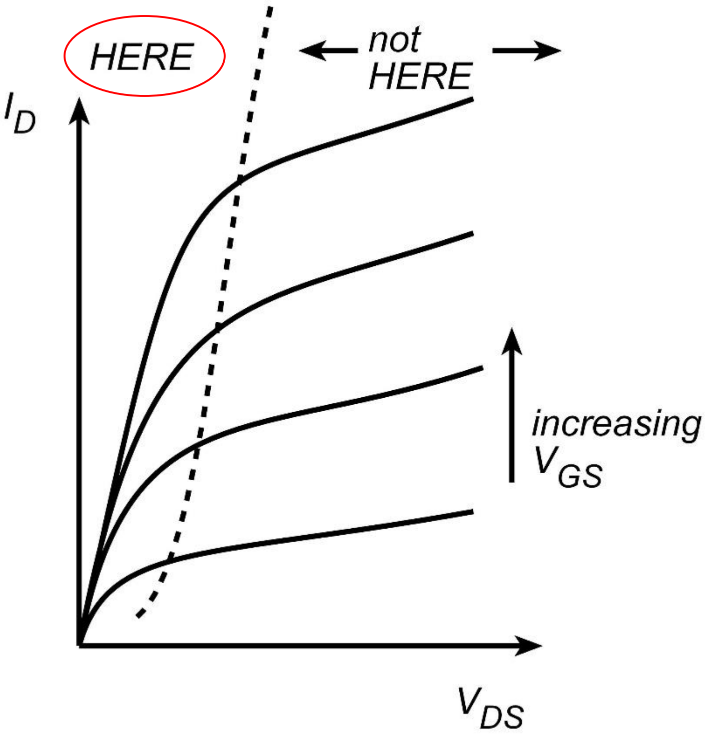
$1/\lambda \sim 3\text{V}$

P-channel

$$g_m / W_g = c_{gs} v_{inj} = 0.7 \text{ mS/µm} = 0.7 \text{ S/mm}$$

$1/\lambda \sim 3\text{V}$

DC charactistics in the resistive region



$$I_D \cong 2K_\mu\left((V_{gs}-V_{th})\cdot V_{DS}-V_{DS}^2/2\right)(1+\lambda V_{DS})$$

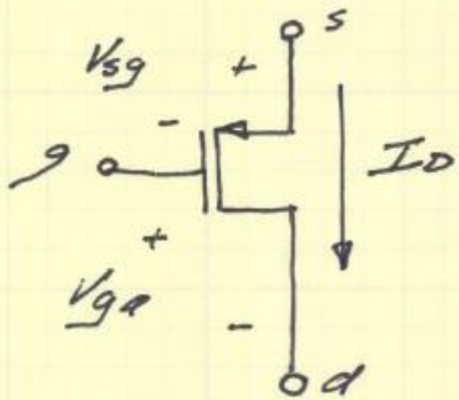
1) this is only approximate

2) this is only for *mobility-limited* operation

Unfortunately, I don't have a derivation in the velocity limit

P-Channel MOSFET

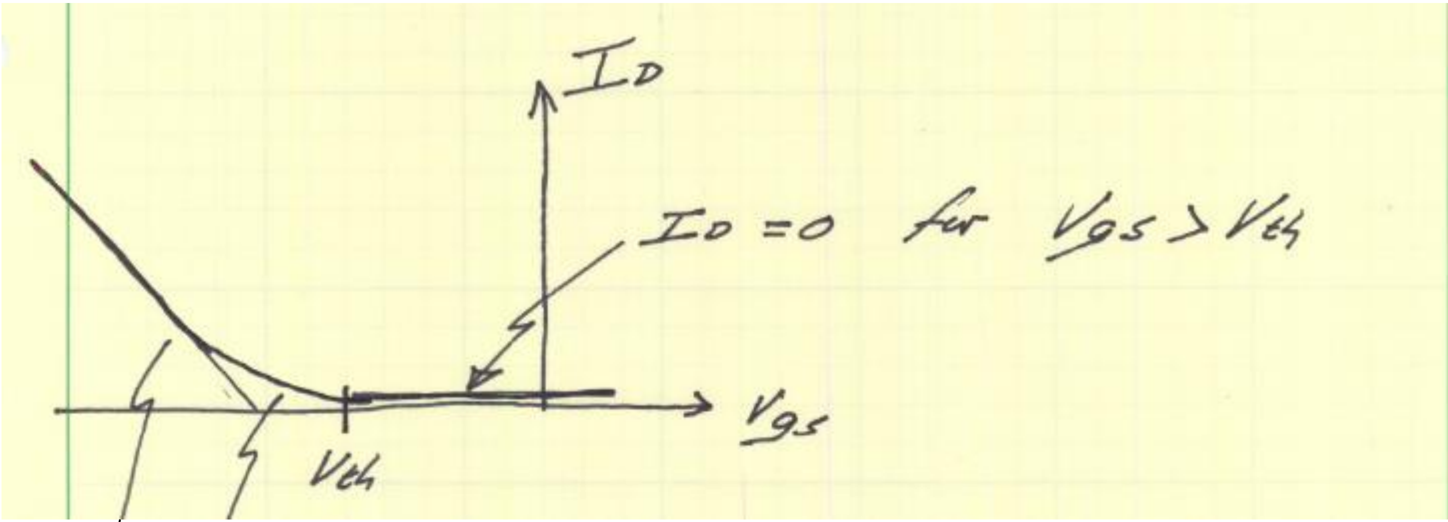
p-channel MOSFET



To turn the device <u>on</u>

the gate must be <u>more</u> <u>negative</u> than the source, by an amount exceeding the threshold voltage V_{th}

P-Channel MOSFET



$$I_{D,\mu} = K_\mu (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})$$

$$I_{D,v} = K_v (1 + \lambda V_{DS})(V_{gs} - V_{th} - \Delta V / 2)$$

$$\Delta V \cong v_{sat} L_g / \mu$$

P-Channel MOSFET

The device is in the constant current region if V_{ds} above the knee voltage.

Example: constant mobility model:

knee @ $V_{Dg} < -V_{th}$

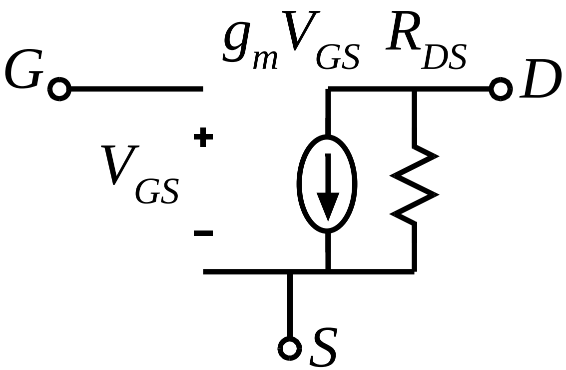
example: suppose the threshold voltage is -1V.

Then the P-MOSFET is in the constant-current region if the drain is <u>at</u> <u>most</u> 1V more positive than the gate.

FET Small-Signal Model: Mobility-Limited

Drain Current

$$I_D = K_\mu (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})$$



Transconductance

$$g_m \equiv \frac{\partial I_D}{\partial V_{GS}} = 2K_\mu (V_{gs} - V_{th})(1 + \lambda V_{DS}) \approx 2K_\mu (V_{gs} - V_{th})$$

Output Conductance

$$G_{ds} = \frac{1}{R_{ds}} \equiv \frac{\partial I_D}{\partial V_{DS}} = \frac{\lambda I_D}{1 + \lambda V_{DS}}$$

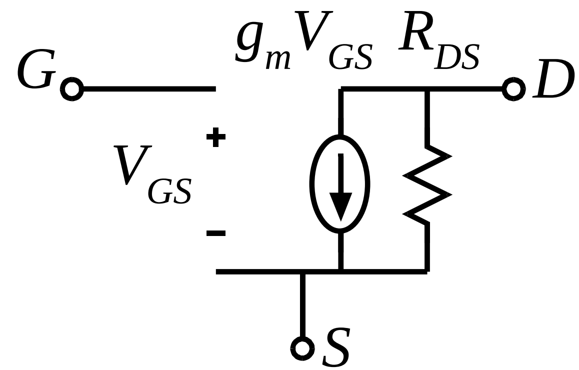
$\cong \lambda I_D$ to within the accuracy of the models we are using

Note that R_{ds} varies roughly as $1/I_D$.

FET transconductance: Mobility-Limited

Drain Current

$$I_D = K_\mu (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})$$



Transconductance

$$g_m \equiv \frac{\partial I_D}{\partial V_{GS}} = 2K_\mu (V_{gs} - V_{th})(1 + \lambda V_{DS})$$

$$\frac{g_m}{I_D} = \frac{2K_\mu (V_{gs} - V_{th})(1 + \lambda V_{DS})}{K_\mu (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})} = \frac{2}{(V_{gs} - V_{th})}$$

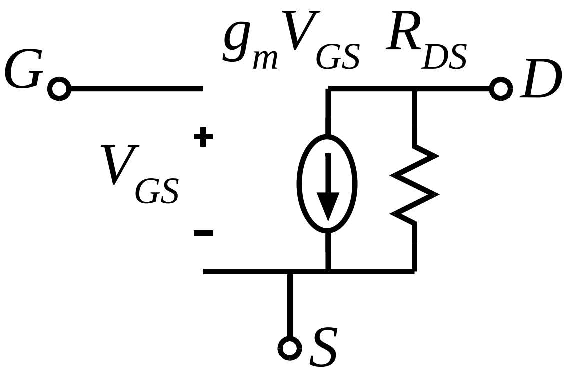
$\rightarrow g_m = 2I_D / (V_{gs} - V_{th})$, but only in mobility-limited case

** and only if $(V_{gs} - V_{th}) > 2V_T = 2kT/q$ **

If $(V_{gs} - V_{th}) \leq 2V_T = 2kT/q$, then the fet is in subthreshold mode and

$g_m \cong I_D / nV_T$, where $n \geq 1$ is some parameter characteristic of the device

FET Small-Signal Model: Velocity-Limited



Drain Current

$$I_D = K_v(1+\lambda V_{DS})(V_{gs} - V_{th} - \Delta V / 2)$$

Transconductance

$$g_m \equiv \frac{\partial I_D}{\partial V_{GS}} = K_v(1+\lambda V_{DS}) \approx K_v$$

Output Conductance

$$G_{ds} = \frac{1}{R_{ds}} \equiv \frac{\partial I_D}{\partial V_{DS}} = \frac{\lambda I_D}{1+\lambda V_{DS}}$$

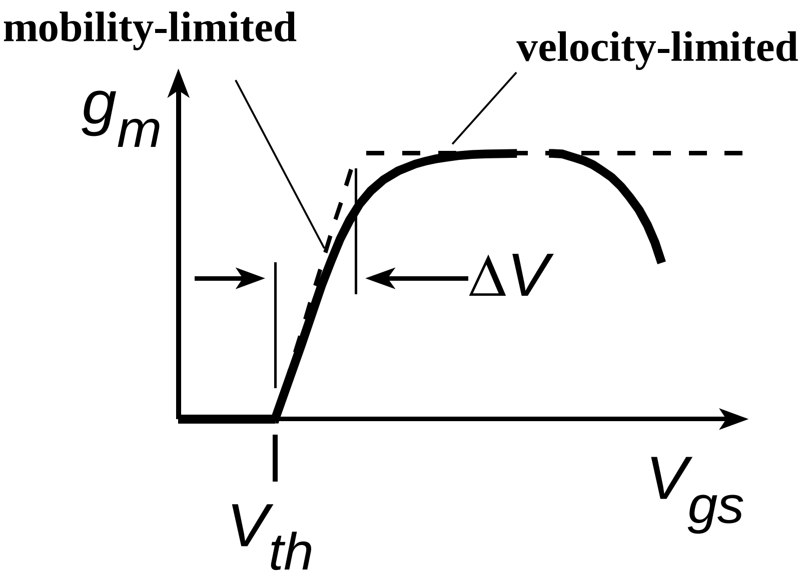
$$\cong \lambda I_D$$

Transconductance vs Vgs

mobility – limited

$$I_{D,\mu} = K_\mu (V_{gs} - V_{th})^2 (1 + \lambda V_{DS})$$

$$\rightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = 2K_\mu (V_{gs} - V_{th})(1 + \lambda V_{DS})$$

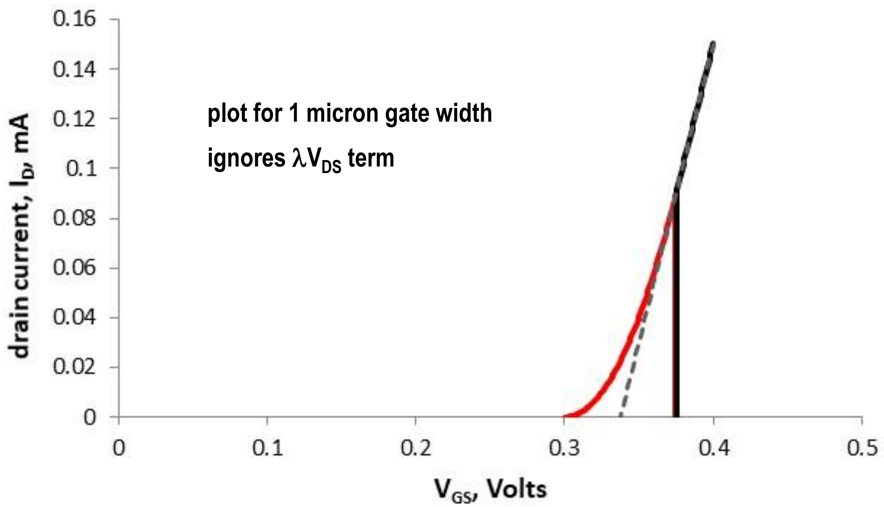


velocity – limited

$$I_D = K_v (V_{gs} - V_{th} - \Delta V / 2)(1 + \lambda V_{DS})$$

$$\rightarrow g_m = \frac{\partial I_D}{\partial V_{GS}} = K_v (1 + \lambda V_{DS})$$

Example #1: NMOS @ 15nm gate length



Assume the following

$L_g = 15\text{nm}$

$c_{gs} = 2.4 \cdot 10^{-2}\,\text{F/m}^2\ (T_{ox} = 1\text{nm},\ \varepsilon_{r,ox} = 3.8,\ c_{semi} = 8.6 \cdot 10^{-2}\,\text{F/m}^2)$

$\mu = 200\ \text{cm}^2/\text{V-s}$

$V_{th} = 0.3\ \text{V}$

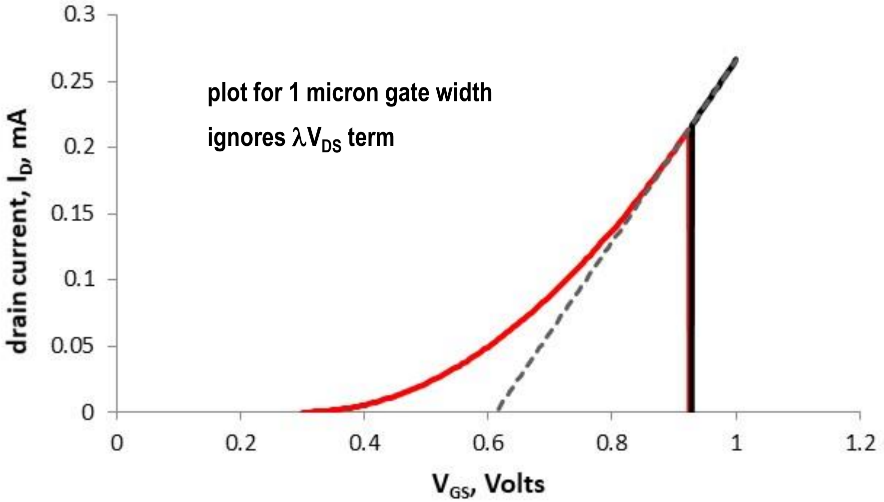
Then:

$K_\mu = \mu c_{gs} W_g / 2L_g = 16\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$

$K_v = c_{gs} v_{inj} W_g = 2.40\text{mA/V} \cdot (W_g / 1\mu\text{m})$

$\Delta V = v_{inj} L_g / \mu = 75\text{mV}$: Note how small is the mobility-limited region

Example #2: 250 nm NMOS:



Assume the following

$$L_g = 250\text{nm}$$

$$c_{gs} = 6.9\cdot 10^{-3}\,\text{F/m}^2\ \ (T_{ox} = 4.9\text{nm},\ \varepsilon_{r,ox} = 3.8,\ c_{semi} = 8.6\cdot 10^{-2}\,\text{F/m}^2)$$

$$\mu = 400\ \text{cm}^2/\text{V-s}$$

$$V_{th} = 0.3\text{V}$$

Then:

$$K_\mu = \mu c_{gs} W_g / 2L_g = 0.55\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$$

$$K_v = c_{gs} v_{inj} W_g = 0.69\text{mA/V} \cdot (W_g / 1\mu\text{m})$$

$\Delta V = v_{inj} L_g / \mu = 0.625\text{V}$ Note how *large* is the mobility-limited region

Example #3: for easy hand calculations

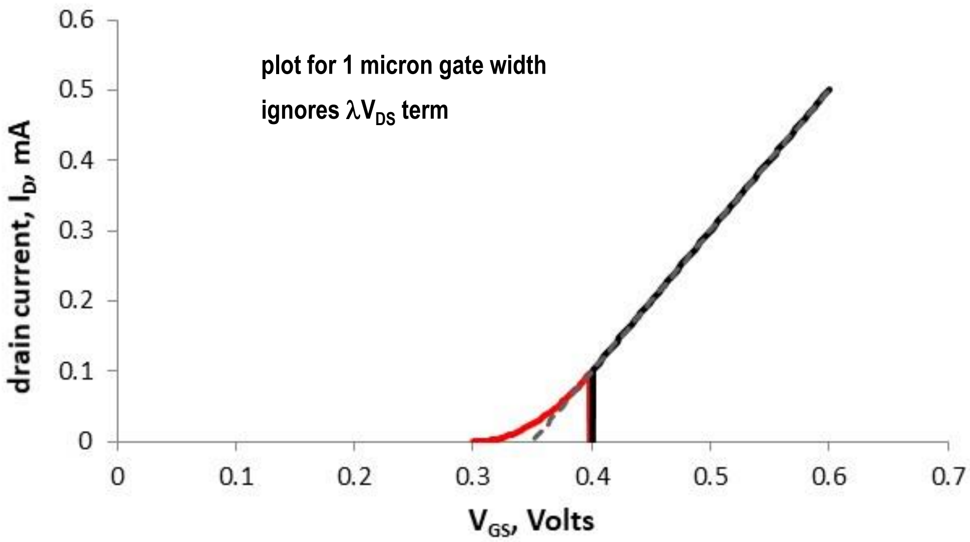
For easy hand calculations with examples in the notes, we will often use:

$$K_\mu = 10\text{mA/V}^2 \cdot (W_g / 1\mu\text{m})$$

$$K_v = 2\text{mA/V} \cdot (W_g / 1\mu\text{m})$$

$$\Delta V = 0.1\text{V}$$

$$V_{th} = 0.3\text{V}$$



I suppose this might be roughly the characteristics of a MOSFET with ~30nm physical gate length.

Caution regarding examples 1,2 and 3

To keep analysis simple, we have ignored the effect of R_S and R_D.

We therefore considerably over-estimated the MOSFET drain current at a given V_{gs}.

In the interest of simplicity, we will accept this limitation in this class.

A proper analysis would need to include the effect of R_S and R_D,
by treating R_S and R_D as separate external resistances,
or by adjusting the FET model parmeters to fit the overall DC charateristics.

MOSFET model: comments

1) MOS models in most undergraduate texts ignore injection velocity limits, yet this is a huge effect in modern MOSFETs.

2) Given (1), there is no consensus on how to teach velocity limited operatoin in undergraduate classes.

3) The 137ab method, here, is my attempt at a reasonably accurate yet simple model.

4) The more accurate expression given here is derived by assuming an exit velocity v_{inj} at the drain end of the channel.

5) Even the more accurate expression is only very approximate for highly scaled MOSFETs: for detailed design, we use foundry CAD models.

6) See publications by M. Lundstrom and D. Antoniadis for good derivations of modern FET I-V characteristics.